OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE
MATHEMATICS (MEI) 4754A

Applications of Advanced Mathematics (C4) Paper A

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
- Scientific or graphical calculator

Wednesday 9 June 2010
Afternoon

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

NOTE

- This paper will be followed by **Paper B: Comprehension**.

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2R–9L24

Section A (36 marks)

1 Express $\frac{x}{x^2 - 1} + \frac{2}{x+1}$ as a single fraction, simplifying your answer. **[3]**

2 Fig. 2 shows the curve $y = \sqrt{1 + x^2}$.

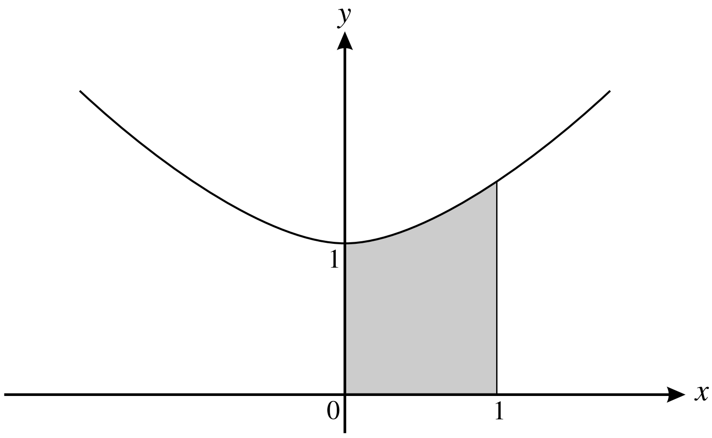

Fig. 2

(i) The following table gives some values of x and y.

x	0	0.25	0.5	0.75	1
y	1	1.0308		1.25	1.4142

Find the missing value of y, giving your answer correct to 4 decimal places.

Hence show that, using the trapezium rule with four strips, the shaded area is approximately 1.151 square units. **[3]**

(ii) Jenny uses a trapezium rule with 8 strips, and obtains a value of 1.158 square units. Explain why she must have made a mistake. **[2]**

(iii) The shaded area is rotated through 360° about the x-axis. Find the exact volume of the solid of revolution formed. **[3]**

3 The parametric equations of a curve are

$$x = \cos 2\theta, \quad y = \sin\theta\cos\theta \quad \text{for } 0 \leqslant \theta < \pi.$$

Show that the cartesian equation of the curve is $x^2 + 4y^2 = 1$.

Sketch the curve. **[5]**

4 Find the first three terms in the binomial expansion of $\sqrt{4 + x}$ in ascending powers of x.

State the set of values of x for which the expansion is valid. **[5]**

5 **(i)** Express $\dfrac{3}{(y-2)(y+1)}$ in partial fractions. **[3]**

(ii) Hence, given that x and y satisfy the differential equation

$$\frac{dy}{dx} = x^2(y-2)(y+1),$$

show that $\dfrac{y-2}{y+1} = Ae^{x^3}$, where A is a constant. **[5]**

6 Solve the equation $\tan(\theta + 45°) = 1 - 2\tan\theta$, for $0° \leqslant \theta \leqslant 90°$. **[7]**

Section B (36 marks)

7 A straight pipeline AB passes through a mountain. With respect to axes Oxyz, with Ox due East, Oy due North and Oz vertically upwards, A has coordinates (–200, 100, 0) and B has coordinates (100, 200, 100), where units are metres.

(i) Verify that $\overrightarrow{AB} = \begin{pmatrix} 300 \\ 100 \\ 100 \end{pmatrix}$ and find the length of the pipeline. **[3]**

(ii) Write down a vector equation of the line AB, and calculate the angle it makes with the vertical. **[6]**

A thin flat layer of hard rock runs through the mountain. The equation of the plane containing this layer is $x + 2y + 3z = 320$.

(iii) Find the coordinates of the point where the pipeline meets the layer of rock. **[4]**

(iv) By calculating the angle between the line AB and the normal to the plane of the layer, find the angle at which the pipeline cuts through the layer. **[5]**

[Question 8 is printed overleaf.]

8 Part of the track of a roller-coaster is modelled by a curve with the parametric equations

$$x = 2\theta - \sin\theta, \quad y = 4\cos\theta \quad \text{for } 0 \leqslant \theta \leqslant 2\pi.$$

This is shown in Fig. 8. B is a minimum point, and BC is vertical.

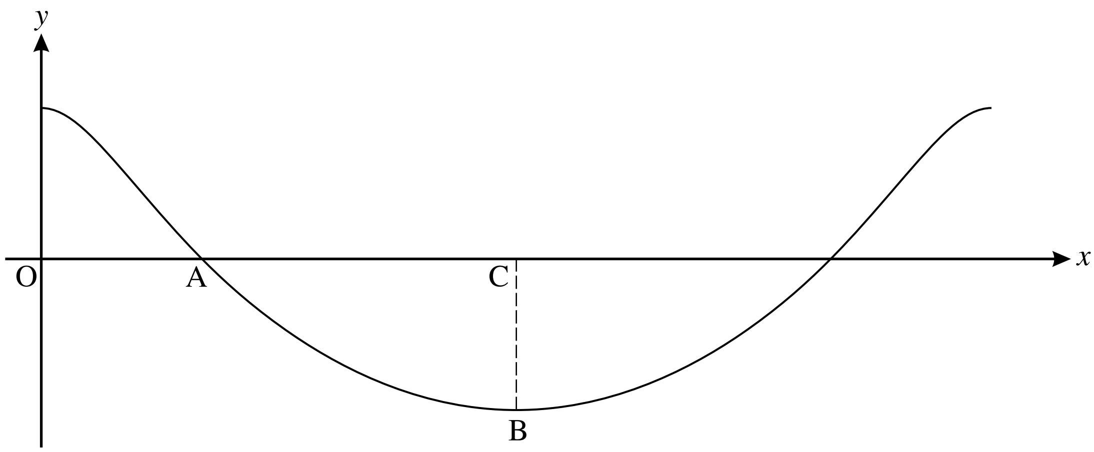

Fig. 8

(i) Find the values of the parameter at A and B.

Hence show that the ratio of the lengths OA and AC is $(\pi - 1) : (\pi + 1)$. **[5]**

(ii) Find $\frac{dy}{dx}$ in terms of θ. Find the gradient of the track at A. **[4]**

(iii) Show that, when the gradient of the track is 1, θ satisfies the equation

$$\cos\theta - 4\sin\theta = 2.$$ **[2]**

(iv) Express $\cos\theta - 4\sin\theta$ in the form $R\cos(\theta + \alpha)$.

Hence solve the equation $\cos\theta - 4\sin\theta = 2$ for $0 \leqslant \theta \leqslant 2\pi$. **[7]**

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ADVANCED GCE

MATHEMATICS (MEI) 4754B

Applications of Advanced Mathematics (C4) Paper B: Comprehension

Candidates answer on the Question Paper

OCR Supplied Materials:
- Insert (inserted)
- MEI Examination Formulae and Tables (MF2)

Other Materials Required:
- Rough paper
- Scientific or graphical calculator

Wednesday 9 June 2010
Afternoon

Duration: Up to 1 hour

Candidate Forename		Candidate Surname	

Centre Number						Candidate Number				

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the boxes above.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Write your answer to each question in the space provided. Additional paper may be used if necessary but you must clearly show your Candidate Number, Centre Number and question number(s).
- You are permitted to use a graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- The insert contains the text for use with the questions.
- You may find it helpful to make notes and do some calculations as you read the passage.
- You are **not** required to hand in these notes with your question paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **18**.
- This document consists of **4** pages. Any blank pages are indicated.

Examiner's Use Only:	
1	
2	
3	
4	
5	
Total	

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- This paper should be attached to the candidate's paper A script before sending to the examiner.

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4R–0B20

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1 The train journey from Swansea to London is 307 km and that by road is 300 km. Carry out the calculations performed on the First Great Western website to estimate how much lower the carbon dioxide emissions are when travelling by rail rather than road. **[2]**

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2 The equation of the curve in Fig. 3 is

$$y = \frac{1}{10^4}\left(x^3 - 100x^2 - 10\,000x + 2\,100\,100\right).$$

Calculate the speed at which the car has its lowest carbon dioxide emissions and the value of its emissions at that speed. **[6]**

[An answer obtained from the graph will be given no marks.]

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3 **(i)** In line 109 the carbon dioxide emissions for a particular train journey from Exeter to London are estimated to be 3.7 tonnes. Obtain this figure. **[2]**

(ii) The text then goes on to state that the emissions per extra passenger on this journey are less than $\frac{1}{2}$ kg. Justify this figure. **[1]**

(i) ....

....

(ii) ....

....

For Examiner's Use

4 The daily number of trains, n, on a line in another country may be modelled by the function defined below, where P is the annual number of passengers.

$n = 10$ for $0 \leqslant P < 10^6$
$n = 11$ for $10^6 \leqslant P < 1.5 \times 10^6$
$n = 12$ for $1.5 \times 10^6 \leqslant P < 2 \times 10^6$
$n = 13$ for $2 \times 10^6 \leqslant P < 2.5 \times 10^6$
$n = 14$ for $2.5 \times 10^6 \leqslant P < 3 \times 10^6$
… and so on …

(i) Sketch the graph of n against P. **[2]**

(ii) Describe, in words, the relationship between the daily number of trains and the annual number of passengers. **[2]**

(i)

(ii) ....

....

....

[Question 5 is printed overleaf.]

For Examiner's Use

5 The FGW website gives the conversion factor for miles to kilometres to 7 significant figures.

> "We got the distance between the two stations by road from theaa.com. We then converted this distance to kilometres by multiplying it by 1.609344."

Suppose this conversion factor is applied to a distance of exactly 100 miles.

State which one of the following best expresses the level of accuracy for the distance in metric units, justifying your answer.

A : to the nearest millimetre
B : to the nearest 10 centimetres
C : to the nearest metre **[3]**

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RECOGNISING ACHIEVEMENT

ADVANCED GCE

MATHEMATICS (MEI) 4754B

Applications of Advanced Mathematics (C4) Paper B: Comprehension
INSERT

Wednesday 9 June 2010
Afternoon

Duration: Up to 1 hour

INFORMATION FOR CANDIDATES

- This insert contains the text for use with the questions.
- This document consists of **8** pages. Any blank pages are indicated.

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Greener travel

Introduction

As the evidence for global warming becomes more compelling, many people are looking at the opportunities in their daily lives to reduce the impact they are having on the environment. One common choice is whether to travel by train or to drive. This article compares the carbon dioxide (CO_2) emissions from these two forms of transport.

A simple model

Several railway companies' websites include estimates of the reduction in carbon dioxide emissions per passenger for those who travel by train. For example, on the First Great Western website, you can enter the journey, say Exeter to London, and be told "Train 16.83 kg, Car 41.7 kg". The implication is that by choosing to travel this route by train rather than by car you prevent 24.87 kg of carbon dioxide entering the atmosphere.

First Great Western (FGW) also tell you how the calculations are carried out.

We calculated the CO_2 emissions per passenger for trains as follows:

We got the distance between the two stations from the National Rail Timetable. We then converted this distance to kilometres by multiplying it by 1.609344.

We calculated the carbon emissions by multiplying the distance in kilometres by 0.0602, the conversion factor for rail (Defra3).

We calculated the CO_2 emissions per passenger for cars as follows:

We got the distance between the two stations by road from theaa.com.

We then converted this distance to kilometres by multiplying it by 1.609344.

We calculated the carbon emissions for the journey by multiplying the distance in kilometres by 0.2095, the conversion factor for an average petrol car (Defra3).

We then divided this figure by 1.58, the average loading for a car (DfT4), to find the carbon emissions for this journey per passenger.

We calculated the carbon saving of taking the train instead of the car by simply subtracting the rail carbon emissions from the car carbon emissions.

Note Defra3 is a document published by the Department for Food and Rural Affairs.
DfT4 is a document published by the Department for Transport.

The fact that FGW have explained how the calculations are carried out is good practice. It allows people to look carefully at them, as is being done in this article, and to make their own judgements.

Outcomes from the website

The outcomes reported on the website raise questions.

Accuracy

The carbon dioxide emissions are given to 3 significant figures in one case and 4 in the other. This suggests that the figures on which they are based are all very accurate. Is this level of accuracy justified? The discussion in the rest of this article suggests that the answer is “No”. The level of variability is such that it would be hard to justify even 2 significant figures.

The target audience

The website appears to be answering an individual traveller who asks the following question.

> *How much less carbon dioxide will enter the atmosphere if I decide to travel by train rather than by car?*

However, the calculations answer a different question.

> *How much less carbon dioxide on average will enter the atmosphere for a train traveller compared to a car traveller for the journey in question?*

The answers will usually be very different.

Carbon dioxide emissions

This article compares the carbon dioxide emissions that result from travelling by car (using petrol or diesel fuel) and by train (using diesel fuel). When people talk about ‘carbon emissions’ they are talking about carbon dioxide (together with any carbon monoxide that is also produced).

The main elements in diesel are carbon and hydrogen; the numbers of atoms are in the ratio of about 12 : 23. When diesel is burnt, the main products are water and carbon dioxide. One litre of diesel has mass 0.85 kg; when it is burnt it results in about 2.7 kg of carbon dioxide entering the atmosphere. (There are also small quantities of other elements which can have damaging effects on the environment.)

Petrol is lighter; one litre has mass 0.72 kg and produces 2.3 kg of carbon dioxide. A litre of petrol produces less carbon dioxide than a litre of diesel. However, because diesel engines are more efficient, they use less fuel and so create less carbon dioxide, typically by 10% to 20%.

The carbon dioxide emissions for a car

Particular cars

The calculation for the carbon dioxide emissions from a car is based on the Defra figure that an ‘average car’ emits 0.2095 kg of carbon dioxide per kilometre. This raises the question of what is an ‘average car’ in terms of emissions.

At the time of writing, the least polluting cars are rated at 99 grams per kilometre, the most polluting at 495. The results of a recent survey of the emissions of different models of new cars are given in Table 1.

Emission rating ($g\,km^{-1}$)	% of models
0 – 100	0.1
101 – 120	4.5
121 – 150	18.6
151 – 165	17.1
166 – 185	18.8
186 – 225	24.7
> 225	16.1

Table 1 (Source: www.carpages.co.uk)

The First Great Western website appears to tell car travellers the levels of carbon dioxide emissions for which they would be responsible. However, it does not ask them what type of car they would be driving. Table 2 gives the distributions of the carbon dioxide emissions for hatchbacks and SUVs (Sport Utility Vehicles).

Emission rating (g km^{-1})	**% of hatchback models**	**% of SUV models**
0 – 100	0.4	0
101 – 120	12.0	0
121 – 150	37.7	0
151 – 165	23.1	0
166 – 185	15.3	6.8
186 – 225	10.6	42.1
> 225	0.8	51.1

Table 2 (Source: www.carpages.co.uk)

The contrast is striking. It is clear that the carbon dioxide emissions from a car are highly dependent on its type.

Average emissions

The calculation on the FGW website is based on an 'average' emission of 0.2095 kilograms of carbon dioxide per kilometre, the figure provided by Defra.

The median of the distribution in Table 1 is in the class labelled 166 – 185 g km^{-1}, and so is less than the Defra figure. The data, as presented in the table, do not allow a mean value to be calculated, but those who carried out the survey say that it is 186 g km^{-1}; this again is less than the Defra figure.

There are a number of possible reasons for this difference and it may well be that the rather higher Defra figure is actually more realistic.

- The data in Table 1 are for new cars, many of which have lower carbon dioxide emissions than older cars that are still on the road.
- Table 1 gives the distribution of models of cars, rather than that of actual cars on the road.
- The emissions figures in the table would have been obtained when the cars were being driven in test conditions. The emissions depend on the speed at which a car is being driven. Fig. 3 shows a typical graph of the carbon dioxide emissions of a car plotted against its speed.

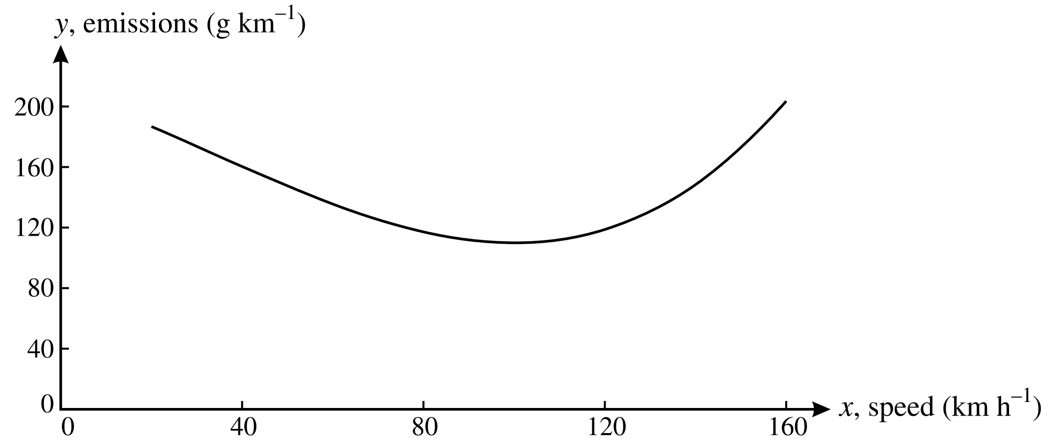

Fig. 3

The carbon dioxide emissions for a train

The figures for train travel also present problems.

Passenger trains vary from single coach vehicles to those with eight or more coaches. Some are powered by diesel, others by electricity. Electricity may be generated by coal-fired power stations with high carbon dioxide emissions; by contrast generation from nuclear, hydro-electric or wind power produces no emissions at all. However, in general electric trains are responsible for much lower emissions.

The FGW website calculation uses the Defra figure of 0.0602 kilograms of carbon dioxide per passenger per kilometre. This too is an average figure, and so one that does not correspond to any particular train journey.

It is, however, possible to be reasonably precise about a particular journey. The example at the start of this article was the journey from Exeter to London with FGW. This defines the type of train being used, in this case an HST (High Speed Train), and, subject to the number of stops it makes on the way, its speed at each part of the journey. The HST uses diesel fuel and so its carbon dioxide emissions may be compared with those from a car.

Three things need to be considered when estimating the carbon dioxide emissions: the distance travelled, the number of passengers (and so their weight) and the number of stops.

All the HSTs run by FGW have recently been fitted with new motors. Reasonable estimates of emissions are shown in Table 4.

Cause	Emission rate
Distance travelled, d km	10 kg per kilometre
Number of passengers, p	1.5 grams per passenger per kilometre
Number of stops, s	200 kg per stop

Table 4

(A lot of energy is required to get a train up to speed after a stop.)

So the total emissions, E kg, are given by the formula: $E = (10 + 0.0015p)d + 200s$.

The distance, by rail, from Exeter to London is 279 km, so the emissions for a journey with 4 stops and 250 passengers (a half full train) are about 3.7 tonnes.

Note that the emissions per extra passenger on the Exeter to London journey are less than $\frac{1}{2}$ kg.

There are two ways of looking at this journey.

- An individual view. Someone considering how to travel can argue

  Since the train is running anyway, I will have almost no effect on its carbon dioxide emissions. By contrast, if I travel by my car, with carbon dioxide emissions of about 140 grams per kilometre, I will be responsible for putting about 44 kg of carbon dioxide into the atmosphere.

  (The journey by road is about 314 km according to theaa.com.)

- An average view. Someone looking at the overall impact of this train on the environment might say

  The total emissions of 3.7 tonnes need to be averaged out over the 250 passengers, giving a figure of 14.8 kg per passenger.

So the outcomes of the calculations can be interpreted in two ways. If you take the individual view, FGW are greatly understating the value to the environment of people travelling by train. On the other hand, if you take the average view, their figure of 16.83 kg of carbon dioxide is similar to the 14.8 kg for the particular journey in the example and so is realistic.

Which of the two is the better interpretation?

To answer that question you need to look at the relationship between overcrowding and the number of trains being run. This is a complex issue; the following section explores only part of it and considers a simple initial model.

Crowded trains

It can be argued that if many more people decided to travel by train, the trains would become overcrowded and eventually this would result in more trains running. At the time of writing, 17 High Speed Trains run every weekday from Exeter to London. An increase in passenger numbers might cause this to be increased to 18 and a further increase to 19 and so on.

Thus, although an individual decision to travel by train results in very little extra carbon dioxide entering the atmosphere, when sufficient people regularly make the same decision and so cause an extra train to run, then their combined decisions have resulted in the extra emissions from the new train.

The daily number of trains might thus be modelled as a step function of the daily number of passengers. Such a step function is illustrated in the graph in Fig. 5. (There is no simple algebraic formula for such a step function.)

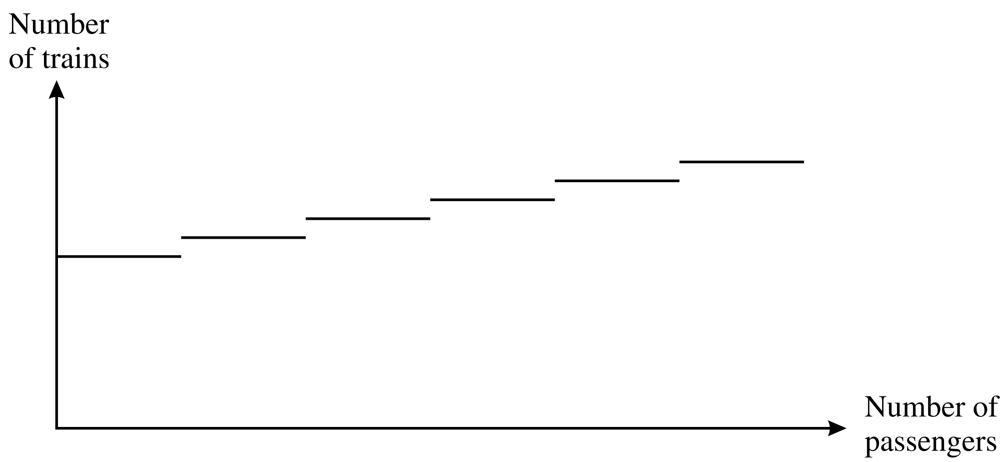

Fig. 5

If this model were realistic, then the average view would probably be preferable. However, this is not the case.

In recent years the number of people travelling by train has increased substantially and is continuing to do so, but this has not resulted in a corresponding increase in the number of trains running; rather they have become more and more crowded. There are two main reasons for this.

- There are not enough suitable trains. Virtually all the available trains in the country are now being fully used.
- In many places the railway lines are used to capacity. To run more trains would require major investment in improved signalling, or even additional railway lines.

In the short term it is likely that the response to overcrowding will often be to run longer trains. Adding an extra coach to a train will certainly increase its carbon dioxide emissions, but to nothing like the extent that would result from running an extra train. So the individual view is probably closer to reality than the average view.

It is also possible that, faced with the necessity of substantial long-term investment to deal with overcrowding, a decision will be made to electrify significant sections of the railway, and this would reduce carbon dioxide emissions.

OCR would like to thank First Great Western for their help in preparing this article.

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GCE

Mathematics (MEI)

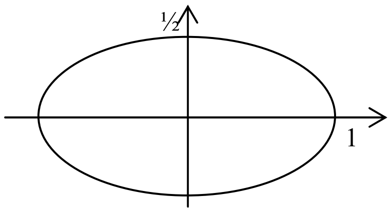
Advanced GCE **4754A**

Applications of Advanced Mathematics (C4) Paper A

Mark Scheme for June 2010

Section A

1 $\frac{x}{x^2-1}+\frac{2}{x+1}=\frac{x}{(x-1)(x+1)}+\frac{2}{x+1}$	B1	$x^2 - 1 = (x + 1)(x - 1)$
$=\frac{x+2(x-1)}{(x-1)(x+1)}$	M1	correct method for addition of fractions
$=\frac{(3x-2)}{(x-1)(x+1)}$	A1	or $\frac{(3x-2)}{x^2-1}$ do not isw for incorrect subsequent cancelling
or $\frac{x}{x^2-1}+\frac{2}{x+1}=\frac{x(x+1)+2(x^2-1)}{(x^2-1)(x+1)}$	M1	correct method for addition of fractions
$=\frac{3x^2+x-2}{(x^2-1)(x+1)}$		
$=\frac{(3x-2)(x+1)}{(x^2-1)(x+1)}$	B1	$(3x - 2)(x + 1)$
$=\frac{(3x-2)}{(x^2-1)}$	A1 [3]	accept denominator as x^2-1 or(x-1)(x+1) do not isw for incorrect subsequent cancelling
2(i) When $x = 0.5$, $y = 1.1180$	B1	4dp
$\Rightarrow$ $A \approx 0.25/2\{1+1.4142+2(1.0308+1.1180+1.25)\}$	M1	
$= 0.25 \times 4.6059 = 1.151475$		(0.125 x 9.2118)
$= 1.151$ (3 d.p.)*	E1 [3]	need evidence
(ii) Explain that the area is an over-estimate. *or* The curve is below the trapezia, so the area is an over- estimate.	B1	or use a diagram to show why
This becomes less with more strips. *or* Greater number of strips improves accuracy so becomes less	B1 [2]	
(iii) $V=\int_0^1 \pi y^2 dx$		
$=\int_0^1 \pi(1+x^2)dx$	M1	allow limits later
$=\pi\left[(x+x^3/3)\right]_0^1$	B1	$x + x^3/3$
$=1\frac{1}{3}\pi$	A1 [3]	exact

3 $y = \sin\theta\cos\theta = \frac{1}{2}\sin 2\theta$ $x = \cos 2\theta$ $\sin^2 2\theta + \cos^2 2\theta = 1$ $\Rightarrow x^2 + (2y)^2 = 1$ $\Rightarrow x^2 + 4y^2 = 1$ *	M1 M1 E1	use of $\sin 2\theta$
or $x^2 + 4y^2 = (\cos 2\theta)^2 + 4(\sin\theta\cos\theta)^2$ $= \cos^2 2\theta + \sin^2 2\theta$ $= 1$ *	M1 M1 E1	substitution use of $\sin 2\theta$
or $\cos 2\theta = 2\cos^2\theta - 1$ $\cos^2\theta = (x+1)/2$ $\cos 2\theta = 1 - 2\sin^2\theta$ $\sin^2\theta = (1-x)/2$ $y^2 = \sin^2\theta\cos^2\theta = \left(\frac{1-x}{2}\right)\left(\frac{x+1}{2}\right)$ $y^2 = (1-x^2)/4$ $x^2 + 4y^2 = 1$*	M1 M1 E1	for both
or $x = \cos 2\theta = \cos^2\theta - \sin^2\theta$ $x^2 = \cos^4\theta - 2\sin^2\theta\cos^2\theta + \sin^4\theta$ $y^2 = \sin^2\theta\cos^2\theta$ $x^2 + 4y^2 = \cos^4\theta - 2\sin^2\theta\cos^2\theta + \sin^4\theta + 4\sin^2\theta\cos^2\theta$ $= (\cos^2\theta + \sin^2\theta)^2$ $= 1$*	M1 M1 E1	correct use of double angle formulae correct squaring and use of $\sin^2\theta + \cos^2\theta = 1$
	M1 A1 [5]	ellipse correct intercepts

4 $\sqrt{4+x}=2(1+\frac{x}{4})^{\frac{1}{2}}$	M1	dealing with √4 (or terms in $4^{\frac{1}{2}}$, $4^{\frac{-1}{2}}$,…etc)
$= 2(1+\frac{1}{2}.\frac{x}{4}+\frac{\frac{1}{2}.-\frac{1}{2}}{2}(\frac{x}{4})^2+...)$	M1 A1	correct binomial coefficients correct **unsimplified** expression for $(1+x/4)^{\frac{1}{2}}$ or $(4+x)^{\frac{1}{2}}$
$= 2(1+\frac{1}{8}x-\frac{1}{128}x^2+...)$		
$= 2+\frac{1}{4}x-\frac{1}{64}x^2+...$	A1	cao
Valid for $-1 < x/4 < 1$ $\Rightarrow$ $-4 < x < 4$	B1 [5]	
5(i) $\frac{3}{(y-2)(y+1)}=\frac{A}{y-2}+\frac{B}{y+1}$ $=\frac{A(y+1)+B(y-2)}{(y-2)(y+1)}$		
$\Rightarrow$ $3 = A(y+1) + B(y-2)$ $y = 2 \Rightarrow 3 = 3A \Rightarrow A = 1$ $y = -1 \Rightarrow 3 = -3B \Rightarrow B = -1$	M1 A1 A1 [3]	substituting, equating coeffs or cover up
(ii) $\frac{dy}{dx}=x^2(y-2)(y+1)$		
$\Rightarrow$ $\int\frac{3\,\mathrm{d}y}{(y-2)(y+1)}=\int 3x^2\,\mathrm{d}x$	M1	separating variables
$\Rightarrow$ $\int(\frac{1}{(y-2)}-\frac{1}{y+1})\,\mathrm{d}y=\int 3x^2\,\mathrm{d}x$		
$\Rightarrow$ $\ln(y-2) - \ln(y+1) = x^3 + c$	B1ft B1	$\ln(y-2) - \ln(y+1)$ ft their A,B $x^3 + c$
$\Rightarrow$ $\ln\left(\frac{y-2}{y+1}\right)=x^3+c$		
$\Rightarrow$ $\frac{y-2}{y+1}=e^{x^3+c}=e^{x^3}.e^c=Ae^{x^3}$ *	M1 E1 [5]	anti-logging including c www
6 $\tan(\theta+45)=\frac{\tan\theta+\tan 45}{1-\tan\theta\tan 45}$	M1	oe using sin/cos
$=\frac{\tan\theta+1}{1-\tan\theta}$	A1	
$\Rightarrow$ $\frac{\tan\theta+1}{1-\tan\theta}=1-2\tan\theta$		
$\Rightarrow$ $1+\tan\theta = (1-2\tan\theta)(1-\tan\theta)$ $= 1 - 3\tan\theta + 2\tan^2\theta$	M1 A1	multiplying up and expanding any correct one line equation
$\Rightarrow$ $0 = 2\tan^2\theta - 4\tan\theta = 2\tan\theta(\tan\theta - 2)$	M1	solving quadratic for $\tan\theta$ oe
$\Rightarrow$ $\tan\theta = 0$ or 2 $\Rightarrow$ $\theta = 0$ or 63.43	A1A1 [7]	www -1 extra solutions in the range

Section B

7(i) $\overrightarrow{AB} = \begin{pmatrix} 100-(-200) \\ 200-100 \\ 100-0 \end{pmatrix} = \begin{pmatrix} 300 \\ 100 \\ 100 \end{pmatrix}$ * AB = $\sqrt{(300^2 + 100^2 + 100^2)}$ = 332 m	E1 M1 A1 [3]	 accept surds
(ii) $\mathbf{r} = \begin{pmatrix} -200 \\ 100 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 300 \\ 100 \\ 100 \end{pmatrix}$ Angle is between $\begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ $\Rightarrow \quad \cos\theta = \frac{3\times 0 + 1\times 0 + 1\times 1}{\sqrt{11}\sqrt{1}} = \frac{1}{\sqrt{11}}$ $\Rightarrow \quad \theta = 72.45°$	B1B1 M1 M1 A1 A1 [6]	oe …and $\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$ complete scalar product method(including cosine) for correct vectors 72.5° or better, accept 1.26 radians
(iii) Meets plane of layer when $(-200+300\lambda) + 2(100+100\lambda) + 3\times 100\lambda = 320$ $\Rightarrow \quad 800\lambda = 320$ $\Rightarrow \quad \lambda = 2/5$ $\mathbf{r} = \begin{pmatrix} -200 \\ 100 \\ 0 \end{pmatrix} + \frac{2}{5}\begin{pmatrix} 300 \\ 100 \\ 100 \end{pmatrix} = \begin{pmatrix} -80 \\ 140 \\ 40 \end{pmatrix}$ so meets layer at (–80, 140, 40)	M1 A1 M1 A1 [4]	
(iv) Normal to plane is $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ Angle is between $\begin{pmatrix} 3 \\ 1 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ $\Rightarrow \quad \cos\theta = \frac{3\times 1 + 1\times 2 + 1\times 3}{\sqrt{11}\sqrt{14}} = \frac{8}{\sqrt{11}\sqrt{14}} = 0.6446..$ $\Rightarrow \quad \theta = 49.86°$ $\Rightarrow \quad$ angle with layer = 40.1°	B1 M1A1 A1 A1 [5]	 complete method ft 90-theirθ accept radians

8(i) At A, $y = 0 \Rightarrow 4\cos\theta = 0$, $\theta = \pi/2$ At B, $\cos\theta = -1$, $\Rightarrow \theta = \pi$ x-coord of A $= 2\times\pi/2 - \sin\pi/2 = \pi - 1$ x-coord of B $= 2\times\pi - \sin\pi = 2\pi$ $\Rightarrow$ OA $= \pi - 1$, AC $= 2\pi - \pi + 1 = \pi + 1$ $\Rightarrow$ ratio is $(\pi - 1):(\pi + 1)$ *	B1 B1 M1 A1 E1 [5]	 for either A or B/C for both A and B/C
(ii) $\dfrac{dy}{d\theta} = -4\sin\theta$ $\dfrac{dx}{d\theta} = 2 - \cos\theta$ $\Rightarrow \dfrac{dy}{dx} = \dfrac{dy/d\theta}{dx/d\theta}$ $= -\dfrac{4\sin\theta}{2-\cos\theta}$ At A, gradient = $-\dfrac{4\sin(\pi/2)}{2-\cos(\pi/2)} = -2$	B1 M1 A1 A1 [4]	either $dx/d\theta$ or $dy/d\theta$ www
(iii) $\dfrac{dy}{dx} = 1 \Rightarrow -\dfrac{4\sin\theta}{2-\cos\theta} = 1$ $\Rightarrow -4\sin\theta = 2 - \cos\theta$ $\Rightarrow \cos\theta - 4\sin\theta = 2$ *	M1 E1 [2]	their $dy/dx = 1$
(iv) $\cos\theta - 4\sin\theta = R\cos(\theta + \alpha)$ $= R(\cos\theta\cos\alpha - \sin\theta\sin\alpha)$ $\Rightarrow R\cos\alpha = 1$, $R\sin\alpha = 4$ $\Rightarrow R^2 = 1^2 + 4^2 = 17$, $R = \sqrt{17}$ $\tan\alpha = 4$, $\alpha = 1.326$ $\Rightarrow \sqrt{17}\cos(\theta + 1.326) = 2$ $\Rightarrow \cos(\theta + 1.326) = 2/\sqrt{17}$ $\Rightarrow \theta + 1.326 = 1.064, 5.219, 7.348$ $\Rightarrow \theta = (-0.262), 3.89, 6.02$	 M1 B1 M1 A1 M1 A1 A1 [7]	 corr pairs accept 76.0°, 1.33 radians inv cos $(2/\sqrt{17})$ ft their R for method -1 extra solutions in the range

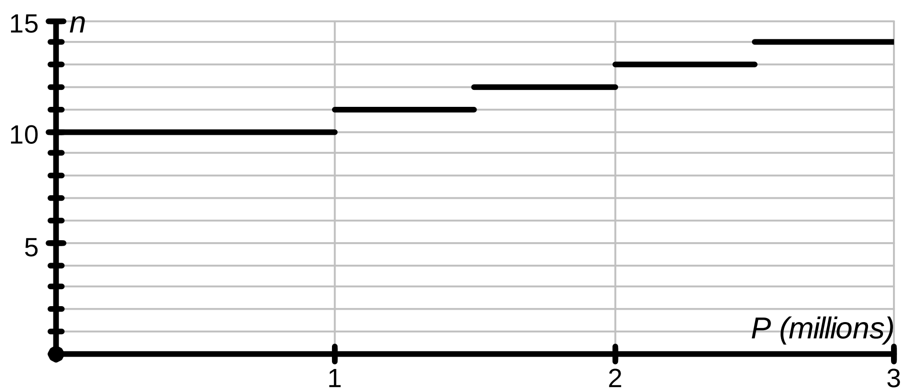
GCE

Mathematics (MEI)

Advanced GCE **4754B**

Applications of Advanced Mathematics (C4) Paper B: Comprehension Greener Travel

Mark Scheme for June 2010

1.		Rail: $307\times0.0602=18.4814=18.5$ kg (3 sf) Road: $300\times0.2095\div1.58=39.77...=39.8$ kg (3 sf) Reduction = 21.3 kg	**B1** for either **B1**
2.		$y=\frac{1}{10^4}(x^3-100x^2-10000x+2100100)$.. $\Rightarrow\frac{dy}{dx}=\frac{1}{10^4}(3x^2-200x-10\ 000)$ $\frac{dy}{dx}=0\ \Rightarrow\ 3x^2-200x-10\ 000=0$ $(3x+100)(x-100)=0$ $x=100$ (or $x=-\frac{100}{3}$) The graph shows the minimum emission occurs at speed of 100 km hour^{-1} Substituting $x=100$ gives y =110.01 Minimum rate of emission is 110 grams per km.	**M1** **A1** **M1** solving quadratic **A1** **A1** or $\frac{d^2y}{dx^2}$ justify min **A1**
3.	**(i)**	Substituting $p=250, d=279, s=4$ in $E=(10+0.0015p)d+200s$ $\Rightarrow E=3694.625$ (in kg) So emissions are 3.7 tonnes to 2 s.f. *	**M1** subst **E1**
	(ii)	Emission rate = 1.5 g km^{-1} Distance = 279 km Emissions = 1.5×279 =418.5 g = 0.42 kg (2 s.f.), and so is less than ½ kg. *or* p=251 in formula gives E=3695.0435, difference=0.4185kg<0.5kg	 **E1**
4.	**(i)**	 **Approximate Step graph** **Correct with scales shown**	 **M1** **A1**
	(ii)	There is a basic service of 10 trains a day for up to 1 million passengers per year. For every half million extra passengers above 1 million, an extra daily train is provided.	**B1** **B1**

5.		100 miles = 1.609344×100 km = 160 km 934 m 40 cm So it appears to give the answer to the nearest 10 cm (option B).	**M1** **A1** **A1** **[18]**