

Tuesday 18 June 2013 – Morning

A2 GCE MATHEMATICS (MEI)

4753/01 Methods for Advanced Mathematics (C3)

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:

- Printed Answer Book 4753/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:

- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets [] at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.



Section A (36 marks)

- 1 Fig. 1 shows the graphs of $y = |x|$ and $y = a|x + b|$, where a and b are constants. The intercepts of $y = a|x + b|$ with the x - and y -axes are $(-1, 0)$ and $(0, \frac{1}{2})$ respectively.

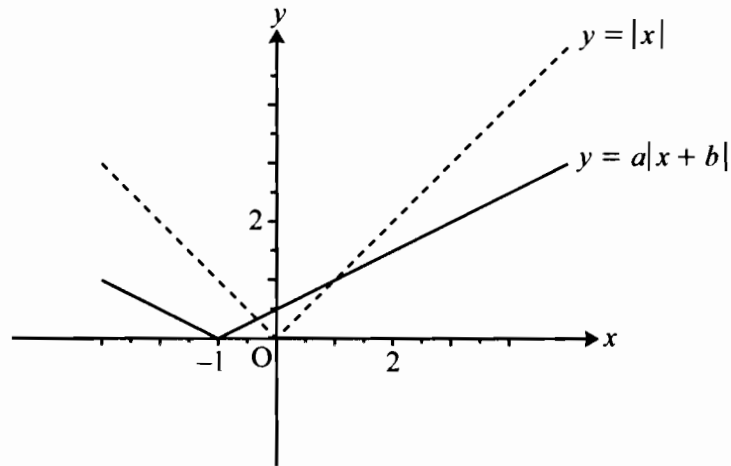


Fig. 1

- (i) Find a and b . [2]
- (ii) Find the coordinates of the two points of intersection of the graphs. [4]
- 2 (i) Factorise fully $n^3 - n$. [2]
- (ii) Hence prove that, if n is an integer, $n^3 - n$ is divisible by 6. [2]

- 3 The function $f(x)$ is defined by $f(x) = 1 - 2 \sin x$ for $-\frac{1}{2}\pi \leq x \leq \frac{1}{2}\pi$. Fig. 3 shows the curve $y = f(x)$.

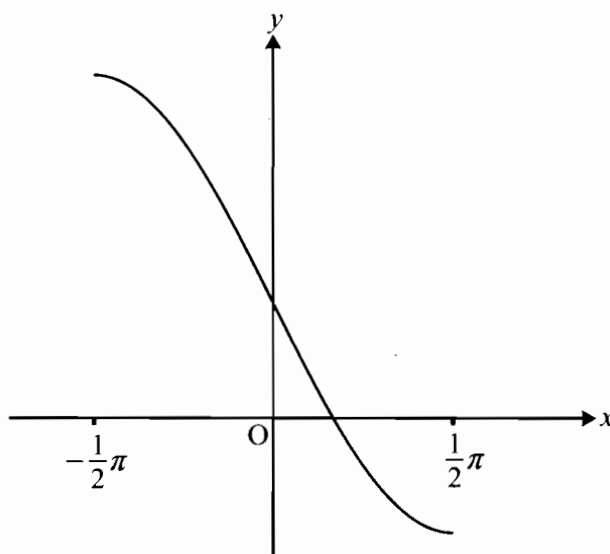


Fig. 3

- (i) Write down the range of the function $f(x)$. [2]
- (ii) Find the inverse function $f^{-1}(x)$. [3]
- (iii) Find $f'(0)$. Hence write down the gradient of $y = f^{-1}(x)$ at the point $(1, 0)$. [3]
- 4 Water flows into a bowl at a constant rate of $10 \text{ cm}^3 \text{ s}^{-1}$ (see Fig. 4).

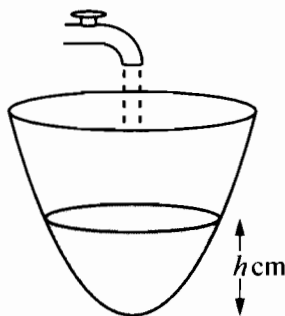


Fig. 4

When the depth of water in the bowl is h cm, the volume of water is $V \text{ cm}^3$, where $V = \pi h^2$. Find the rate at which the depth is increasing at the instant in time when the depth is 5 cm. [5]

- 5 Given that $y = \ln\left(\sqrt{\frac{2x-1}{2x+1}}\right)$, show that $\frac{dy}{dx} = \frac{1}{2x-1} - \frac{1}{2x+1}$. [4]
- 6 Using a suitable substitution or otherwise, show that $\int_0^{\frac{1}{2}\pi} \frac{\sin 2x}{3 + \cos 2x} dx = \frac{1}{2} \ln 2$. [5]

- 7 (i) Show algebraically that the function $f(x) = \frac{2x}{1-x^2}$ is odd. [2]

Fig. 7 shows the curve $y = f(x)$ for $0 \leq x \leq 4$, together with the asymptote $x = 1$.

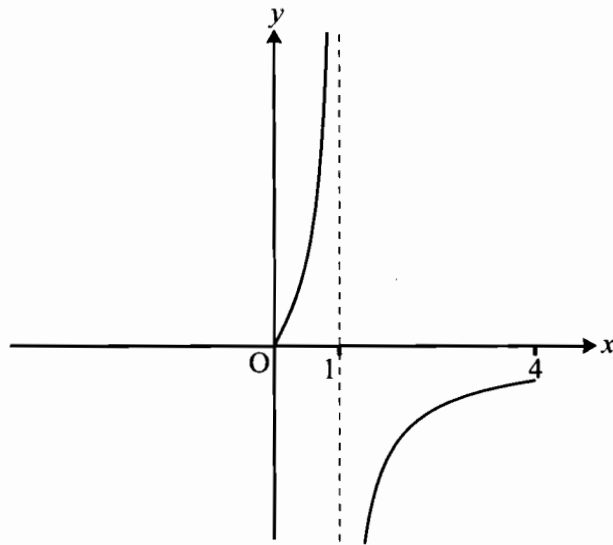


Fig. 7

- (ii) Use the copy of Fig. 7 to complete the curve for $-4 \leq x \leq 4$. [2]

Section B (36 marks)

- 8 Fig. 8 shows the curve $y = f(x)$, where $f(x) = (1 - x)e^{2x}$, with its turning point P.

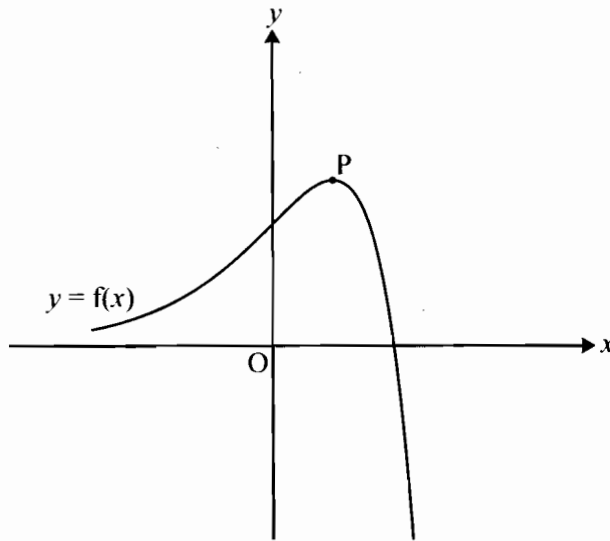


Fig. 8

- (i) Write down the coordinates of the intercepts of $y = f(x)$ with the x - and y -axes. [2]
- (ii) Find the exact coordinates of the turning point P. [6]
- (iii) Show that the exact area of the region enclosed by the curve and the x - and y -axes is $\frac{1}{4}(e^2 - 3)$. [5]

The function $g(x)$ is defined by $g(x) = 3f\left(\frac{1}{2}x\right)$.

- (iv) Express $g(x)$ in terms of x .

Sketch the curve $y = g(x)$ on the copy of Fig. 8, indicating the coordinates of its intercepts with the x - and y -axes and of its turning point. [4]

- (v) Write down the exact area of the region enclosed by the curve $y = g(x)$ and the x - and y -axes. [1]

- 9 Fig. 9 shows the curve with equation $y^3 = \frac{x^3}{2x-1}$. It has an asymptote $x = a$ and turning point P.

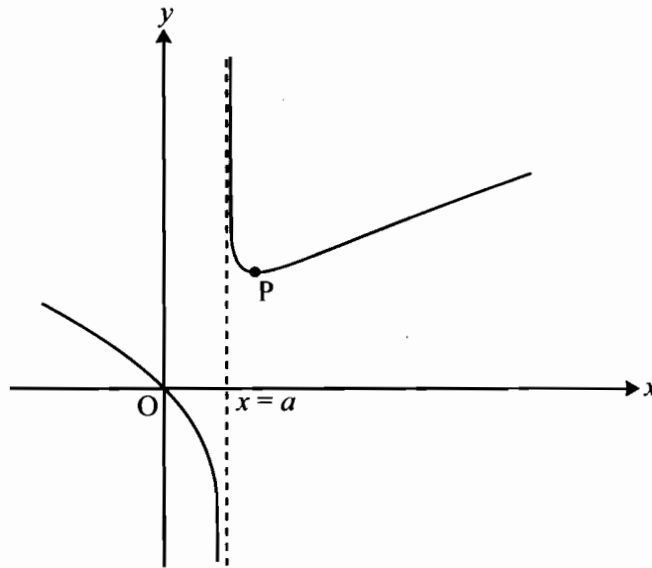


Fig. 9

- (i) Write down the value of a . [1]

(ii) Show that $\frac{dy}{dx} = \frac{4x^3 - 3x^2}{3y^2(2x-1)^2}$.

Hence find the coordinates of the turning point P, giving the y -coordinate to 3 significant figures. [9]

(iii) Show that the substitution $u = 2x - 1$ transforms $\int \frac{x}{\sqrt[3]{2x-1}} dx$ to $\frac{1}{4} \int (u^{\frac{2}{3}} + u^{-\frac{1}{3}}) du$.

Hence find the exact area of the region enclosed by the curve $y^3 = \frac{x^3}{2x-1}$, the x -axis and the lines $x = 1$ and $x = 4.5$. [8]