1) i) $f(x) = x^3 + x^2 - 10x + 8$

$f(1) = 1^3 + 1^2 - 10(1) + 8$

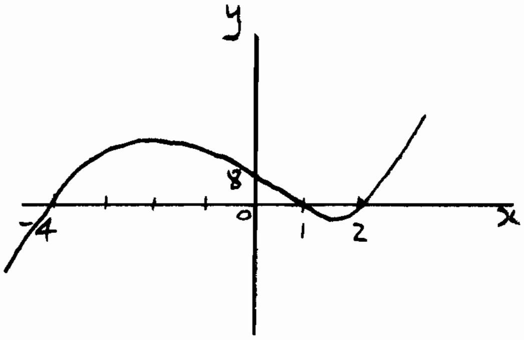
$= 1 + 1 - 10 + 8 = 0$

$\therefore$ by factor theorem $(x-1)$ is a factor of $f(x)$

$$\begin{array}{r|l} & x^2 + 2x - 8 \\ \hline x-1 & x^3 + x^2 - 10x + 8 \\ & x^3 - x^2 \\ \hline & \quad 2x^2 - 10x \\ & \quad 2x^2 - 2x \\ \hline & \qquad -8x + 8 \\ & \qquad -8x + 8 \\ \hline \end{array}$$

$f(x) = (x-1)(x^2 + 2x - 8)$

$f(x) = (x-1)(x-2)(x+4)$



1 ii) $f(x+3)$ is obtained from $f(x)$ by a translation of $\begin{pmatrix} -3 \\ 0 \end{pmatrix}$

Resulting graph

$y = (x+3)^3 + (x+3)^2 - 10(x+3) + 8$

y-intercept given by

$(0+3)^3 + (0+3)^2 - 10(0+3) + 8$

$= 27 + 9 - 30 + 8$

$= 14$

2) i) Roots of $f(x) = 0$ are $x = -1, 2, 5$

Coeff of $x^3 = 1$

$\Rightarrow f(x) = (x+1)(x-2)(x-5)$

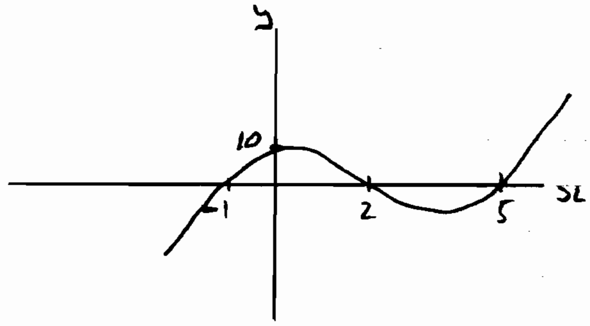
$f(x) = (x^2 + x - 2x - 2)(x-5)$

$= (x^2 - x - 2)(x-5)$

$= x^3 - x^2 - 2x$
$\quad -5x^2 + 5x + 10$

$= x^3 - 6x^2 + 3x + 10$

ii)



y-intercept is $1 \times (-2) \times (-5) = 10$

iii) $f(x) + 10$

$= x^3 - 6x^2 + 3x + 20$

When $x = 4$, $f(x) + 10$

$= 4^3 - 6(4)^2 + 3(4) + 20$

2iii) cont $= 64 - 96 + 12 + 20 = 0$

$\therefore\ x = 4$ is a root

$$\begin{array}{r|l} & x^2 - 2x - 5 \\ \hline x-4 & x^3 - 6x^2 + 3x + 20 \\ & x^3 - 4x^2 \\ \hline & \quad -2x^2 + 3x \\ & \quad -2x^2 + 8x \\ \hline & \qquad -5x + 20 \\ & \qquad -5x + 20 \\ \hline \end{array}$$

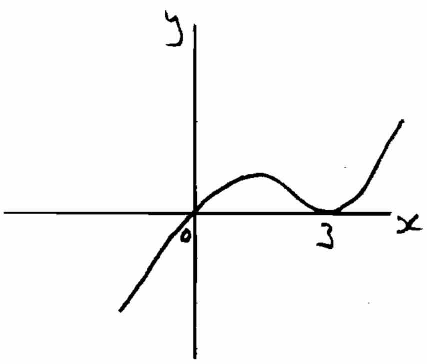
$f(x) + 10 = (x-4)(x^2 - 2x - 5)$

Quadratic eqn which gives other two roots is

$$x^2 - 2x - 5 = 0$$

3) i) $y = x(x-3)^2$

$= x(x-3)(x-3)$



ii) $x(x-3)^2 = 2$

$x(x^2 - 6x + 9) = 2$

$x^3 - 6x^2 + 9x - 2 = 0$

3iii) Let $f(x) = x^3 - 6x^2 + 9x - 2$

$f(2) = 2^3 - 6(2)^2 + 9(2) - 2$

$= 8 - 24 + 18 - 2$

$= 0$

$\therefore\ x = 2$ is a root of $f(x) = 0$

and $(x-2)$ is a factor of $f(x)$

$$\begin{array}{r|l} & x^2 - 4x + 1 \\ \hline x-2 & x^3 - 6x^2 + 9x - 2 \\ & x^3 - 2x^2 \\ \hline & \quad -4x^2 + 9x \\ & \quad -4x^2 + 8x \\ \hline & \qquad x - 2 \\ & \qquad x - 2 \\ \hline \end{array}$$

$f(x) = (x-2)(x^2 - 4x + 1)$

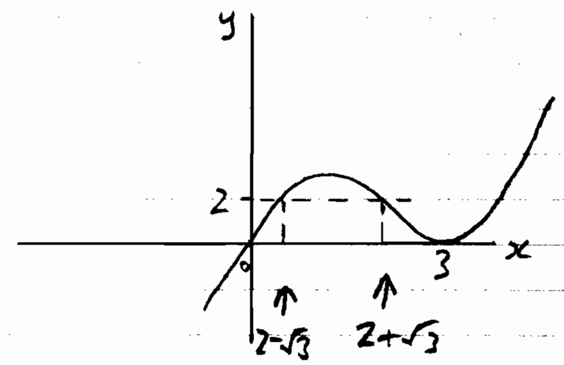
Solve $x^2 - 4x + 1 = 0$

$x = \dfrac{4 \pm \sqrt{4^2 - 4}}{2}$

$x = \dfrac{4 \pm \sqrt{12}}{2}$

$x = \dfrac{4 \pm 2\sqrt{3}}{2}$

$x = 2 \pm \sqrt{3}$



4)

i) $f(x) = x^3 + 9x^2 + 20x + 12$

$f(-2) = (-2)^3 + 9(-2)^2 + 20(-2) + 12$

$= -8 + 36 - 40 + 12$

$= 0$

$\therefore x = -2$ is a root of $f(x) = 0$

ii)

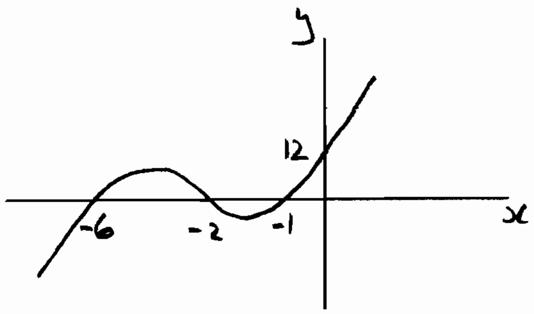
$$\begin{array}{r|l} & x^2 + 3x + 2 \\ \hline x+6 & x^3 + 9x^2 + 20x + 12 \\ & x^3 + 6x^2 \\ \hline & +3x^2 + 20x \\ & +3x^2 + 18x \\ \hline & +2x + 12 \\ & +2x + 12 \\ \hline \end{array}$$

$f(x) \div (x+6) = x^2 + 3x + 2$

iii) $f(x) = (x+6)(x^2 + 3x + 2)$

$f(x) = (x+6)(x+2)(x+1)$

iv)



v) $f(x) = 12$

$\Rightarrow x^3 + 9x^2 + 20x + 12 = 12$

$\Rightarrow x^3 + 9x^2 + 20x = 0$

$x(x^2 + 9x + 20) = 0$

$x(x+5)(x+4) = 0$

$\Rightarrow x = 0,\ x = -5,\ x = -4$

5) $f(x) = x^3 - 5x + 2$

i) If $x = 2$ is a root of $f(x) = 0$

then $(x-2)$ is a factor of $f(x)$

$$\begin{array}{r|l} & x^2 + 2x - 1 \\ \hline x-2 & x^3 \quad\quad - 5x + 2 \\ & x^3 - 2x^2 \\ \hline & +2x^2 - 5x \\ & +2x^2 - 4x \\ \hline & -x + 2 \\ & -x + 2 \\ \hline \end{array}$$

$f(x) = (x-2)(x^2 + 2x - 1)$

Solve $x^2 + 2x - 1 = 0$

$x = \dfrac{-2 \pm \sqrt{2^2 + 4}}{2}$

$x = \dfrac{-2 \pm \sqrt{8}}{2}$

$x = \dfrac{-2 \pm 2\sqrt{2}}{2}$

$x = -1 \pm \sqrt{2}$

Other roots are

$x = -1 \pm \sqrt{2}$

5 ii) $f(x) = x^3 - 5x + 2$

$f(x-3) = (x-3)^3 - 5(x-3) + 2$

$= (x-3)(x^2-6x+9) - 5x + 15 + 2$

$$= \begin{aligned} &x^3 - 6x^2 + 9x \\ &\quad -3x^2 + 18x - 27 \\ &\qquad\quad -5x + 17 \end{aligned}$$

$= x^3 - 9x^2 + 22x - 10$

5 iii) $f(x-3)$ translates graph of $f(x)$ by $\begin{pmatrix} 3 \\ 0 \end{pmatrix}$

so roots are moved by $+3$

New roots are

$2 + 3 = 5$

$-1 + \sqrt{2} + 3 = 2 + \sqrt{2}$

$-1 - \sqrt{2} + 3 = 2 - \sqrt{2}$

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