MEI Core 3 Chain Rule - Rates of Change Questions Jan 05 - May 09

1 Fig. 4 shows a cone. The angle between the axis and the slant edge is 30°. Water is poured into the cone at a constant rate of 2 cm^3 per second. At time t seconds, the radius of the water surface is r cm and the volume of water in the cone is $V\,\text{cm}^3$.

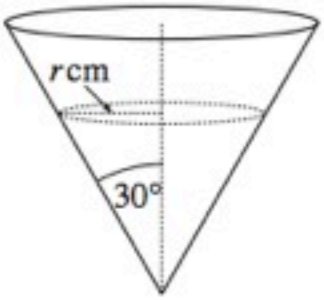

Fig. 4

(i) Write down the value of $\dfrac{dV}{dt}$. [1]

(ii) Show that $V = \dfrac{\sqrt{3}}{3}\pi r^3$, and find $\dfrac{dV}{dr}$. [3]

[You may assume that the volume of a cone of height h and radius r is $\frac{1}{3}\pi r^2 h$.]

(iii) Use the results of parts (i) and (ii) to find the value of $\dfrac{dr}{dt}$ when $r = 2$. [3]

2 Fig. 4 is a diagram of a garden pond.

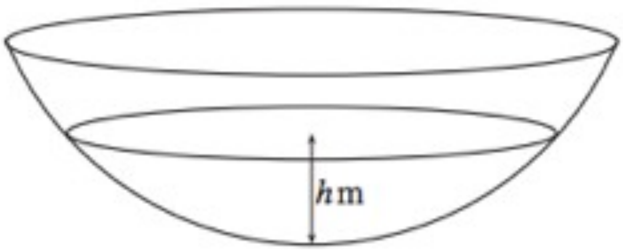

Fig. 4

The volume $V\,\text{m}^3$ of water in the pond when the depth is h metres is given by

$$V = \tfrac{1}{3}\pi h^2(3 - h).$$

(i) Find $\dfrac{dV}{dh}$. [2]

Water is poured into the pond at the rate of 0.02 m^3 per minute.

(ii) Find the value of $\dfrac{dh}{dt}$ when $h = 0.4$. [4]

3 When the gas in a balloon is kept at a constant temperature, the pressure P in atmospheres and the volume $V\ \mathrm{m}^3$ are related by the equation

$$P = \frac{k}{V},$$

where k is a constant. [This is known as Boyle's Law.]

When the volume is $100\ \mathrm{m}^3$, the pressure is 5 atmospheres, and the volume is increasing at a rate of $10\ \mathrm{m}^3$ per second.

(i) Show that $k = 500$. [1]

(ii) Find $\frac{\mathrm{d}P}{\mathrm{d}V}$ in terms of V. [2]

(iii) Find the rate at which the pressure is decreasing when $V = 100$. [4]

4 The variables x and y satisfy the equation $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 5$. **[4]**

(i) Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = -\left(\frac{y}{x}\right)^{\frac{1}{3}}$.

Both x and y are functions of t.

(ii) Find the value of $\frac{\mathrm{d}y}{\mathrm{d}t}$ when $x = 1$, $y = 8$ and $\frac{\mathrm{d}x}{\mathrm{d}t} = 6$. **[3]**