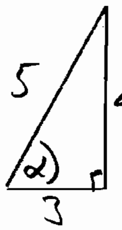
1)

$$3\cos\theta + 4\sin\theta$$

$$= 5\left(\frac{3}{5}\cos\theta + \frac{4}{5}\sin\theta\right)$$



$$= 5\cos(\theta - \alpha)$$

where $\alpha = \tan^{-1}\left(\frac{4}{3}\right)$

$\alpha = 0.927$ radians

Answer: $5\cos(\theta - 0.927)$

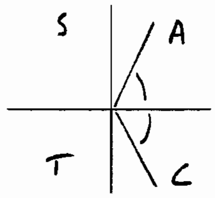
$$3\cos\theta + 4\sin\theta = 2$$

$$5\cos(\theta - 0.927) = 2$$

$$\cos(\theta - 0.927) = \frac{2}{5}$$

$$\theta - 0.927 = \cos^{-1}\left(\frac{2}{5}\right)$$

$$\theta - 0.927 = 1.159, \ -1.159$$



$\Rightarrow \theta = 2.086$ radians
or $\theta = -0.232$ radians

2)

$$\frac{1}{\sqrt{1-2x}} = (1-2x)^{-\frac{1}{2}}$$

$$\approx 1 + -\tfrac{1}{2}(-2x) + \frac{-\frac{1}{2}\cdot-\frac{3}{2}}{1\cdot 2}(-2x)^2$$

$$= 1 + x + \frac{3}{2}x^2 + \ldots$$

Valid for $|2x| < 1$

$$\Rightarrow -\frac{1}{2} < x < \frac{1}{2}$$

---

2 ii)

$$\frac{1+2x}{\sqrt{1-2x}} = (1+2x)(1-2x)^{-\frac{1}{2}}$$

$$\approx (1+2x)\left(1 + x + \frac{3}{2}x^2\right)$$

$$\approx 1 + 2x + x + 2x^2 + \frac{3}{2}x^2$$

$$= 1 + 3x + \frac{7}{2}x^2$$

---

3) Volume $= \displaystyle\int_1^2 \pi x^2\,dy$

Since $y = 1 + x^2$

$$y - 1 = x^2$$

$\therefore$ Volume $= \pi\displaystyle\int_1^2 (y-1)\,dy$

$$= \pi\left[\frac{y^2}{2} - y\right]_1^2$$

$$= \pi\left[\left(\frac{4}{2} - 2\right) - \left(\frac{1}{2} - 1\right)\right]$$

$$= \frac{\pi}{2}$$

---

4)

$$\sin(\theta + 45^\circ) = \cos\theta$$

$$\Rightarrow \sin\theta\cos 45^\circ + \cos\theta\sin 45^\circ = \cos\theta$$

$$\Rightarrow \frac{1}{\sqrt{2}}\sin\theta + \frac{1}{\sqrt{2}}\cos\theta = \cos\theta$$

$$\Rightarrow \sin\theta + \cos\theta = \sqrt{2}\cos\theta$$

4 cont)

$\sin\theta = \sqrt{2}\cos\theta - \cos\theta$

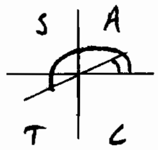
$\sin\theta = (\sqrt{2}-1)\cos\theta$

$\frac{\sin\theta}{\cos\theta} = \tan\theta = \sqrt{2}-1$

4ii)

$\theta = \tan^{-1}(\sqrt{2}-1)$

$\theta = 22.5^\circ,\ 202.5^\circ$



5)

Let $\frac{4}{x(x^2+4)} \equiv \frac{A}{x} + \frac{Bx+C}{x^2+4}$

$\Rightarrow 4 \equiv A(x^2+4) + (Bx+C)x$

When $x = 0$

$4 = 4A \quad \Rightarrow A = 1$

Equating coeffs of x^2

$0 = A + B$

$0 = 1 + B \quad \Rightarrow B = -1$

Equating coeffs of x

$0 = C \quad \Rightarrow C = 0$

$\therefore \frac{4}{x(x^2+4)} \equiv \frac{1}{x} - \frac{1}{x^2+4}$

6)

$\operatorname{cosec}\theta = 3$

$\Rightarrow \frac{1}{\sin\theta} = 3$

$\Rightarrow \sin\theta = \frac{1}{3}$

$\theta = \sin^{-1}(\frac{1}{3})$

$\theta = 19.5^\circ,\ 160.5^\circ$

7)
i) $C(15, 0, 0)$, $D(9, 6, 24)$ $\quad \vec{CD} = \begin{pmatrix}-6\\6\\24\end{pmatrix}$

$C(15, 0, 0)$, $B(15, 20, 0)$ $\quad \vec{CB} = \begin{pmatrix}0\\20\\0\end{pmatrix}$

7ii) $|CD| = \sqrt{(15-9)^2 + (0-6)^2 + (0-24)^2}$

$|CD| = 25.46$ cm

7iii) $\begin{pmatrix}4\\0\\1\end{pmatrix} . \begin{pmatrix}-6\\6\\24\end{pmatrix} = -24 + 0 + 24 = 0$

$\begin{pmatrix}4\\0\\1\end{pmatrix} . \begin{pmatrix}0\\20\\0\end{pmatrix} = 0 + 0 + 0 = 0$

$\begin{pmatrix}4\\0\\1\end{pmatrix}$ is $\perp$ to two non-parallel vectors in plane BCDE

It is $\therefore$ normal to plane.

Eqn of form $4x + 0y + z = d$

C on plane so $4(15) + 0(0) + 0 = d$

$60 = d$

Plane is $4x + z = 60$

7iv) For OG

$\underline{r} = \begin{pmatrix}0\\0\\0\end{pmatrix} + \lambda\begin{pmatrix}3\\6\\24\end{pmatrix}$ or $\underline{r} = \lambda\begin{pmatrix}1\\2\\8\end{pmatrix}$

7iv) cont)

For AF $\underline{r} = \begin{pmatrix} 0 \\ 20 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ -6 \\ 24 \end{pmatrix}$

or $\underline{r} = \begin{pmatrix} 0 \\ 20 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -2 \\ 8 \end{pmatrix}$

For OG with $\lambda = 5$

$$\underline{r} = 5\begin{pmatrix} 1 \\ 2 \\ 8 \end{pmatrix} = \begin{pmatrix} 5 \\ 10 \\ 40 \end{pmatrix}$$

For AF with $\mu = 5$

$$\underline{r} = \begin{pmatrix} 0 \\ 20 \\ 0 \end{pmatrix} + 5\begin{pmatrix} 1 \\ -2 \\ 8 \end{pmatrix} = \begin{pmatrix} 5 \\ 10 \\ 40 \end{pmatrix}$$

$\therefore$ OG and AF meet at $(5, 10, 40)$

7v)

Volume of Pyramid POABC

$= \frac{1}{3} \times$ Area of Base $\times$ height

$= \frac{1}{3} \times (15 \times 20) \times 40$

$= 4000 \text{ cm}^3$

Volume of Pyramid PDEFG

$= \frac{1}{3} \times (6 \times 8) \times (40 - 24)$

$= 256 \text{ cm}^3$

Volume of ornament

$= 4000 - 256 = 3744 \text{ cm}^3$

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8) i)

$x = k\cos\theta, \quad y = \frac{1}{2}k\sin\theta$

$\Rightarrow \frac{x}{k} = \cos\theta, \quad \frac{2y}{k} = \sin\theta$

$\Rightarrow \frac{x^2}{k^2} + \frac{4y^2}{k^2} = \cos^2\theta + \sin^2\theta = 1$

$\Rightarrow x^2 + 4y^2 = k^2$

$\therefore x = k\cos\theta$ and $y = \frac{1}{2}k\sin\theta$ are parametric equns for the given curve

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8ii)

$$x^2 + 4y^2 = k^2$$

Differentiate implicitly

$$2x + 8y\frac{dy}{dx} = 0$$

$$8y\frac{dy}{dx} = -2x$$

$$\frac{dy}{dx} = -\frac{2x}{8y} = -\frac{x}{4y}$$

8ii) alternative method

$$\frac{dy}{d\theta} = \frac{1}{2}k\cos\theta$$

$$\frac{dx}{d\theta} = -k\sin\theta$$

$$\frac{dy}{dx} = \frac{dy}{d\theta} \Big/ \frac{dx}{d\theta} = \frac{\frac{1}{2}k\cos\theta}{-k\sin\theta}$$

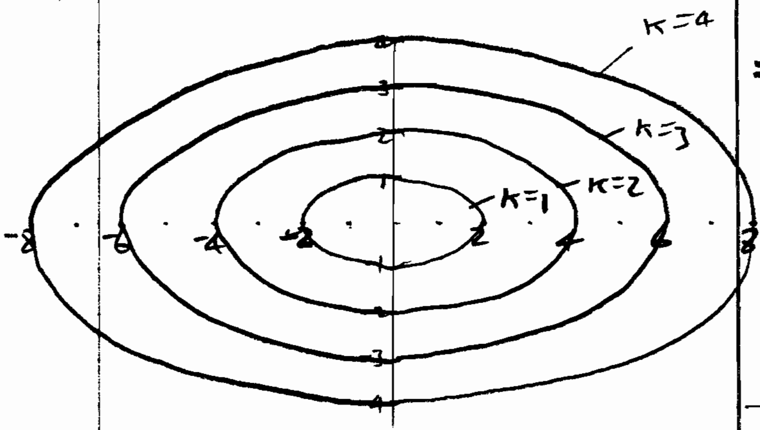
$$= \frac{\frac{1}{2}x}{-k \times \frac{2y}{k}} = -\frac{x}{4y}$$

8iii) Point $(2,0)$ on curve

$$2^2 + 4(0^2) = k^2$$

$$\Rightarrow k = 2$$

8iv)



8v)

If path of stream $\perp$ to contour

Gradient of stream path on map $\times$ gradient of contour $= -1$

$$\therefore \frac{dy}{dx}_{\text{stream}} = -\frac{1}{\frac{dy}{dx}_{\text{contour}}}$$

$$= -\frac{1}{-\frac{x}{4y}} = \frac{4y}{x}$$

8vi)

$$\frac{dy}{dx} = \frac{4y}{x}$$

$$\int \frac{1}{y}\,dy = \int \frac{4}{x}\,dx$$

$$\ln y = 4\ln x + c$$

Given $(2,1)$ on path

$$\ln(1) = 4\ln(2) + c$$

$$\Rightarrow c = -4\ln 2$$

$$\therefore \ln y = 4\ln x - 4\ln 2$$

$$\Rightarrow \ln y = 4\ln\left(\frac{x}{2}\right)$$

$$\Rightarrow \ln y = \ln\left(\frac{x}{2}\right)^4$$

$$\Rightarrow y = \left(\frac{x}{2}\right)^4$$

$$\Rightarrow y = \frac{x^4}{16}$$