OCR

Wednesday 18 June 2014 – Afternoon

A2 GCE MATHEMATICS (MEI)

4754/01A Applications of Advanced Mathematics (C4) Paper A

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4754/01A
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.
- This paper will be followed by **Paper B: Comprehension**.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Section A (36 marks)

1 Express $\dfrac{3x}{(2-x)(4+x^2)}$ in partial fractions. **[5]**

2 Find the first three terms in the binomial expansion of $(4+x)^{\frac{3}{2}}$. State the set of values of x for which the expansion is valid. **[5]**

3 Fig. 3 shows the curve $y = x^3 + \sqrt{(\sin x)}$ for $0 \leqslant x \leqslant \frac{\pi}{4}$.

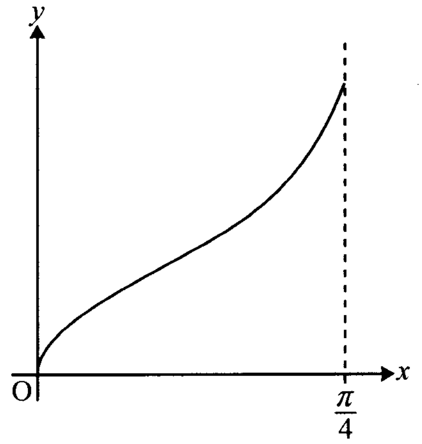

Fig. 3

(i) Use the trapezium rule with 4 strips to estimate the area of the region bounded by the curve, the x-axis and the line $x = \frac{\pi}{4}$, giving your answer to 3 decimal places. **[4]**

(ii) Suppose the number of strips in the trapezium rule is increased. Without doing further calculations, state, with a reason, whether the area estimate increases, decreases, or it is not possible to say. **[1]**

4 **(i)** Show that $\cos(\alpha+\beta) = \dfrac{1-\tan\alpha\tan\beta}{\sec\alpha\sec\beta}$. **[3]**

(ii) Hence show that $\cos 2\alpha = \dfrac{1-\tan^2\alpha}{1+\tan^2\alpha}$. **[2]**

(iii) Hence or otherwise solve the equation $\dfrac{1-\tan^2\theta}{1+\tan^2\theta} = \dfrac{1}{2}$ for $0° \leqslant \theta \leqslant 180°$. **[3]**

5 A curve has parametric equations $x = e^{3t}, y = te^{2t}$.

(i) Find $\dfrac{dy}{dx}$ in terms of t. Hence find the exact gradient of the curve at the point with parameter $t = 1$. **[4]**

(ii) Find the cartesian equation of the curve in the form $y = ax^b \ln x$, where a and b are constants to be determined. **[3]**

6 Fig. 6 shows the region enclosed by the curve $y = (1 + 2x^2)^{\frac{1}{3}}$ and the line $y = 2$.

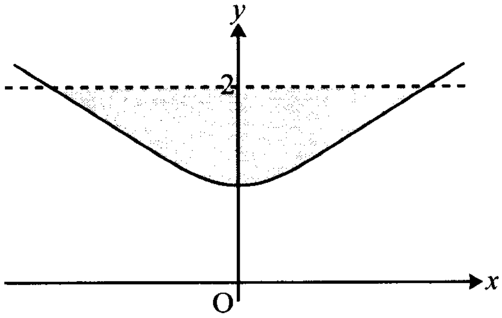

Fig. 6

This region is rotated about the y-axis. Find the volume of revolution formed, giving your answer as a multiple of π. **[6]**

Question 7 begins on page 4.

Section B (36 marks)

7 Fig. 7 shows a tetrahedron ABCD. The coordinates of the vertices, with respect to axes $Oxyz$, are A(–3, 0, 0), B(2, 0, –2), C(0, 4, 0) and D(0, 4, 5).

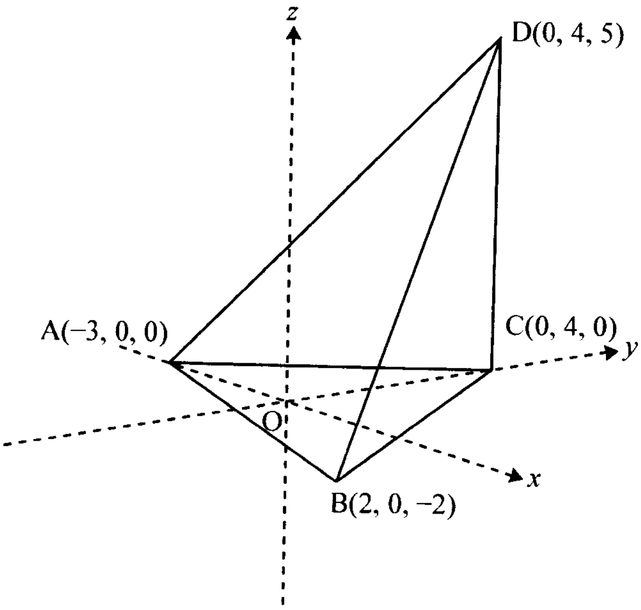

Fig. 7

(i) Find the lengths of the edges AB and AC, and the size of the angle CAB. Hence calculate the area of triangle ABC. **[7]**

(ii) (*A*) Verify that $4\mathbf{i} - 3\mathbf{j} + 10\mathbf{k}$ is normal to the plane ABC. **[2]**

(*B*) Hence find the equation of this plane. **[2]**

(iii) Write down a vector equation for the line through D perpendicular to the plane ABC. Hence find the point of intersection of this line with the plane ABC. **[5]**

The volume of a tetrahedron is $\frac{1}{3}$ × area of base × height.

(iv) Find the volume of the tetrahedron ABCD. **[2]**

8 Fig. 8.1 shows an upright cylindrical barrel containing water. The water is leaking out of a hole in the side of the barrel.

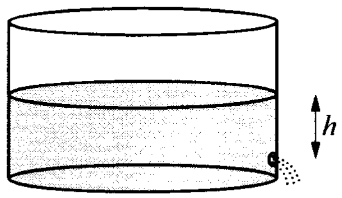

Fig. 8.1

The height of the water surface above the hole t seconds after opening the hole is h metres, where

$$\frac{\mathrm{d}h}{\mathrm{d}t} = -A\sqrt{h}$$

and where A is a positive constant. Initially the water surface is 1 metre above the hole.

(i) Verify that the solution to this differential equation is

$$h = \left(1 - \tfrac{1}{2}At\right)^2.$$ **[3]**

The water stops leaking when $h = 0$. This occurs after 20 seconds.

(ii) Find the value of A, and the time when the height of the water surface above the hole is 0.5 m. **[4]**

Fig. 8.2 shows a similar situation with a different barrel; h is in metres.

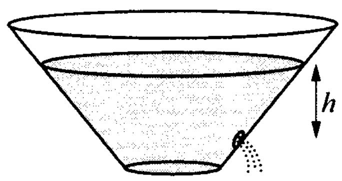

Fig. 8.2

For this barrel,

$$\frac{\mathrm{d}h}{\mathrm{d}t} = -B\frac{\sqrt{h}}{(1+h)^2},$$

where B is a positive constant. When $t = 0$, $h = 1$.

(iii) Solve this differential equation, and hence show that

$$h^{\frac{1}{2}}(30 + 20h + 6h^2) = 56 - 15Bt.$$ **[7]**

(iv) Given that $h = 0$ when $t = 20$, find B.

Find also the time when the height of the water surface above the hole is 0.5 m. **[4]**

END OF QUESTION PAPER