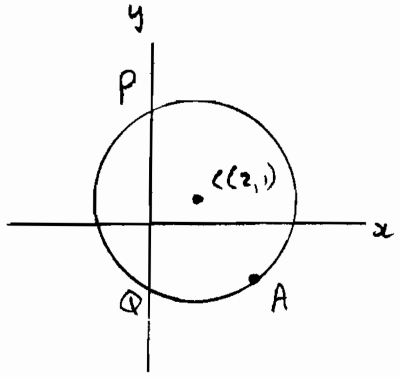
MEI CORE 1 CIRCLES REVISION

1)



i) Circle $(x-2)^2+(y-1)^2=5^2$

$x^2-4x+4+y^2-2y+1=25$

$x^2+y^2-4x-2y-20=0$

ii) Cuts y axis when $x=0$

$\Rightarrow \quad y^2-2y-20=0$

$y = \dfrac{+2 \pm \sqrt{4+80}}{2}$

$y = \dfrac{+2 \pm \sqrt{84}}{2}$

$y = \dfrac{+2 \pm 2\sqrt{21}}{2}$

$y = 1 \pm \sqrt{21}$

$P(0, 1+\sqrt{21}) \quad Q(0, 1-\sqrt{21})$

iii) Subst $A(5,-3)$ in circle

$(5-2)^2+(-3-1)^2$

$= 3^2 + (-4)^2$

$= 9 + 16 = 25 = 5^2$ ✓

$\therefore$ A is on circle

Find gradient of AC

$m = \dfrac{y_2-y_1}{x_2-x_1} = \dfrac{-3-1}{5-2}$

$= \dfrac{-4}{3} = -\dfrac{4}{3}$

$\therefore$ gradient of tgt at A $= +\dfrac{3}{4}$

Using $y-y_1=m(x-x_1)$

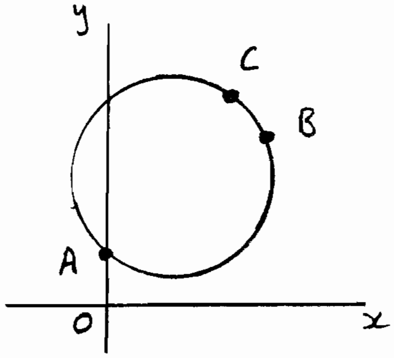
$y--3 = \dfrac{3}{4}(x-5)$

$y+3 = \dfrac{3}{4}x - \dfrac{15}{4}$

$y = \dfrac{3}{4}x - \dfrac{27}{4}$

$\Rightarrow 4y = 3x - 27$

2)



$A(0,2)$
$B(7,9)$
$C(6,10)$

i) $|AC| = \sqrt{(6-0)^2+(10-2)^2}$

$= \sqrt{36+64} = \sqrt{100}$

$|AC| = 10$

Gradient of $AB = \dfrac{y_2-y_1}{x_2-x_1}$

$= \dfrac{9-2}{7-0} = \dfrac{7}{7}$

$= 1$

Gradient of $BC = \dfrac{10-9}{6-7} = \dfrac{1}{-1}$

$= -1$

AB and BC $\perp$ since $1 \times -1 = -1$

$\therefore \angle ABC = 90°$

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ii) $\angle$ in semi-circle $= 90°$

$\therefore$ AC is a diameter

Centre is midpoint of AC

Centre $= \left(\dfrac{x_1+x_2}{2}, \dfrac{y_1+y_2}{2}\right)$

Centre $= \left(\dfrac{0+6}{2}, \dfrac{2+10}{2}\right) = (3,6)$

Since $|AC| = 10$, radius $= 5$

Circle $(x-3)^2+(y-6)^2 = 5^2$

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iii) Gradient of $AC = \dfrac{y_2-y_1}{x_2-x_1}$

$= \dfrac{10-2}{6-0} = \dfrac{8}{6} = \dfrac{4}{3}$

$\therefore$ gradient of tgt at C is $-\dfrac{3}{4}$

Using $y-y_1 = m(x-x_1)$

$y-10 = -\frac{3}{4}(x-6)$

$y-10 = -\frac{3}{4}x + \frac{18}{4}$

$y = -\frac{3}{4}x + \frac{58}{4}$

$4y + 3x = 58$

When $y=0$, $3x = 58$

$x = \dfrac{58}{3}$

Crosses x-axis at $\left(\frac{58}{3}, 0\right)$

When $x=0$, $4y = 58$

$y = \dfrac{58}{4} = \dfrac{29}{2}$

Crosses y-axis at $\left(0, \frac{29}{2}\right)$

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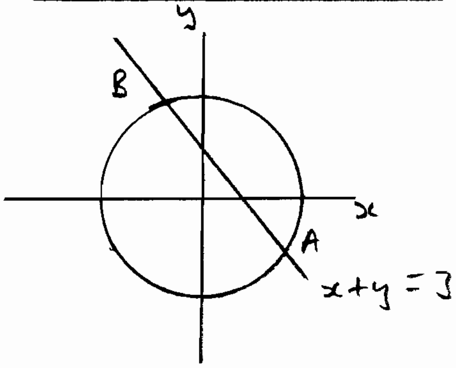
3) $x^2 + y^2 = 45$

i) Centre $(0,0)$ radius $\sqrt{45}$

ii)



Solve $x^2 + y^2 = 45$ ①

$x + y = 3$ ②

From ② $y = 3 - x$

Sub in ①

$x^2 + (3-x)^2 = 45$

$x^2 + 9 - 6x + x^2 = 45$

$2x^2 - 6x - 36 = 0$

$x^2 - 3x - 18 = 0$

$(x-6)(x+3) = 0$

Either $x - 6 = 0$

$\Rightarrow x = 6$

or $x + 3 = 0$

$\Rightarrow x = -3$

When $x = 6$ $y = 3 - 6$

$y = -3$

So $A(6,-3)$

When $x = -3$, $y = 3 - -3$

$y = 6$

So $B(-3,6)$

$|AB| = \sqrt{(x_1-x_2)^2 + (y_1-y_2)^2}$

$|AB| = \sqrt{(6--3)^2 + (-3-6)^2}$

$|AB| = \sqrt{81 + 81}$

$|AB| = \sqrt{162}$

4) $A(9,8)$
$B(5,0)$
$C(3,1)$

i) gradient $AB = \dfrac{y_2 - y_1}{x_2 - x_1}$

$= \dfrac{8-0}{9-5} = \dfrac{8}{4}$

$= 2$

gradient $BC = \dfrac{0-1}{5-3} = -\dfrac{1}{2}$

AB and BC are $\perp$ since

$2 \times -\frac{1}{2} = -1$

ii) Centre is midpoint of AC

$= \left(\dfrac{x_1+x_2}{2}, \dfrac{y_1+y_2}{2}\right)$

$= \left(\dfrac{9+3}{2}, \dfrac{8+1}{2}\right) = \left(6, \dfrac{9}{2}\right)$

Diameter $AC = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{(9-3)^2+(8-1)^2}$

$= \sqrt{36+49} = \sqrt{85}$

Radius $= \dfrac{\sqrt{85}}{2}$

Eqn of circle

$(x-6)^2 + \left(y - \frac{5}{2}\right)^2 = \left(\frac{\sqrt{85}}{2}\right)^2$

$(x-6)^2 + \left(y - \frac{9}{2}\right)^2 = \dfrac{85}{4}$

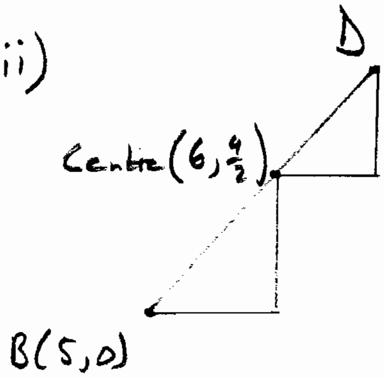
Show B on circle $B(5,0)$

$(5-6)^2 + \left(0 - \frac{9}{2}\right)^2$

$= 1 + \dfrac{81}{4} = \dfrac{85}{4}$ ✓

$\therefore$ B on circle

iii)

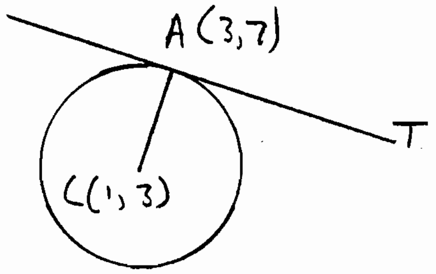


By similar Δs

$D\left(6+1, \frac{9}{2}+\frac{9}{2}\right)$

$D(7, 9)$

5)



i) Gradient $AC = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{7-3}{3-1} = \frac{4}{2} = 2$

$\therefore$ gradient of tgt at $A = -\frac{1}{2}$

Using $y - y_1 = m(x - x_1)$

$y - 7 = -\frac{1}{2}(x - 3)$

$y - 7 = -\frac{1}{2}x + \frac{3}{2}$

$y = -\frac{1}{2}x + \frac{17}{2}$

$\Rightarrow \quad 2y = -x + 17$

$\Rightarrow \quad x + 2y = 17$

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ii) Solve $\begin{cases} x + 2y = 17 & (1) \\ y = 2x - 9 & (2) \end{cases}$

Sub for y in (1)

$x + 2(2x - 9) = 17$

$x + 4x - 18 = 17$

$5x = 35$

$x = 7$

$\Rightarrow \quad y = 2(7) - 9 = 14 - 9 = 5$

Intersect at $(7, 5)$

so $T(7,5)$

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iii) $(x-1)^2 + (y-3)^2 = 20$

$y = 2x - 9$ is a tgt if

there is a single point of intersection

$(x-1)^2 + (y-3)^2 = 20 \quad (1)$

$y = 2x - 9 \quad (2)$

Sub for y in (1)

$(x-1)^2 + (2x - 9 - 3)^2 = 20$

$x^2 - 2x + 1 + (2x - 12)^2 = 20$

$x^2 - 2x + 1 + 4x^2 - 48x + 144 = 20$

$5x^2 - 50x + 125 = 0$

$x^2 - 10x + 25 = 0$

$(x-5)(x-5) = 0$

$\Rightarrow \quad x = 5$

When $x = 5$, $y = 2(5) - 9 = 1$

Only 1 point of intersection

so $y = 2x - 9$ is a tangent

to the circle at $(5, 1)$
