OCR
RECOGNISING ACHIEVEMENT

Tuesday 17 January 2012 – Morning

A2 GCE MATHEMATICS (MEI)

4754A Applications of Advanced Mathematics (C4) Paper A

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4754A
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

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INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.
- This paper will be followed by **Paper B: Comprehension**.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Section A (36 marks)

1 Express $\dfrac{x+1}{x^2(2x-1)}$ in partial fractions. **[5]**

2 Solve, correct to 2 decimal places, the equation $\cot 2\theta = 3$ for $0° \leqslant \theta \leqslant 180°$. **[4]**

3 Express $3\sin x + 2\cos x$ in the form $R\sin(x+\alpha)$, where $R > 0$ and $0 < \alpha < \frac{\pi}{2}$.

Hence find, correct to 2 decimal places, the coordinates of the maximum point on the curve $y = \mathrm{f}(x)$, where

$$\mathrm{f}(x) = 3\sin x + 2\cos x, \quad 0 \leqslant x \leqslant \pi.$$ **[7]**

4 **(i)** Complete the table of values for the curve $y = \sqrt{\cos x}$.

x	0	$\frac{\pi}{8}$	$\frac{\pi}{4}$	$\frac{3\pi}{8}$	$\frac{\pi}{2}$
y		0.9612	0.8409		

Hence use the trapezium rule with strip width $h = \frac{\pi}{8}$ to estimate the value of the integral $\int_0^{\frac{\pi}{2}} \sqrt{\cos x}\,\mathrm{d}x$, giving your answer to 3 decimal places. **[3]**

Fig. 4 shows the curve $y = \sqrt{\cos x}$ for $0 \leqslant x \leqslant \frac{\pi}{2}$.

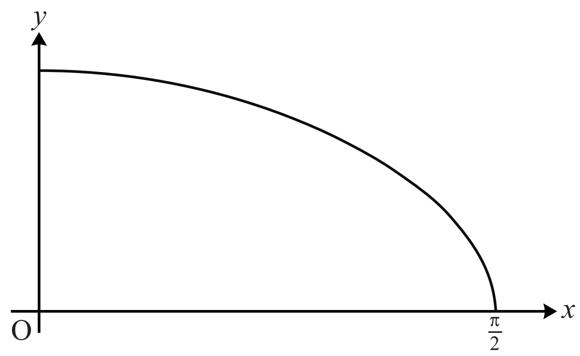

Fig. 4

(ii) State, with a reason, whether the trapezium rule with a strip width of $\frac{\pi}{16}$ would give a larger or smaller estimate of the integral. **[1]**

5 Verify that the vector $2\mathbf{i} - \mathbf{j} + 4\mathbf{k}$ is perpendicular to the plane through the points A(2, 0, 1), B(1, 2, 2) and C(0, –4, 1). Hence find the cartesian equation of the plane. **[5]**

6 Given the binomial expansion $(1 + qx)^p = 1 - x + 2x^2 + \ldots$, find the values of p and q. Hence state the set of values of x for which the expansion is valid. **[6]**

7 Show that the straight lines with equations $\mathbf{r} = \begin{pmatrix} 4 \\ 2 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} 3 \\ 0 \\ 1 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} -1 \\ 4 \\ 9 \end{pmatrix} + \mu \begin{pmatrix} -1 \\ 1 \\ 3 \end{pmatrix}$ meet.

Find their point of intersection. **[5]**

Section B (36 marks)

8 Fig. 8 shows a cross-section of a car headlight whose inside reflective surface is modelled, in suitable units, by the curve

$$x = 2t^2,\ y = 4t, \quad -\sqrt{2} \leqslant t \leqslant \sqrt{2}\ .$$

$P(2t^2, 4t)$ is a point on the curve with parameter t. TS is the tangent to the curve at P, and PR is the line through P parallel to the x-axis. Q is the point (2, 0). The angles that PS and QP make with the positive x-direction are θ and ϕ respectively.

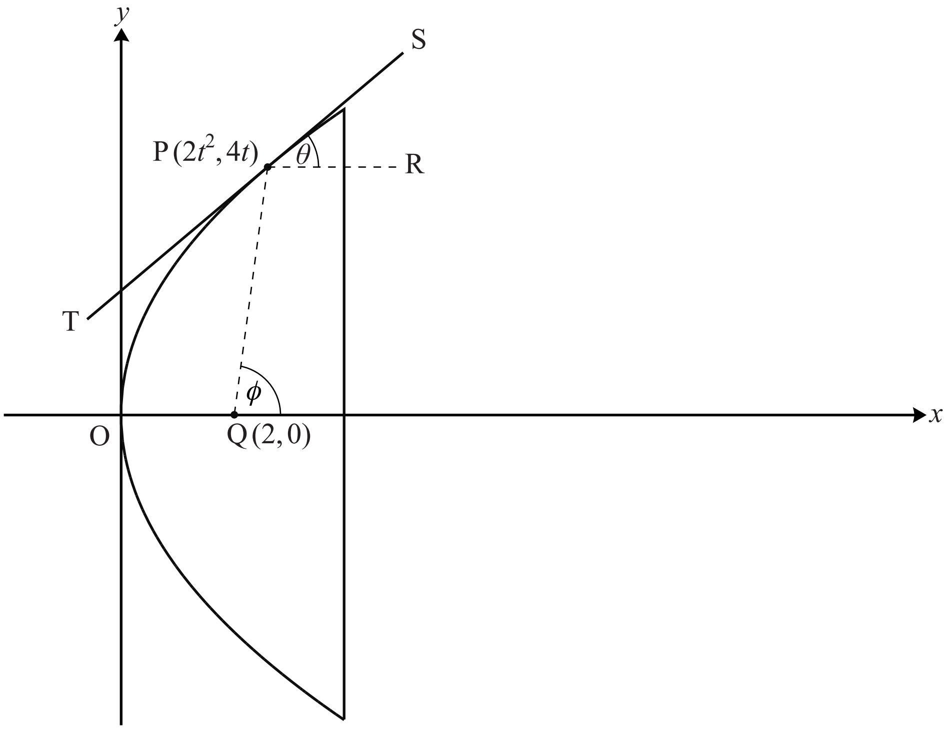

Fig. 8

(i) By considering the gradient of the tangent TS, show that $\tan\theta = \dfrac{1}{t}$. **[3]**

(ii) Find the gradient of the line QP in terms of t. Hence show that $\phi = 2\theta$, and that angle TPQ is equal to θ. **[8]**

[The above result shows that if a lamp bulb is placed at Q, then the light from the bulb is reflected to produce a parallel beam of light.]

The inside surface of the headlight has the shape produced by rotating the curve about the x-axis.

(iii) Show that the curve has cartesian equation $y^2 = 8x$. Hence find the volume of revolution of the curve, giving your answer as a multiple of π. **[7]**

9

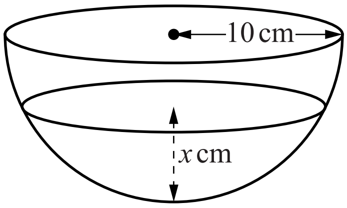

Fig. 9

Fig. 9 shows a hemispherical bowl, of radius 10 cm, filled with water to a depth of x cm. It can be shown that the volume of water, V cm^3, is given by

$$V = \pi(10x^2 - \tfrac{1}{3}x^3).$$

Water is poured into a leaking hemispherical bowl of radius 10 cm. Initially, the bowl is empty. After t seconds, the volume of water is changing at a rate, in cm^3 s^{-1}, given by the equation

$$\frac{\mathrm{d}V}{\mathrm{d}t} = k(20 - x),$$

where k is a constant.

(i) Find $\frac{\mathrm{d}V}{\mathrm{d}x}$, and hence show that $\pi x \frac{\mathrm{d}x}{\mathrm{d}t} = k$. **[4]**

(ii) Solve this differential equation, and hence show that the bowl fills completely after T seconds, where $T = \frac{50\pi}{k}$. **[5]**

Once the bowl is full, the supply of water to the bowl is switched off, and water then leaks out at a rate of kx cm^3 s^{-1}.

(iii) Show that, t seconds later, $\pi(20 - x)\frac{\mathrm{d}x}{\mathrm{d}t} = -k$. **[3]**

(iv) Solve this differential equation.

Hence show that the bowl empties in $3T$ seconds. **[6]**