RECOGNISING ACHIEVEMENT

ADVANCED GCE **4754/01A**

MATHEMATICS (MEI)

Applications of Advanced Mathematics (C4) Paper A

TUESDAY 22 JANUARY 2008 Afternoon

Time: 1 hour 30 minutes

Additional materials: Answer Booklet (8 pages)
Graph paper
MEI Examination Formulae and Tables (MF2)

INSTRUCTIONS TO CANDIDATES

- Write your name in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 72.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

NOTE

- This paper will be followed by **Paper B: Comprehension**.

This document consists of **4** printed pages.

Section A (36 marks)

1 Express $3\cos\theta + 4\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \frac{1}{2}\pi$.

Hence solve the equation $3\cos\theta + 4\sin\theta = 2$ for $-\pi \leqslant \theta \leqslant \pi$. [7]

2 **(i)** Find the first three terms in the binomial expansion of $\dfrac{1}{\sqrt{1-2x}}$. State the set of values of x for which the expansion is valid. [5]

(ii) Hence find the first three terms in the series expansion of $\dfrac{1+2x}{\sqrt{1-2x}}$. [3]

3 Fig. 3 shows part of the curve $y = 1 + x^2$, together with the line $y = 2$.

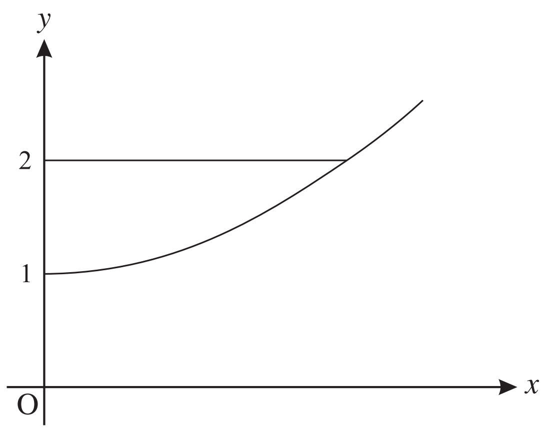


Fig. 3

The region enclosed by the curve, the y-axis and the line $y = 2$ is rotated through 360° about the y-axis. Find the volume of the solid generated, giving your answer in terms of π. [5]

4 The angle θ satisfies the equation $\sin(\theta + 45°) = \cos\theta$.

(i) Using the exact values of $\sin 45°$ and $\cos 45°$, show that $\tan\theta = \sqrt{2} - 1$. [5]

(ii) Find the values of θ for $0° < \theta < 360°$. [2]

5 Express $\dfrac{4}{x(x^2+4)}$ in partial fractions. [6]

6 Solve the equation $\operatorname{cosec}\theta = 3$, for $0° < \theta < 360°$. [3]

Section B (36 marks)

7 A glass ornament OABCDEFG is a truncated pyramid on a rectangular base (see Fig. 7). All dimensions are in centimetres.

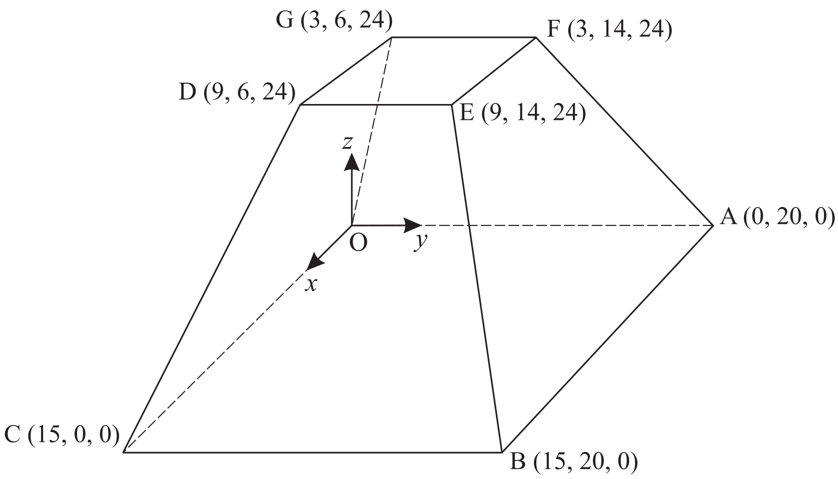


Fig. 7

(i) Write down the vectors $\overrightarrow{CD}$ and $\overrightarrow{CB}$. [2]

(ii) Find the length of the edge CD. [2]

(iii) Show that the vector $4\mathbf{i} + \mathbf{k}$ is perpendicular to the vectors $\overrightarrow{CD}$ and $\overrightarrow{CB}$. Hence find the cartesian equation of the plane BCDE. [5]

(iv) Write down vector equations for the lines OG and AF.

Show that they meet at the point P with coordinates (5, 10, 40). [5]

You may assume that the lines CD and BE also meet at the point P.

The volume of a pyramid is $\frac{1}{3} \times$ area of base $\times$ height.

(v) Find the volumes of the pyramids POABC and PDEFG.

Hence find the volume of the ornament. [4]

8 A curve has equation

$$x^2 + 4y^2 = k^2,$$

where k is a positive constant.

(i) Verify that

$$x = k\cos\theta, \qquad y = \tfrac{1}{2}k\sin\theta,$$

are parametric equations for the curve. [3]

(ii) Hence or otherwise show that $\dfrac{dy}{dx} = -\dfrac{x}{4y}$. [3]

(iii) Fig. 8 illustrates the curve for a particular value of k. Write down this value of k. [1]

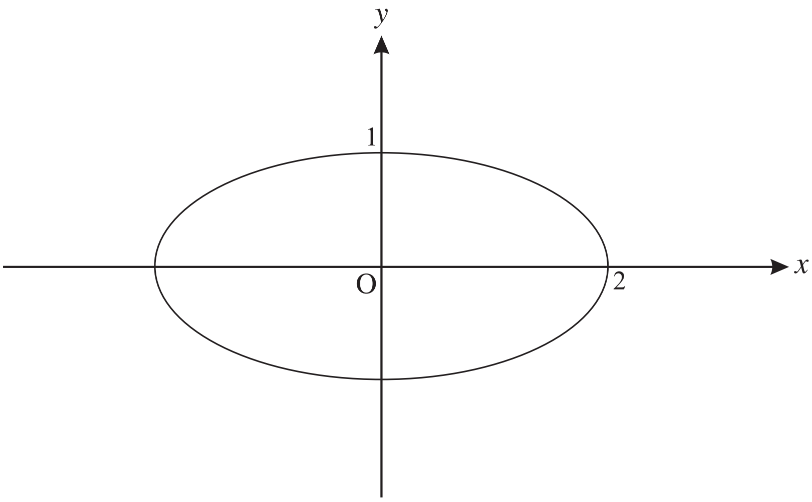


Fig. 8

(iv) Copy Fig. 8 and on the same axes sketch the curves for $k = 1$, $k = 3$ and $k = 4$. [3]

On a map, the curves represent the contours of a mountain. A stream flows down the mountain. Its path on the map is always at right angles to the contour it is crossing.

(v) Explain why the path of the stream is modelled by the differential equation

$$\frac{dy}{dx} = \frac{4y}{x}.$$ [2]

(vi) Solve this differential equation.

Given that the path of the stream passes through the point (2, 1), show that its equation is $y = \dfrac{x^4}{16}$. [6]

OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE **4754/01B**

MATHEMATICS (MEI)

Applications of Advanced Mathematics (C4) Paper B: Comprehension

TUESDAY 22 JANUARY 2008 Afternoon

Time: Up to 1 hour

Additional materials: Rough paper
MEI Examination Formulae and Tables (MF2)

Candidate Forename	
Candidate Surname	
Centre Number	
Candidate Number	

INSTRUCTIONS TO CANDIDATES

- Write your name in capital letters, your Centre Number and Candidate Number in the boxes above.
- Read each question carefully and make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Write your answers in the spaces provided on the question paper.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets [ ] at the end of each question or part question.
- The total number of marks for this paper is 18.
- The insert contains the text for use with the questions.
- You may find it helpful to make notes and do some calculations as you read the passage.
- You are **not** required to hand in these notes with your question paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.

FOR EXAMINER'S USE	
1	
2	
3	
4	
5	
6	
7	
TOTAL	

This document consists of **4** printed pages and an insert.

For Examiner's Use

1 An additive sequence has third term 5 and fourth term 6. Complete the sequence up to its sixth term. [2]

____ , ____ , 5 , 6 , ____ , ____ .

....

....

2 The Lucas sequence on line 32 is 2, 1, 3, 4, 7, 11, 18, 29, … .

State, with reasoning, whether the hundredth term will be odd or even. [2]

....

....

....

3 On lines 56 and 57 it is stated that the sequence $1, \phi, \phi^2, \phi^3, \phi^4, \phi^5, \ldots$ can be written in the form $1, \phi, \phi + 1, 2\phi + 1, 3\phi + 2, 5\phi + 3, \ldots$.

Write down the next term, ϕ^6, in the form $a\phi + b$ where a and b are integers. [1]

....

....

4 Find the length of HF in Fig. 3 in surd form, simplifying your answer. [3]

....

....

....

....

....

5 On lines 84 to 87 it is stated that the equation of the line BD is $\phi y + x = \phi$ and the equation of the line CF is $(\phi - 1)y = x - 1$.

(i) Write down the gradients of these lines in terms of ϕ. [2]

(ii) Show that these lines are perpendicular. [2]

(i) Gradient of line BD is ....

Gradient of line CF is ....

(ii) ....

....

....

....

6 On line 90 it is stated that the point Q has coordinates $\left(\frac{\phi + 1}{2\phi - 1}, \frac{\phi - 1}{2\phi - 1}\right)$, where $\phi = \frac{1 + \sqrt{5}}{2}$.

Show that the x-coordinate is $\frac{5 + 3\sqrt{5}}{10}$. [3]

....

....

....

....

7 The arithmetic sequence a, $a + d$, $a + 2d$, $a + 3d$, $\ldots$ is an additive sequence. Prove that $a = 0$ and $d = 0$. [3]

....

....

....

....

RECOGNISING ACHIEVEMENT

ADVANCED GCE **4754/01B**

MATHEMATICS (MEI)

Applications of Advanced Mathematics (C4) Paper B: Comprehension

INSERT

TUESDAY 22 JANUARY 2008 Afternoon

Time: Up to 1 hour

INSTRUCTIONS TO CANDIDATES

- This insert contains the text for use with the questions.

This document consists of **8** printed pages.

The Golden Ratio

Introduction

In mathematics there are many important constants. You have met some of these, such as π, the ratio of the circumference of a circle to its diameter, and e, the base of natural logarithms.

In this article, you will be introduced to another important constant and some of its properties, one of which is linked to sequences.

Geometric sequences

The sequence

$$1,\ x,\ x^2,\ x^3,\ x^4,\ x^5,\ \ldots \qquad (*)$$

is a geometric sequence with first term 1 and common ratio x. You can substitute various values for x and each one will generate a new geometric sequence.

For example, $x = 3$ generates the geometric sequence

$$1,\ 3,\ 9,\ 27,\ 81,\ 243,\ \ldots$$

and $x = -\frac{1}{2}$ generates the geometric sequence

$$1,\ -\tfrac{1}{2},\ \tfrac{1}{4},\ -\tfrac{1}{8},\ \tfrac{1}{16},\ -\tfrac{1}{32},\ \ldots\ .$$

Notice that neither of these is an arithmetic sequence, prompting the following question.

Are there any values of x for which the geometric sequence generated is also an arithmetic sequence?

In order to answer this question, it is helpful to think about the first three terms of the sequence $(*)$.

Suppose that $1,\ x,\ x^2$ are the first three terms of an arithmetic sequence. Then the difference between the first two terms, $x - 1$, is equal to the difference between the next two terms, $x^2 - x$, so that

$$x - 1 = x^2 - x.$$

This equation can be written in the form $x^2 - 2x + 1 = 0$. This has a repeated root of $x = 1$; the only value of x for which $1,\ x,\ x^2,\ \ldots$ is both an arithmetic and a geometric sequence is $x = 1$.

You will notice that $x = 1$ leads to the rather uninteresting sequence $1,\ 1,\ 1,\ 1,\ 1,\ \ldots$.

Additive sequences

In the Fibonacci sequence, the first two terms are given as $1,\ 1$. After that, each term is the sum of the two previous terms. The Fibonacci sequence is

$$1,\ 1,\ 2,\ 3,\ 5,\ 8,\ 13,\ 21,\ 34,\ \ldots\ .$$

Another sequence with the same property, that each term is the sum of the two previous terms, is the Lucas sequence

$$2,\ 1,\ 3,\ 4,\ 7,\ 11,\ 18,\ 29,\ \ldots\ .$$

In general, you can choose any two numbers for the first two terms, call them u_1 and u_2, and then use the rule

$$u_{n+1} = u_n + u_{n-1} \text{ for } n \geqslant 2$$

to generate subsequent terms. For the purposes of this article, any sequence with this property will be called an *additive sequence*.

Now think again about the geometric sequence (∗). Are there any values of x for which this sequence is an additive sequence?

To be an additive sequence, x must satisfy each of the following equations.

$$\begin{aligned} x^2 &= x + 1 \\ x^3 &= x^2 + x \\ x^4 &= x^3 + x^2 \\ &\vdots \end{aligned}$$

The first equation shows that $x \neq 0$. So each of the other equations reduces to the quadratic equation $x^2 = x + 1$. The two roots of this quadratic equation are $x = \frac{1+\sqrt{5}}{2}$ and $x = \frac{1-\sqrt{5}}{2}$. Therefore these are the only two values of x for which the sequence (∗) is both a geometric sequence and an additive sequence.

The Golden Ratio

The first of these numbers, $\frac{1+\sqrt{5}}{2}$, is called the Golden Ratio and is denoted by the Greek letter ϕ, pronounced 'phi'.

Substituting $x = \phi$ in the sequence (∗) gives

$$1,\ \phi,\ \phi^2,\ \phi^3,\ \phi^4,\ \phi^5,\ \ldots\ .$$

When written in this form, it is not at all obvious that it is an additive sequence. However, using the fact that $\phi^2 = \phi + 1$, the third term can be written as $\phi + 1$ and the fourth term, ϕ^3, can be written as $\phi^2 + \phi$ which simplifies to $2\phi + 1$. Continuing in this way, the sequence can be written as

$$1,\ \phi,\ \phi + 1,\ 2\phi + 1,\ 3\phi + 2,\ 5\phi + 3,\ \ldots\ ,$$

confirming the additive property $u_{n+1} = u_n + u_{n-1}$.

The Golden Rectangle

The rectangle ABCD in Fig. 1 has width $\phi = \dfrac{1+\sqrt{5}}{2}$ units and height 1 unit.

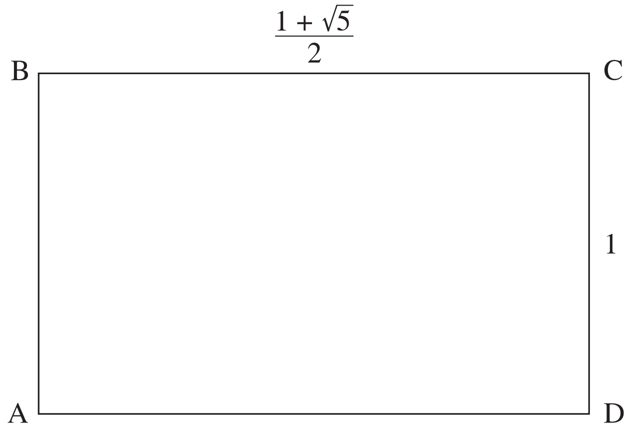


Fig. 1

The ratio of the sides of this rectangle is $\dfrac{1+\sqrt{5}}{2} : 1$.

Many artists and architects believe that, of all rectangles, ones with this ratio are particularly pleasing to the human eye. For this reason approximations to these rectangles have been used extensively in art and architecture.

Any rectangle with sides in the ratio $\dfrac{1+\sqrt{5}}{2} : 1$ is called a Golden Rectangle.

Golden Rectangles have many interesting properties, some of which will be described here.

First remove a unit square, ABEF, from a Golden Rectangle, as shown in Fig. 2.

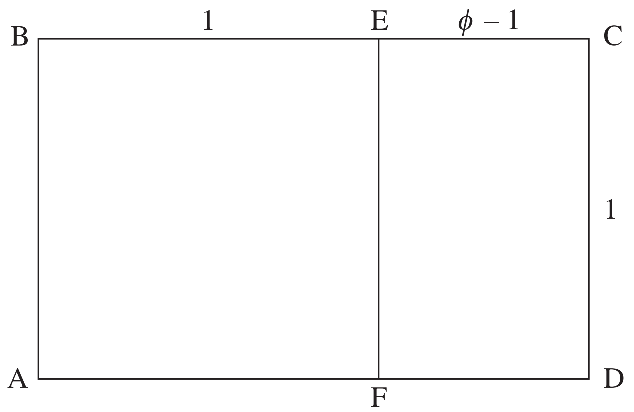


Fig. 2

Then using $\phi^2 = \phi + 1$, the ratio of the sides of rectangle ECDF may be found as follows.

$$\begin{aligned} \text{CD} : \text{DF} &= 1 : \phi - 1 \\ &= \phi : \phi^2 - \phi \\ &= \phi : 1 \end{aligned}$$

Therefore the ratio is the same for rectangle ECDF as for rectangle ABCD. So rectangle ECDF is also a Golden Rectangle.

This shows a property of a Golden Rectangle: any Golden Rectangle can be divided into a square and a smaller Golden Rectangle.

So if you remove a square ECGH from the Golden Rectangle ECDF, as shown in Fig. 3, then the ratio of the sides of rectangle FHGD is the same as that for rectangle ABCD.

This implies that the point H must lie on the line BD.

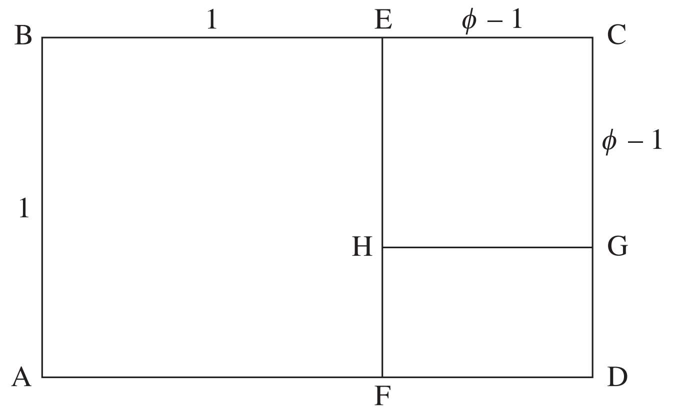


Fig. 3

Continuing in this way, removing a square from the Golden Rectangle just constructed, you can form a sequence of Golden Rectangles as shown in Fig. 4. The lines BD and CF are also shown.

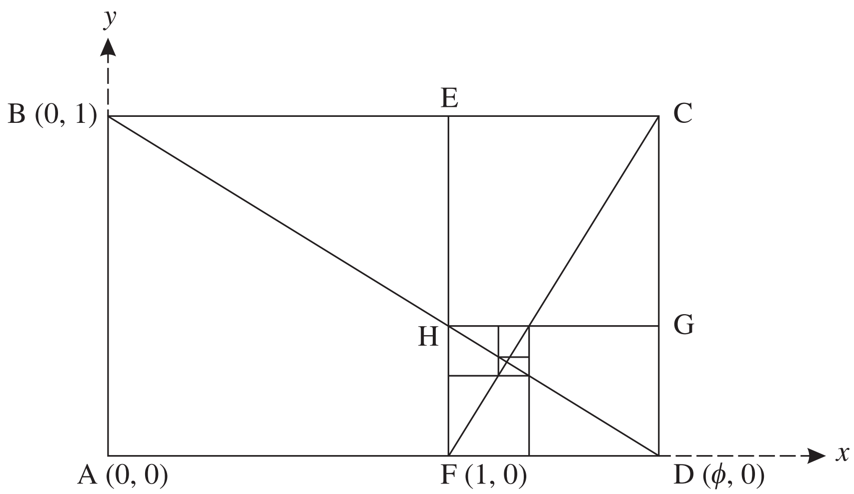


Fig. 4

This process can be continued indefinitely, each rectangle generated being similar to the original rectangle ABCD. Each square has a corner on the line BD and another corner on the line CF.

Taking A as the origin, AD as the positive x-axis and AB as the positive y-axis, as in Fig. 4, it can be shown that the equation of the line BD is

$$\phi y + x = \phi$$

and the equation of the line CF is

$$(\phi - 1)y = x - 1,$$

where $\phi = \dfrac{1 + \sqrt{5}}{2}$.

It follows that lines BD and CF are perpendicular and Q, the point of intersection of these two lines, has coordinates $\left(\frac{\phi + 1}{2\phi - 1}, \frac{\phi - 1}{2\phi - 1}\right)$.

Spirals

In Fig. 5, circular arcs have been drawn in each square. Arc AE has centre F, arc EG has centre H, and so on.

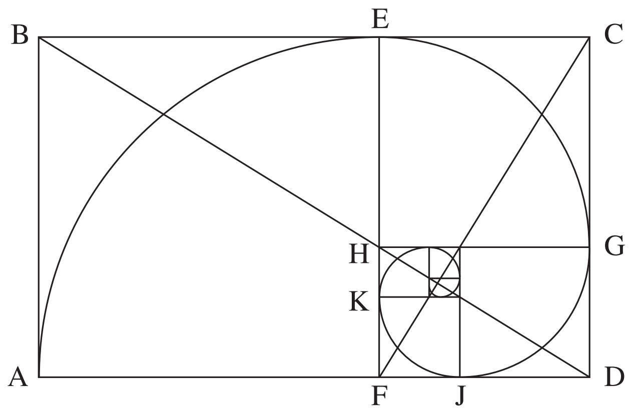


Fig. 5

In this spiral, the radius changes abruptly as it moves into each new square. For example, at the point E the radius changes from 1 unit in square ABEF to $\frac{-1 + \sqrt{5}}{2}$ units in square ECGH.

It is, however, possible to construct a spiral through points A, E, G, J, K and so on (see Fig. 6) in a different way so that these 'jumps' are smoothed out.

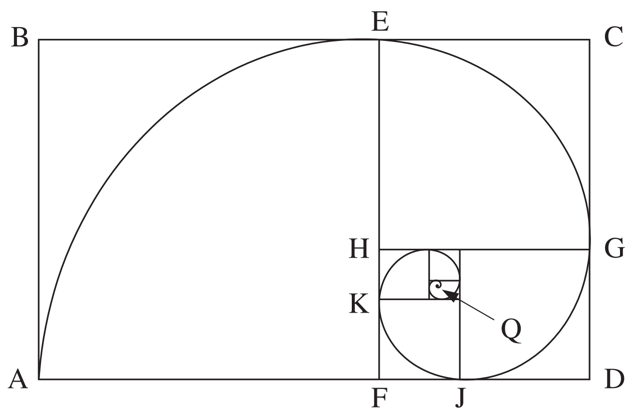


Fig. 6

As a point P moves along the resulting spiral from A towards Q, the distance QP decreases uniformly. During each full turn, the distance QP decreases by a constant factor of $\phi^4 = \left(\frac{1 + \sqrt{5}}{2}\right)^4 = \frac{7 + 3\sqrt{5}}{2} \approx 6.9$.

For example, if you draw a straight line from Q, such as the one shown in Fig. 7, and measure the distances QP_1 and QP_2 you will find that $QP_1 \approx 6.9 \times QP_2$.

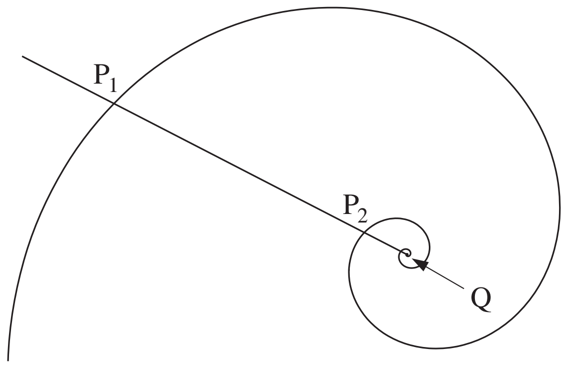


Fig. 7

This spiral is called a logarithmic spiral. It has the property that, although any two segments of the spiral differ in size, they have the same shape. If you zoomed in near the centre of the spiral and then enlarged this view, the shape would fit exactly onto the original spiral.

Fig. 8

Fig. 8 is a photograph of the shell of a nautilus, a marine creature. The shape of the shell forms a logarithmic spiral. This is just one of many examples where the Golden Ratio may be found in the natural world.

Acknowledgements:

Fig. 8 © iStockphoto.com / Horst Gossmann

4754 (C4) Applications of Advanced Mathematics

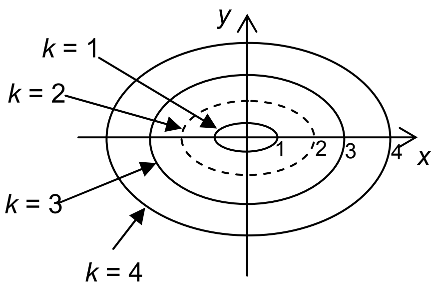
Section A

1 $3\cos\theta + 4\sin\theta = R\cos(\theta - \alpha)$ $= R(\cos\theta\cos\alpha + \sin\theta\sin\alpha)$ $\Rightarrow R\cos\alpha = 3,\ R\sin\alpha = 4$ $\Rightarrow R^2 = 3^2 + 4^2 = 25 \Rightarrow R = 5$ $\tan\alpha = 4/3 \Rightarrow \alpha = 0.9273$ $5\cos(\theta - 0.9273) = 2$ $\Rightarrow \cos(\theta - 0.9273) = 2/5$ $\theta - 0.9273 = 1.1593, -1.1593$ $\Rightarrow \theta = 2.087, -0.232$	M1 B1 M1A1 M1 A1 A1 [7]	$R = 5$ cwo and no others in the range
2(i) $(1-2x)^{-\frac{1}{2}} = 1 - \frac{1}{2}(-2x) + \frac{(-\frac{1}{2}).(-\frac{3}{2})}{2!}(-2x)^2 + ...$ $= 1 + x + \frac{3}{2}x^2 + ...$ Valid for $-1 < -2x < 1 \Rightarrow -\frac{1}{2} < x < \frac{1}{2}$	M1 A1 A1 M1 A1 [5]	binomial expansion with $p = -\frac{1}{2}$ correct expression cao
(ii) $\frac{1+2x}{\sqrt{1-2x}} = (1+2x)(1+x+\frac{3}{2}x^2+...)$ $= 1 + x + \frac{3}{2}x^2 + 2x + 2x^2 + ...$ $= 1 + 3x + \frac{7}{2}x^2 + ...$	M1 A1ft A1 [3]	substitituting their $1 + x + \frac{3}{2}x^2 + ...$ and expanding cao
3 $V = \int_1^2 \pi x^2 dy$ $y = 1 + x^2 \Rightarrow x^2 = y - 1$ $\Rightarrow V = \int_1^2 \pi(y-1)dy$ $= \pi\left[\frac{1}{2}y^2 - y\right]_1^2$ $= \pi(2 - 2 - \frac{1}{2} + 1)$ $= \frac{1}{2}\pi$	B1 M1 B1 M1 A1 [5]	 $\left[\frac{1}{2}y^2 - y\right]$ substituting limits into integrand

4(i) $\sin(\theta + 45°) = \cos\theta$ $\Rightarrow \sin\theta\cos 45 + \cos\theta\sin 45 = \cos\theta$ $\Rightarrow (1/\sqrt{2})\sin\theta + (1/\sqrt{2})\cos\theta = \cos\theta$ $\Rightarrow \sin\theta + \cos\theta = \sqrt{2}\cos\theta$ $\Rightarrow \sin\theta = (\sqrt{2} - 1)\cos\theta$ $\Rightarrow \frac{\sin\theta}{\cos\theta} = \tan\theta = \sqrt{2} - 1$ *	M1 B1 A1 M1 E1 [5]	compound angle formula sin 45 = 1/√2, cos 45 = 1/√2 collecting terms
(ii) $\tan\theta = \sqrt{2} - 1$ $\Rightarrow \theta = 22.5°,$ $202.5°$	B1 B1 [2]	 and no others in the range
5 $\frac{4}{x(x^2+4)} = \frac{A}{x} + \frac{Bx+C}{x^2+4}$ $= \frac{A(x^2+4)+(Bx+C)x}{x(x^2+4)}$ $\Rightarrow 4 = A(x^2+4)+(Bx+C)x$ $x = 0 \Rightarrow 4 = 4A \Rightarrow A = 1$ coefft of x^2: $0 = A + B \Rightarrow B = -1$ coeffts of x: $0 = C$ $\Rightarrow \frac{4}{x(x^2+4)} = \frac{1}{x} - \frac{x}{x^2+4}$	M1 M1 B1 DM1 A1 A1 [6]	correct partial fractions A=1 Substitution or equating coeffts B= –1 C= 0
6 $\text{cosec}\,\theta = 3$ $\Rightarrow \sin\theta = 1/3$ $\Rightarrow \theta = 19.47°,$ $160.53°$	M1 A1 A1 [3]	 and no others in the range

Section B

7(i) $\overrightarrow{CD} = \begin{pmatrix} -6 \\ 6 \\ 24 \end{pmatrix}$ $\overrightarrow{CB} = \begin{pmatrix} 0 \\ 20 \\ 0 \end{pmatrix}$.	B1 B1 [2]	
(ii) $\sqrt{(-6)^2 + 6^2 + 24^2}$ = 25.46 cm	M1 A1 [2]	
(iii) $\overrightarrow{CD}.\begin{pmatrix} 4 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -6 \\ 6 \\ 24 \end{pmatrix}.\begin{pmatrix} 4 \\ 0 \\ 1 \end{pmatrix} = -24 + 0 + 24 = 0$ $\overrightarrow{CB}.\begin{pmatrix} 4 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 20 \\ 0 \end{pmatrix}.\begin{pmatrix} 4 \\ 0 \\ 1 \end{pmatrix} = 0 + 0 + 0 = 0$ $\Rightarrow$ plane BCDE is $4x + z = c$ At C (say) $4\times15 + 0 = c \Rightarrow c = 60$ $\Rightarrow$ plane BCDE is $4x + z = 60$	M1 A1 B1 M1 A1 [5]	using scalar product or other equivalent methods
(iv) OG: $\mathbf{r} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 6 \\ 24 \end{pmatrix}$ AF: $\mathbf{r} = \begin{pmatrix} 0 \\ 20 \\ 0 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ -6 \\ 24 \end{pmatrix}$ At (5, 10, 40), $3\lambda = 5 \Rightarrow \lambda = 5/3$ $\Rightarrow 6\lambda = 10$, $24\lambda = 40$, so consistent. At (5, 10, 40), $3\mu = 5 \Rightarrow \mu = 5/3$ $\Rightarrow 20 - 6\mu = 10$, $24\mu = 40$, so consistent. So lines meet at (5, 10, 40)*	B1 B1 M1 E1 E1 [5]	 evaluating parameter and checking consistency. [or other methods, e.g. solving]
(v) h=40 POABC: $V = 1/3 \times 20 \times 15 \times 40$ $= 4000\text{ cm}^3$. PDEFG: $V = 1/3 \times 8 \times 6 \times (40-24)$ $= 256\text{ cm}^3$ $\Rightarrow$ vol of ornament $= 4000 - 256 = 3744\text{ cm}^3$	B1 M1 A1 A1 [4]	soi 1/3 x w x d x h used for either –condone one error both volumes correct cao

8(i) $\cos\theta = \frac{x}{k}, \sin\theta = \frac{2y}{k}$ $\cos^2\theta + \sin^2\theta = 1$ $\Rightarrow \left(\frac{x}{k}\right)^2 + \left(\frac{2y}{k}\right)^2 = 1$ $\Rightarrow \frac{x^2}{k^2} + \frac{4y^2}{k^2} = 1$ $\Rightarrow x^2 + 4y^2 = k^2$ *	M1 M1 E1 [3]	Used substitution
(ii) $\frac{dx}{d\theta} = -k\sin\theta, \frac{dy}{d\theta} = \frac{1}{2}k\cos\theta$ $\frac{dy}{dx} = \frac{dy/d\theta}{dx/d\theta} = -\frac{\frac{1}{2}k\cos\theta}{k\sin\theta}$ $= -½ \cot\theta$ $-\frac{x}{4y} = -\frac{2k\cos\theta}{4k\sin\theta} = -\frac{1}{2}\cot\theta = \frac{dy}{dx}$	M1 A1 E1	oe
or, by differentiating implicitly $2x + 8y\, dy/dx = 0$ $\Rightarrow dy/dx = -2x/8y = -x/4y$*	M1 A1 E1 [3]	
(iii) $k = 2$	B1 [1]	
(iv) 	B1 B1 B1 [3]	1 correct curve –shape and position 2 or more curves correct shape- in concentric form all 3 curves correct
(v) grad of stream path = –1/grad of contour $\Rightarrow \frac{dy}{dx} = -\frac{1}{(-x/4y)} = \frac{4y}{x}$ *	M1 E1 [2]	
(vi) $\frac{dy}{dx} = \frac{4y}{x} \Rightarrow \int\frac{dy}{y} = \int\frac{4dx}{x}$ $\Rightarrow \ln y = 4\ln x + c = \ln e^c x^4$ $\Rightarrow y = Ax^4$ where $A = e^c$. When $x = 2, y = 1 \Rightarrow 1 = 16A \Rightarrow A = 1/16$ $\Rightarrow y = x^4/16$ *	M1 A1 M1 M1 A1 E1 [6]	Separating variables $\ln y = 4 \ln x$ $(+c)$ antilogging correctly (at any stage) substituting $x = 2, y = 1$ evaluating a correct constant www

Paper B Comprehension 4754 (C4)

1	4, 1, 5, 6, 11, 17	B1 B1	for 11 and 17 for 1 and 4
2	Even, odd, odd, even, odd, odd recurs 100th term is therefore even	M1 A1	for reason www
3	$\phi^6 = (3\phi+2)+(5\phi+3) = 8\phi+5$	B1	
4	$1-EH = 1-CG = 1-(\phi-1)$ $= 2-\phi = 2-\left(\frac{1+\sqrt{5}}{2}\right)$ $= \frac{3-\sqrt{5}}{2}$	M1 A1 A1	oe
5	(i)Gradients $-\frac{1}{\phi}$ and $\frac{1}{\phi-1}$ (ii) Product of gradients: $-\frac{1}{\phi}\times\frac{1}{\phi-1} = -\frac{1}{\phi^2-\phi}$ $= -\frac{1}{1} = -1$	B1 B1 M1 E1	
6	$\frac{\phi+1}{2\phi-1} = \frac{\frac{1+\sqrt{5}}{2}+1}{1+\sqrt{5}-1}$ $= \frac{3+\sqrt{5}}{2\sqrt{5}}$ $= \frac{(3+\sqrt{5})\sqrt{5}}{2\sqrt{5}\times\sqrt{5}} = \frac{3\sqrt{5}+5}{10}$	M1 A1 E1	
7	$a+(a+d) = a+2d \quad \Rightarrow \quad a = d$ $(a+d)+(a+2d) = a+3d \quad \Rightarrow \quad a = 0$ $a = d = 0$ *	M1 M1 E1 [18]	