1)

$$\frac{1}{x} + \frac{x}{x+2} = 1$$

$$\frac{(x+2)+x^2}{x(x+2)} = 1$$

$$x+2+x^2 = x(x+2)$$

$$x+2+\cancel{x^2} = \cancel{x^2}+2x$$

$$2 = 2x - x$$

$$\Rightarrow \quad x = 2$$

2)

$$y = \sqrt{1+x^3}$$

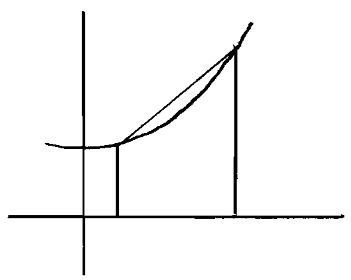
i)

	x_0	x_1	x_2	x_3	x_4
x	0	0·5	1·0	1·5	2·0
y	1	1.060	1.414	2.091	3

$$A \approx \frac{0\cdot5}{2}\left[1+2(1.060+1\cdot414+2\cdot092)+3\cdot0\right]$$

$$A \approx 3.28$$

ii)



Concave curve

Trapezium rule gives over-estimate

$\therefore$ 3.25 will be correct

3 i) $\sin(\theta+\phi)$

$$= \sin\theta\cos\phi + \cos\theta\sin\phi$$

$$\sin(45^\circ+60^\circ)$$

$$= \sin 45^\circ\cos 60^\circ + \cos 45^\circ \sin 60^\circ$$

$$= \frac{1}{\sqrt2}\times\frac{1}{2} + \frac{1}{\sqrt2}\times\frac{\sqrt3}{2}$$

$$= \frac{1+\sqrt3}{2\sqrt2}$$

3 ii) Sine Rule

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\Rightarrow \frac{1}{\sin 30^\circ} = \frac{AC}{\sin 105^\circ}$$

$$AC = \frac{\sin 105^\circ}{\sin 30^\circ}$$

$$= \frac{\frac{1+\sqrt3}{2\sqrt2}}{\frac{1}{2}}$$

$$= \frac{(1+\sqrt3)}{2\sqrt2}\times\frac{2}{1}$$

$$= \frac{1+\sqrt3}{\sqrt2}$$

4

$$\frac{1+\tan^2\theta}{1-\tan^2\theta}$$

$$= \frac{1+\frac{\sin^2\theta}{\cos^2\theta}}{1-\frac{\sin^2\theta}{\cos^2\theta}}$$

4cont)

$$= \frac{\dfrac{\cos^2\theta + \sin^2\theta}{\cos^2\theta}}{\dfrac{\cos^2\theta - \sin^2\theta}{\cos^2\theta}}$$

$$= \frac{\cos^2\theta + \sin^2\theta}{\cos^2\theta - \sin^2\theta}$$

$$= \frac{1}{\cos 2\theta}$$

$$= \sec 2\theta$$

Solve $\dfrac{1+\tan^2\theta}{1-\tan^2\theta} = 2$

$\Rightarrow \sec 2\theta = 2$

$\Rightarrow \dfrac{1}{\cos 2\theta} = 2$

$\Rightarrow \cos 2\theta = \dfrac{1}{2}$

$\Rightarrow 2\theta = 60^\circ, 300^\circ$

$\Rightarrow \theta = 30^\circ, 150^\circ$

5) $(1+3x)^{\frac{1}{3}}$

$$\approx 1 + \tfrac{1}{3}(3x) + \frac{\tfrac{1}{3}\cdot -\tfrac{2}{3}}{1.2}(3x)^2 + \frac{\tfrac{1}{3}\cdot -\tfrac{2}{3}\cdot -\tfrac{5}{3}}{1.2.3}(3x)^3$$

$$= 1 + x - x^2 + \tfrac{5}{3}x^3$$

Valid for $|3x| < 1$

$-\frac{1}{3} < x < \frac{1}{3}$

6) i)

Let $\dfrac{1}{(2x+1)(x+1)} \equiv \dfrac{A}{2x+1} + \dfrac{B}{x+1}$

$\Rightarrow 1 \equiv A(x+1) + B(2x+1)$

when $x = -1$

$1 = B(2(-1)+1)$

$1 = -B \quad \Rightarrow B = -1$

when $x = -\frac{1}{2}$

$1 = A(-\frac{1}{2}+1)$

$1 = \frac{1}{2}A \quad \Rightarrow A = 2$

$\therefore \dfrac{1}{(2x+1)(x+1)} = \dfrac{2}{2x+1} - \dfrac{1}{x+1}$

6ii)

$$\frac{dy}{dx} = \frac{y}{(2x+1)(x+1)}$$

$$\int \frac{1}{y}\,dy = \int \frac{1}{(2x+1)(x+1)}\,dx$$

$$\int \frac{1}{y}\,dy = \int \left(\frac{2}{2x+1} - \frac{1}{x+1}\right) dx$$

$\ln y = \ln(2x+1) - \ln(x+1) + c$

Passes thro $(0,2)$

$\Rightarrow \ln 2 = \ln 1 - \ln 1 + c$

$\Rightarrow \ln 2 = c$

6ii cont) $\therefore \ln y = \ln(2x+1) - \ln(x+1) + \ln 2$

$\ln y = \ln\left(\dfrac{2(2x+1)}{x+1}\right)$

$\Rightarrow \quad y = \dfrac{4x+2}{x+1}$

Section B

7) i) $x = \cos\theta \qquad y = \sin\theta - \frac{1}{8}\sin 2\theta$

At A, $x = 1$

$\Rightarrow \cos\theta = 1$

$\Rightarrow \theta = 0$

At B, $x = -1$

$\Rightarrow \cos\theta = -1$

$\Rightarrow \theta = \pi$

At C, $x = 0$

$\Rightarrow \cos\theta = 0$

$\Rightarrow \theta = \frac{\pi}{2}$ or $\frac{3\pi}{2}$

when $\theta = \frac{\pi}{2}$, $y = 1 - 0 = 1$

when $\theta = \frac{3\pi}{2}$, $y = -1 - 0 = -1$

$\therefore$ C is point $(0, 1)$

7ii)

$\dfrac{dy}{d\theta} = \cos\theta - \frac{1}{4}\cos 2\theta$

$\dfrac{dx}{d\theta} = -\sin\theta$

$\dfrac{dy}{dx} = \dfrac{dy}{d\theta} \Big/ \dfrac{dx}{d\theta}$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{\cos\theta - \frac{1}{4}\cos 2\theta}{-\sin\theta}$

$\dfrac{dy}{dx} = \dfrac{4\cos\theta - \cos 2\theta}{-4\sin\theta}$

$\dfrac{dy}{dx} = \dfrac{\cos 2\theta - 4\cos\theta}{4\sin\theta}$

At max pt $\dfrac{dy}{dx} = 0$

$\Rightarrow \cos 2\theta - 4\cos\theta = 0$

$\Rightarrow 2\cos^2\theta - 1 - 4\cos\theta = 0$

7iii)

$2\cos^2\theta - 4\cos\theta - 1 = 0$

$\cos\theta = \dfrac{4 \pm \sqrt{16+8}}{4}$

$\cos\theta = \cancel{2.225}$ or -0.2247

$\Rightarrow \theta = 1.7975$ radians

$\Rightarrow y = \sin(1.7975) - \frac{1}{8}\sin(2 \times 1.7975)$

$y = 1.029$ to 4 s.f.

7iv)

$y = \frac{1}{4}(4-x)\sqrt{1-x^2}$

Volume $= \displaystyle\int_{-1}^{1} \pi y^2\,dx$

$= \pi \displaystyle\int_{-1}^{1} \frac{1}{16}(4-x)^2(1-x^2)\,dx$

$= \dfrac{\pi}{16} \displaystyle\int_{-1}^{1} (16 - 8x + x^2)(1-x^2)\,dx$

7iv) cont

$$= \frac{\pi}{16}\int_{-1}^{1}\left(16 - 8x + x^2 - 16x^2 + 8x^3 - x^4\right)dx$$

$$= \frac{\pi}{16}\int_{-1}^{1}\left(16 - 8x - 15x^2 + 8x^3 - x^4\right)dx$$

$$= \frac{\pi}{16}\left[16x - 4x^2 - 5x^3 + 2x^4 - \frac{x^5}{5}\right]_{-1}^{1}$$

$$= \frac{\pi}{16}\left[\left(16 - 4 - 5 + 2 - \frac{1}{5}\right) - \left(-16 - 4 + 5 + 2 + \frac{1}{5}\right)\right]$$

$$= \frac{\pi}{16}\left[32 - 10 - \frac{2}{5}\right]$$

$$= \frac{27\pi}{20} = 4.24$$

8) i)

$A(0, -40, 0)$

$B(40, 0, -20)$

$$|AB| = \sqrt{(40-0)^2 + (0--40)^2 + (-20-0)^2}$$

$$|AB| = 60\text{ m}$$

8ii)

$$\cos\angle ABC = \frac{\vec{BA}\cdot\vec{BC}}{|\vec{BA}||\vec{BC}|}$$

$$\cos A\hat{B}C = \frac{\begin{pmatrix}-40\\-40\\20\end{pmatrix}\cdot\begin{pmatrix}3\\4\\1\end{pmatrix}}{60 \times \sqrt{3^2+4^2+1^2}}$$

$$\cos A\hat{B}C = \frac{-120 - 160 + 20}{60\sqrt{26}}$$

$$\angle ABC = 148.2°$$

8iii)

$$\underset{\sim}{r} = \begin{pmatrix}40\\0\\-20\end{pmatrix} + \lambda\begin{pmatrix}3\\4\\1\end{pmatrix}$$

At C

$$\begin{pmatrix}a\\b\\0\end{pmatrix} = \begin{pmatrix}40+3\lambda\\4\lambda\\-20+\lambda\end{pmatrix}$$

$\Rightarrow -20 + \lambda = 0 \Rightarrow \lambda = 20$

$\therefore a = 40 + 3\times 20 = 100$

$b = 4\times 20 = 80$

8iv)

$$\begin{pmatrix}6\\-5\\2\end{pmatrix}\cdot\begin{pmatrix}-40\\-40\\20\end{pmatrix} = -240 + 200 + 40 = 0$$

$$\begin{pmatrix}6\\-5\\2\end{pmatrix}\cdot\begin{pmatrix}3\\4\\1\end{pmatrix} = 18 - 20 + 2 = 0$$

Since $\begin{pmatrix}6\\-5\\2\end{pmatrix}$ is $\perp$ to two non-parallel lines in plane it is $\perp$ to plane.

Plane of form

$$6x - 5y + 2z = d$$

$A(0,-40,0)$ on plane so

$$6(0) - 5(-40) + 2(0) = d$$

$$200 = d$$

Plane is

$$6x - 5y + 2z = 200$$

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