RECOGNISING ACHIEVEMENT

GCE

Mathematics (MEI)

Advanced GCE

Unit **4754A:** Applications of Advanced Mathematics: Paper A

Mark Scheme for January 2011

Oxford Cambridge and RSA Examinations

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Section A

1(i) x: -2, -1, 0, 1, 2 y: 1.0655, 1.1696, 1.4142, 1.9283, 2.8964 $A \approx$ ½×1{1.0655+2.8964+2(1.1696+1.4142+1.9283)} = 6.493	B2,1,0 M1 A1 [4]	table values formula 6.5 or better www
(ii) Smaller, as the trapezium rule is an over-estimate in this case and the error is less with more strips	B1 B1 [2]	
2 $x=\dfrac{1}{1+t} \Rightarrow 1+t=\dfrac{1}{x}$	M1	attempt to solve for t
$\Rightarrow$ $t=\dfrac{1}{x}-1$	A1	oe
$y=\dfrac{1-t}{1+2t}=\dfrac{1-\frac{1}{x}+1}{1+\frac{2}{x}-2}$	M1	substituting for t in terms of x
$=\dfrac{2-\frac{1}{x}}{\frac{2}{x}-1}=\dfrac{2x-1}{2-x}$	M1 A1 [5]	clearing subsidiary fractions
3 $(3-2x)^{-3}=3^{-3}(1-\frac{2}{3}x)^{-3}$	M1	dealing with the '3'
$=\dfrac{1}{27}(1+(-3)(-\frac{2}{3}x)+\dfrac{(-3)(-4)}{2}(-\frac{2}{3}x)^2+\ldots)$	B1	correct binomial coeffs
$=\dfrac{1}{27}(1+2x+\dfrac{8}{3}x^2+\ldots)$	B2,1,0	1, 2, 8/3 oe
$=\dfrac{1}{27}+\dfrac{2}{27}x+\dfrac{8}{81}x^2+\ldots$	A1	cao
Valid for $-1<-\dfrac{2}{3}x<1$	M1	
$\Rightarrow$ $-\dfrac{3}{2}<x<\dfrac{3}{2}$	A1 [7]	

4(i) $\overrightarrow{AB} = \begin{pmatrix} 2 \\ 3 \\ -5 \end{pmatrix}, \overrightarrow{BC} = \begin{pmatrix} 5 \\ 0 \\ 2 \end{pmatrix}$ $\overrightarrow{AB}.\overrightarrow{BC} = \begin{pmatrix} 2 \\ 3 \\ -5 \end{pmatrix}.\begin{pmatrix} 5 \\ 0 \\ 2 \end{pmatrix} = 2\times5+3\times0+(-5)\times2=0$ $\Rightarrow$ AB is perpendicular to BC.	B1 B1 M1E1 [4]	
(ii) AB = $\sqrt{(2^2 + 3^2 + (-5)^2)} = \sqrt{38}$ BC = $\sqrt{(5^2 + 0^2 + 2^2)} = \sqrt{29}$ Area = ½ $\times\sqrt{38} \times \sqrt{29}$ = ½ $\sqrt{1102}$ or 16.6 units^2	M1 B1 A1 [3]	complete method ft lengths of both AB, BC oe www
5 LHS = $\dfrac{2\sin\theta\cos\theta}{1+2\cos^2\theta-1}$ $=\dfrac{2\sin\theta\cos\theta}{2\cos^2\theta}$ $=\dfrac{\sin\theta}{\cos\theta}=\tan\theta=\text{RHS}$	M1 M1 E1 [3]	one correct double angle formula used cancelling cos θ's
6(i) $\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -8-3\lambda \\ -2 \\ 6+\lambda \end{pmatrix}$ Substituting into plane equation: $2(-8-3\lambda)-3(-2)+6+\lambda=11$ $\Rightarrow$ $-16-6\lambda+6+6+\lambda=11$ $\Rightarrow$ $5\lambda=-15$, $\lambda=-3$ So point of intersection is (1, –2, 3)	B1 M1 A1 A1ft [4]	
(ii) Angle between $\begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} -3 \\ 0 \\ 1 \end{pmatrix}$ $\cos\theta = \dfrac{2\times(-3)+(-3)\times0+1\times1}{\sqrt{14}\sqrt{10}}$ = (–)0.423 $\Rightarrow$ acute angle = 65°	B1 M1 A1 A1 [4]	 allow M1 for a complete method only for any vectors

Section B

7(i) When $t = 0$, $v = 5(1 - e^0) = 0$ As $t \to \infty$, $e^{-2t} \to 0$, $\Rightarrow$ $v \to 5$ When $t = 0.5$, $v = 3.16$ m s^{-1}	E1 E1 B1 [3]	
(ii) $\frac{dv}{dt} = 5\times(-2)e^{-2t} = 10e^{-2t}$ $10 - 2v = 10 - 10(1 - e^{-2t}) = 10e^{-2t}$ $\Rightarrow$ $\frac{dv}{dt} = 10 - 2v$	B1 M1 E1 [3]	
(iii) $\frac{dv}{dt} = 10 - 0.4v^2$ $\Rightarrow$ $\frac{10}{100-4v^2}\frac{dv}{dt} = 1$ $\Rightarrow$ $\frac{10}{25-v^2}\frac{dv}{dt} = 4$ $\Rightarrow$ $\frac{10}{(5-v)(5+v)}\frac{dv}{dt} = 4$ * $\frac{10}{(5-v)(5+v)} = \frac{A}{5-v} + \frac{B}{5+v}$ $\Rightarrow$ $10 = A(5+v) + B(5-v)$ $v = 5 \Rightarrow 10 = 10A \Rightarrow A = 1$ $v = -5 \Rightarrow 10 = 10B \Rightarrow B = 1$ $\Rightarrow$ $\frac{10}{(5-v)(5+v)} = \frac{1}{5-v} + \frac{1}{5+v}$ $\Rightarrow$ $\int (\frac{1}{5-v} + \frac{1}{5+v})\,dv = 4\int dt$ $\Rightarrow$ $\ln(5+v) - \ln(5-v) = 4t + c$ when $t = 0$, $v = 0$, $\Rightarrow 0 = 4\times0 + c \Rightarrow c = 0$ $\Rightarrow$ $\ln\left(\frac{5+v}{5-v}\right) = 4t$ $\Rightarrow$ $t = \frac{1}{4}\ln\left(\frac{5+v}{5-v}\right)$ *	M1 E1 M1 A1 M1 A1 A1 E1 [8]	for both A=1,B=1 separating variables correctly and indicating integration ft their $A,B,$ condone absence of c ft finding c from an expression of correct form
(iv) When $t \to \infty$, $e^{-4t} \to 0$, $\Rightarrow$ $v \to 5/1 = 5$ when $t = 0.5$, $t = \frac{5(1-e^{-2})}{1+e^{-2}} = 3.8\,\text{m s}^{-1}$	E1 M1A1 [3]	
(v) The first model	E1 [1]	www

8(i) AC = 5sec α $\Rightarrow$ CF = AC tan β = 5sec α tan β $\Rightarrow$ GF = 2CF = 10sec α tan β *	B1 M1 E1 [3]	oe ACtanβ
(ii) CE = BE – BC = 5 tan($\alpha + \beta$) – 5 tan α = 5(tan($\alpha + \beta$) – tan α) $= 5\left(\frac{\tan\alpha + \tan\beta}{1-\tan\alpha\tan\beta} - \tan\alpha\right)$ $= 5\left(\frac{\tan\alpha + \tan\beta - \tan\alpha + \tan^2\alpha\tan\beta}{1-\tan\alpha\tan\beta}\right)$ $= \frac{5(1+\tan^2\alpha)\tan\beta}{1-\tan\alpha\tan\beta}$ $= \frac{5\tan\beta\sec^2\alpha}{1-\tan\alpha\tan\beta}$ *	 E1 M1 M1 DM1 E1 [5]	 compound angle formula combining fractions $\sec^2 = 1 + \tan^2$
(iii) $\sec^2 45° = 2$, tan 45° = 1 $\Rightarrow$ $\text{CE} = \frac{5t\times 2}{1-t} = \frac{10t}{1-t}$ $\text{CD} = \frac{10t}{1+t}$ $\Rightarrow$ $\text{DE} = \frac{10t}{1-t} + \frac{10t}{1+t} = 10t(\frac{1}{1-t} + \frac{1}{1+t})$ $= 10t\left(\frac{1+t+1-t}{(1-t)(1+t)}\right) = \frac{20t}{1-t^2}$ *	B1 M1 A1 M1 E1 [5]	used substitution for both in CE or CD oe for both adding their CE and CD
(iv) cos 45° = 1/√2 $\Rightarrow$ sec α = √2 $\Rightarrow$ GF = 10√2 tan β = 10√2 t	M1 E1 [2]	
(v) DE = 2GF $\Rightarrow$ $\frac{20t}{1-t^2} = 20\sqrt{2}t$ $\Rightarrow$ $1 - t^2 = 1/\sqrt{2} \Rightarrow t^2 = 1 - 1/\sqrt{2}$ * $\Rightarrow$ $t = 0.541$ $\Rightarrow$ $\beta = 28.4°$	 E1 M1 A1 [3]	 invtan t

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