Section A

1)

$$\frac{2x}{x-2} - \frac{4x}{x+1} = 3$$

$$2x(x+1) - 4x(x-2) = 3(x-2)(x+1)$$

$$2x^2 + 2x - 4x^2 + 8x$$
$$= 3(x^2 - 2x + x - 2)$$

$$-2x^2 + 10x = 3x^2 - 3x - 6$$

$$5x^2 - 13x - 6 = 0$$

$$(5x+2)(x-3) = 0$$

$$\Rightarrow x = -\frac{2}{5}$$

or $x = 3$

2)

$$x = t - \ln t \qquad (t > 0)$$
$$y = t + \ln t$$

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

$$\frac{dy}{dx} = \frac{1 + \frac{1}{t}}{1 - \frac{1}{t}}$$

When $t = 2$

$$\frac{dy}{dx} = \frac{1 + \frac{1}{2}}{1 - \frac{1}{2}}$$
$$= \frac{\frac{3}{2}}{\frac{1}{2}} = 3$$

3) $A(-2, 4, 1)$
$B(2, 3, 4)$
$C(4, 8, 3)$

$$\vec{BA} = \begin{pmatrix} -4 \\ 1 \\ -3 \end{pmatrix}$$

$$\vec{BC} = \begin{pmatrix} 2 \\ 5 \\ -1 \end{pmatrix}$$

$$\vec{BA}.\vec{BC} = \begin{pmatrix} -4 \\ 1 \\ -3 \end{pmatrix} . \begin{pmatrix} 2 \\ 5 \\ -1 \end{pmatrix}$$

$$= -8 + 5 + 3 = 0$$

$\therefore \vec{BA}$ and $\vec{BC}$ are $\perp$

$\angle ABC = 90°$

Area of $\triangle ABC$ =

$\frac{1}{2} \times |BA| \times |BC|$

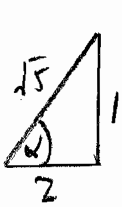
$$= \frac{1}{2} \times \sqrt{16+1+9}\,\sqrt{4+25+1}$$

$$= \frac{1}{2} \times \sqrt{26}\,\sqrt{30}$$

$$\approx 13.964 \text{ units}^2$$

4)

$$2\sin 2\theta + \cos 2\theta = 1$$



$$\sqrt{5}\sin(2\theta + \alpha) = 1$$

where $\tan\alpha = \frac{1}{2}$
$\alpha = 26.6°$

4 cont

$$\sqrt{5}\sin(2\theta + 26.6^\circ) = 1$$

$$\sin(2\theta + 26.6^\circ) = \frac{1}{\sqrt{5}}$$

$$2\theta + 26.6^\circ = 26.6^\circ,\ 153.4^\circ$$

$$386.6^\circ,\ 513.4^\circ$$

$$2\theta = 0,\ 126.8^\circ,\ 360^\circ,\ 486.8^\circ$$

$$\theta = 0,\ 63.4^\circ,\ 180^\circ,\ 243.4^\circ$$

5) i) Point $(2, -1, 4)$

$$\underline{n} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$$

Plane of form

$$x - y + 2z = c$$

Subst point

$$2 - (-1) + 2(4) = c$$

$$2 + 1 + 8 = c$$

$$11 = c$$

Plane is

$$x - y + 2z = 11$$

ii)

$$\underline{r} = \begin{pmatrix} 7 \\ 12 \\ 9 \end{pmatrix} + \lambda \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix}$$

$$\underline{r} = \begin{pmatrix} 7+\lambda \\ 12+3\lambda \\ 9+2\lambda \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 7+\lambda \\ 12+3\lambda \\ 9+2\lambda \end{pmatrix}$$

Subst in plane

$$7 + \lambda - (12 + 3\lambda) + 2(9 + 2\lambda) = 11$$

$$7 + \lambda - 12 - 3\lambda + 18 + 4\lambda = 11$$

$$13 + 2\lambda = 11$$

$$2\lambda = -2$$

$$\lambda = -1$$

Point of intersection is

$$(6, 9, 7)$$

6) i)

$$\frac{1}{\sqrt{4 - x^2}} = \frac{1}{2\sqrt{1 - \frac{x^2}{4}}}$$

$$= \frac{1}{2}\left(1 - \frac{x^2}{4}\right)^{-\frac{1}{2}}$$

$$\approx \frac{1}{2}\left[1 + \frac{-\frac{1}{2}}{1}\left(\frac{-x^2}{4}\right) + \frac{-\frac{1}{2}\cdot -\frac{3}{2}}{1.2}\left(\frac{-x^2}{4}\right)^2\right]$$

$$\approx \frac{1}{2}\left[1 + \frac{x^2}{8} + \frac{3}{128}x^4\right]$$

$$= \frac{1}{2} + \frac{x^2}{16} + \frac{3x^4}{256}$$

6 ii) $\displaystyle\int_0^1 \frac{1}{\sqrt{4-x^2}}\,dx$

$$\approx \int_0^1 \left(\frac{1}{2} + \frac{x^2}{16} + \frac{3x^4}{256}\right)dx$$

$$\approx \left[\frac{x}{2} + \frac{x^3}{48} + \frac{3x^5}{1280}\right]$$

$$\approx \frac{1}{2} + \frac{1}{48} + \frac{3}{1280}$$

$$\approx 0.5232 \quad \text{to 4 sig fig.}$$

6iii)

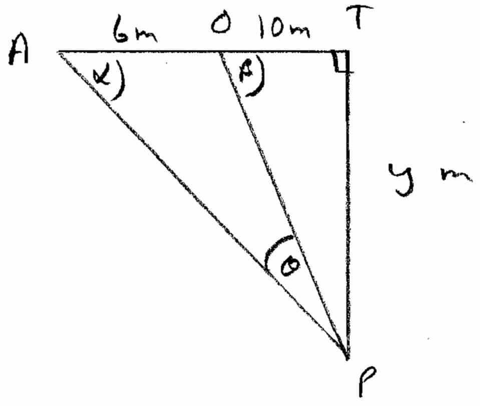
$$\int_0^1 \frac{1}{\sqrt{4-x^2}}\,dx = \left[\arcsin\left(\frac{x}{2}\right)\right]_0^1$$

$$= \arcsin\left(\tfrac{1}{2}\right) - \arcsin(0)$$

$$\approx 0.5236 \quad \text{to 4 sig fig}$$

Section B

7 i)



In $\triangle OTP$

$$\angle TPO = 90 - \beta$$

In $\triangle ATP$

$$\alpha + 90 + 90 - \beta + \theta = 180^\circ$$

$$\Rightarrow \alpha - \beta + \theta = 0$$

$$\Rightarrow \theta = \beta - \alpha$$

$$\tan\theta = \tan(\beta - \alpha)$$

$$= \frac{\tan\beta - \tan\alpha}{1 + \tan\beta\tan\alpha}$$

But $\tan\beta = \dfrac{y}{10}$ and $\tan\alpha = \dfrac{y}{16}$

$$\therefore \tan\theta = \frac{\frac{y}{10} - \frac{y}{16}}{1 + \frac{y}{10}\times\frac{y}{16}}$$

$$\tan\theta = \frac{\frac{16y}{160} - \frac{10y}{160}}{1 + \frac{y^2}{160}}$$

$$\tan\theta = \frac{\frac{6y}{160}}{\frac{160 + y^2}{160}}$$

$$= \frac{6y}{160 + y^2}$$

When $y = 6$

$$\tan\theta = \frac{36}{196}$$

$$\theta = 10.4^\circ$$

6ii) $\tan\theta = \dfrac{6y}{160+y^2}$

$$\sec^2\theta \frac{d\theta}{dy} = \frac{(160+y^2)6-6y(2y)}{(160+y^2)^2}$$

$$\frac{1}{\cos^2\theta}\frac{d\theta}{dy} = \frac{6(160+y^2)-12y^2}{(160+y^2)^2}$$

$$\frac{1}{\cos^2\theta}\frac{d\theta}{dy} = \frac{6(160+y^2-2y^2)}{(160+y^2)^2}$$

$$\frac{d\theta}{dy} = \frac{6(160-y^2)}{(160+y^2)^2}\cos^2\theta$$

6iii) θ a max when $\dfrac{d\theta}{dy} = 0$

$\dfrac{d\theta}{dy} = 0$ when $160-y^2 = 0$

$\Rightarrow y^2 = 160$

$\Rightarrow y = \sqrt{160}$

$y \approx 12.65$ m

to 4 sig fig.

8) $x = \dfrac{a}{1+kt}$

Given when $t=0$, $x = 2.5$

i) $\dfrac{dx}{dt} = \dfrac{(1+kt)0 - ak}{(1+kt)^2}$

$$\frac{dx}{dt} = \frac{-ak}{(1+kt)^2}$$

$$= \frac{-ka^2}{a(1+kt)^2} = \frac{-kx^2}{a}$$

ii) When $t=0$, $x = 2.5$

$\Rightarrow 2.5 = \dfrac{a}{1+0}$

$\Rightarrow a = 2.5$

when $t = 1$, $x = 1.6$

$\Rightarrow 1.6 = \dfrac{2.5}{1+k}$

$(1+k)1.6 = 2.5$

$1+k = \dfrac{2.5}{1.6}$

$k = \dfrac{2.5}{1.6} - 1$

$k = 0.5625$

iii) $x = \dfrac{2.5}{1+0.5625t}$

As $t \to \infty$, $x \to 0$

In the long term this predicts red squirrels die out

8 iv)

$$\frac{1}{2y-y^2} = \frac{1}{y(2-y)}$$

$$\frac{1}{y(2-y)} \equiv \frac{A}{y} + \frac{B}{2-y}$$

$$1 \equiv A(2-y) + By$$

when $y=2$

$$1 = 2B \Rightarrow B = \tfrac{1}{2}$$

when $y=0$

$$1 = 2A \Rightarrow A = \tfrac{1}{2}$$

$$\therefore \frac{1}{2y-y^2} \equiv \frac{1}{2y} + \frac{1}{2(2-y)}$$

v)

$$\frac{dy}{dt} = 2y - y^2$$

$$\int \frac{1}{2y-y^2}\,dy = \int 1\,dt$$

$$\int \left(\frac{1}{2y} + \frac{1}{2(2-y)}\right) dy = \int 1\,dt$$

$$\tfrac{1}{2}\ln y - \tfrac{1}{2}\ln(2-y) = t + c$$

Given $t=0$, $y=1$

$$\tfrac{1}{2}\ln 1 - \tfrac{1}{2}\ln 1 = 0 + c$$

$$\Rightarrow c = 0$$

$$\therefore \tfrac{1}{2}\ln y - \tfrac{1}{2}\ln(2-y) = t$$

$$\tfrac{1}{2}\ln\left(\frac{y}{2-y}\right) = t$$

$$\ln\left(\frac{y}{2-y}\right) = 2t$$

$$\frac{y}{2-y} = e^{2t}$$

$$y = e^{2t}(2-y)$$

$$y = 2e^{2t} - ye^{2t}$$

$$y + ye^{2t} = 2e^{2t}$$

$$y(1+e^{2t}) = 2e^{2t}$$

$$y = \frac{2e^{2t}}{(1+e^{2t})}$$

$$\Rightarrow \quad y = \frac{2}{\frac{1}{e^{2t}} + 1}$$

$$\Rightarrow \quad y = \frac{2}{1+e^{-2t}}$$

vi) As $t \to \infty$, $y \to 2$

Long term population of grey squirrels $\to$ 2000