1) $\dfrac{3x+2}{x(x^2+1)} \equiv \dfrac{A}{x} + \dfrac{Bx+C}{x^2+1}$

$\Rightarrow 3x+2 \equiv A(x^2+1) + (Bx+C)x$

$x=0$

$\Rightarrow 2 = A$

Coeff of x^2

$0 = A + B \quad \Rightarrow B = -2$

$x=1$

$5 = 2(2) + (-2+C)1$

$5 = 4 - 2 + C$

$\Rightarrow 3 = C$

Partial fractions

$\dfrac{2}{x} + \dfrac{3-2x}{x^2+1}$

2) $(1+2x)^{\frac{1}{3}}$

$= 1 + \frac{1}{3}(2x) + \dfrac{\frac{1}{3}\cdot-\frac{2}{3}}{1.2}(2x)^2$

$+ \dfrac{\frac{1}{3}\cdot-\cancel{\frac{2}{3}}\cdot-\frac{5}{3}}{1.\cancel{2}.3}(2x)^3 + +$

$\approx 1 + \frac{2}{3}x - \frac{4}{9}x^2 + \frac{40}{81}x^3$

Valid for $|2x| < 1$

$\Rightarrow -\frac{1}{2}\ x < \frac{1}{2}$

3) $\underline{a} = \begin{pmatrix}2\\1\\-1\end{pmatrix} \quad \underline{b} = \begin{pmatrix}4\\-2\\1\end{pmatrix}$

Require $\lambda\begin{pmatrix}2\\1\\-1\end{pmatrix} + \mu\begin{pmatrix}4\\-2\\1\end{pmatrix} = \begin{pmatrix}0\\4\\-3\end{pmatrix}$

$2\lambda + 4\mu = 0$

$2\lambda = -4\mu$

$\lambda = -2\mu$

Subst in 2nd row

$-2\mu - 2\mu = 4$

$\Rightarrow -4\mu = 4$

$\Rightarrow \mu = -1$

$\therefore \lambda = -2(-1) = 2$

Check in third row

$2(-1) - 1(1) = -3$ ✓

Answer $\lambda = 2, \ \mu = -1$

4) Prove $\cot\beta - \cot\alpha = \dfrac{\sin(\alpha-\beta)}{\sin\alpha\sin\beta}$

$\dfrac{\sin(\alpha-\beta)}{\sin\alpha\sin\beta} = \dfrac{\sin\alpha\cos\beta - \cos\alpha\sin\beta}{\sin\alpha\sin\beta}$

$= \dfrac{\sin\alpha\cos\beta}{\sin\alpha\sin\beta} - \dfrac{\cos\alpha\sin\beta}{\sin\alpha\sin\beta}$

$= \dfrac{\cos\beta}{\sin\beta} - \dfrac{\cos\alpha}{\sin\alpha}$

$= \cot\beta - \cot\alpha$

5) i) Plane $2x - y + z = 2$

Normal $\begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$

Plane $x - z = 1$

Normal $\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$

Angle between planes = angle between normals

$$\cos\theta = \frac{\begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}}{\left|\begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}\right| \left|\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}\right|}$$

$$\cos\theta = \frac{2 + 0 - 1}{\sqrt{4+1+1}\ \sqrt{1+0+1}}$$

$$\cos\theta = \frac{1}{\sqrt{12}}$$

$$\theta = 73.2^\circ$$

5ii) $$\underline{r} = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 + 2\lambda \\ -\lambda \\ 1 + \lambda \end{pmatrix}$$

Subst in plane

$$2(2 + 2\lambda) - (-\lambda) + (1 + \lambda) = 2$$

$$4 + 4\lambda + \lambda + 1 + \lambda = 2$$

$$6\lambda = -3$$

$$\lambda = -\tfrac{1}{2}$$

Subst back in line

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2 + 2(-\frac{1}{2}) \\ -(-\frac{1}{2}) \\ 1 + (-\frac{1}{2}) \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ \frac{1}{2} \\ \frac{1}{2} \end{pmatrix}$$

Point of intersection $\left(1, \tfrac{1}{2}, \tfrac{1}{2}\right)$

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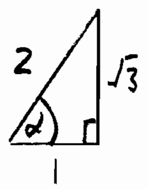
6) i) $\cos\theta + \sqrt{3}\sin\theta$



$$= 2\left(\frac{1}{2}\cos\theta + \frac{\sqrt{3}}{2}\sin\theta\right)$$

$$= 2\cos(\theta - \alpha)$$

where $\alpha = \tan^{-1}\sqrt{3} = \frac{\pi}{3}$

$$\cos\theta + \sqrt{3}\sin\theta = 2\cos\left(\theta - \frac{\pi}{3}\right)$$

6ii) $$\frac{d}{d\theta}\tan\theta = \sec^2\theta$$

$$\int_0^{\frac{\pi}{3}} \frac{1}{(\cos\theta + \sqrt{3}\sin\theta)^2}\, d\theta$$

$$= \int_0^{\frac{\pi}{3}} \frac{1}{4\cos^2(\theta - \frac{\pi}{3})}\, d\theta$$

$$= \frac{1}{4}\int_0^{\frac{\pi}{3}} \sec^2\left(\theta - \frac{\pi}{3}\right) d\theta$$

6ii) cont

$$= \frac{1}{4}\left[\tan\left(\theta - \frac{\pi}{3}\right)\right]_0^{\frac{\pi}{3}}$$

$$= \frac{1}{4}\left[\tan 0 - \tan\left(-\frac{\pi}{3}\right)\right]$$

$$= \frac{1}{4}\left[0 - (-\sqrt{3})\right]$$

$$= \frac{\sqrt{3}}{4}$$

7)i) A) $\frac{98-89}{1\cdot5} = 6$ hours

B) $\frac{98-80}{1\cdot5} = 12$ hours

7ii)

$$\frac{d\theta}{dt} = -k(\theta - \theta_0)$$

$$\Rightarrow \int \frac{1}{\theta - \theta_0}\,d\theta = \int -k\,dt$$

$$\Rightarrow \ln(\theta - \theta_0) = -kt + c$$

$$\Rightarrow \theta - \theta_0 = e^{-kt+c}$$

$$\theta - \theta_0 = e^{-kt} \times e^{c}$$

Writing e^c as A

$$\theta = Ae^{-kt} + \theta_0$$

7iii) $\theta_0 = 50$, when $t = 0$, $\theta = 98$

$$\Rightarrow 98 = Ae^0 + 50$$

$$\Rightarrow A = 48$$

When $t = 0$, $\theta = 98$, $\frac{d\theta}{dt} = -1.5$

$$\Rightarrow -1.5 = -k(98 - 50)$$

$$\Rightarrow k = \frac{1.5}{48} = 0.03125$$

7iv) A) $\theta = 48e^{-0.03125t} + 50$

$t = 0$, $\theta = 98$

$$89 = 48e^{-0.03125t} + 50$$

$$\frac{39}{48} = e^{-0.03125t}$$

$$t = \frac{\ln\left(\frac{39}{48}\right)}{-0.03125}$$

$t = 6.64$ hours

B)

$$80 = 48e^{-0.03125t} + 50$$

$$\frac{30}{48} = e^{-0.03125t}$$

$$t = \frac{\ln\left(\frac{30}{48}\right)}{-0.03125}$$

$t = 15.04$ hours

v) Models disagree more as time progresses and temperature falls.

8) $x = 2 + 2\sin\theta$

$y = 2\cos\theta + \sin 2\theta$

$(0 \le \theta \le 2\pi)$

i) $\frac{dy}{d\theta} = -2\sin\theta + 2\cos 2\theta$

$\frac{dx}{d\theta} = 2\cos\theta$

$\frac{dy}{dx} = \frac{dy}{d\theta} \Big/ \frac{dx}{d\theta}$

$\frac{dy}{dx} = \frac{2\cos 2\theta - 2\sin\theta}{2\cos\theta}$

$\frac{dy}{dx} = \frac{\cos 2\theta - \sin\theta}{\cos\theta}$

8 ii) When $\theta = \frac{\pi}{6}$

$\frac{dy}{dx} = \frac{\cos\frac{\pi}{3} - \sin\frac{\pi}{6}}{\cos\frac{\pi}{6}}$

$= \frac{\frac{1}{2} - \frac{1}{2}}{\frac{\sqrt{3}}{2}} = 0$

When $\theta = \frac{\pi}{6}$

$y = 2\cos\frac{\pi}{6} + \sin\frac{\pi}{3} = \sqrt{3} + \frac{\sqrt{3}}{2}$

$= \frac{3\sqrt{3}}{2}$

$x = 2 + 2\sin\frac{\pi}{6} = 3$

B is point $\left(3, \frac{3\sqrt{3}}{2}\right)$

Assuming curve is symmetrical about x-axis

$|BC| = 2 \times \frac{3\sqrt{3}}{2} = 3\sqrt{3}$ m

8 iii)

A) $x\cos\theta = 2\cos\theta + 2\sin\theta\cos\theta$

$= 2\cos\theta + \sin 2\theta = y$

B) $2\sin\theta = x - 2$

$\sin\theta = \frac{x-2}{2}$

$\Rightarrow \cos^2\theta = (1 - \sin^2\theta) = 1 - \left(\frac{x-2}{2}\right)^2$

$\Rightarrow \cos^2\theta = 1 - \frac{(x^2 - 4x + 4)}{4}$

$= x - \frac{x^2}{4}$

c) $y^2 = x^2\cos^2\theta = x^2\left(x - \frac{x^2}{4}\right)$

$\Rightarrow y^2 = x^3 - \frac{x^4}{4}$

iv) $V = \pi\int_0^4 y^2\,dx$

$V = \pi\int_0^4 \left(x^3 - \frac{x^4}{4}\right) dx$

$V = \pi\left[\frac{x^4}{4} - \frac{x^5}{20}\right]_0^4$

$V = \pi\left[(64 - 51.2) - (0 - 0)\right]$

$V = 12.8\pi$ or 40.2 m^3