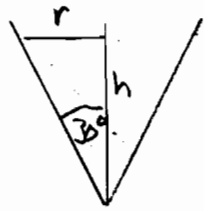


1)



$$i) \quad \frac{dV}{dt} = 2 \text{ cm}^3 \text{ s}^{-1}$$

$$ii) \quad V = \frac{1}{3} \pi r^2 h$$

$$\text{but } \tan 30^\circ = \frac{r}{h}$$

$$\text{so } \frac{1}{\sqrt{3}} = \frac{r}{h}$$

$$\Rightarrow h = \sqrt{3} r$$

$$\therefore V = \frac{1}{3} \pi r^2 \times \sqrt{3} r$$

$$V = \frac{\sqrt{3} \pi r^3}{3}$$

$$\frac{dV}{dr} = 3 \frac{\sqrt{3} \pi r^2}{3}$$

$$\frac{dV}{dr} = \sqrt{3} \pi r^2$$

$$iii) \quad \text{Find } \frac{dr}{dt} \text{ when } r = 2$$

$$\frac{dr}{dt} = \frac{dr}{dV} \times \frac{dV}{dt}$$

$$= \frac{1}{\frac{dV}{dr}} \times \frac{dV}{dt}$$

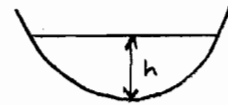
$$\frac{dr}{dt} = \frac{1}{\sqrt{3} \pi r^2} \times \frac{dV}{dt}$$

$$\text{When } r = 2 \text{ cm and } \frac{dV}{dt} = 2 \text{ cm}^3 \text{ s}^{-1}$$

$$\frac{dr}{dt} = \frac{1}{\sqrt{3} \pi \times 2^2} \times 2 \text{ cm s}^{-1}$$

$$\begin{aligned} \frac{dr}{dt} &= 0.091888 \text{ cm s}^{-1} \\ &= 0.0919 \text{ cm s}^{-1} \\ &\quad \text{to 3 s.f.} \end{aligned}$$

2)



$$\text{Given } V = \frac{1}{3} \pi h^2 (3-h)$$

$$i) \quad V = \frac{1}{3} \pi (3h^2 - h^3)$$

$$\frac{dV}{dh} = \frac{\pi}{3} (6h - 3h^2)$$

$$\frac{dV}{dh} = \pi (2h - h^2)$$

$$ii) \quad \text{Given } \frac{dV}{dt} = 0.02 \text{ m}^3 \text{ per min}$$

$$\text{Find } \frac{dh}{dt} \text{ when } h = 0.4$$

2ii) cont)

$$\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt}$$

$$= \frac{1}{\frac{dV}{dh}} \times \frac{dV}{dt}$$

$$\frac{dh}{dt} = \frac{1}{\pi(2h-h^2)} \times \frac{dV}{dt}$$

When $h = 0.4$, and $\frac{dV}{dt} = 0.02$

$$\frac{dh}{dt} = \frac{1}{\pi(2 \times 0.4 - 0.4^2)} \times 0.02$$

$$\frac{dh}{dt} = 0.009947 \text{ m per min}$$

$$\frac{dh}{dt} = 0.00995 \text{ m per min}$$

to 3 s.f.

3) $P = \frac{k}{V}$

i) Given when $V = 100 \text{ m}^3$, $P = 5$

so $5 = \frac{k}{100}$

$\Rightarrow k = 500$

ii) $P = 500V^{-1}$

$$\frac{dP}{dV} = -500V^{-2} = -\frac{500}{V^2}$$

iii) Find $\frac{dP}{dt}$ when $V = 100$

Also given that $\frac{dV}{dt} = 10 \text{ m}^3 \text{ s}^{-1}$

Now $\frac{dP}{dt} = \frac{dP}{dV} \times \frac{dV}{dt}$

$$\Rightarrow \frac{dP}{dt} = -\frac{500}{V^2} \times 10$$

When $V = 100$

$$\frac{dP}{dt} = \frac{-500}{100^2} \times 10 \text{ atmos s}^{-1}$$

$$= 0.5 \text{ atmos s}^{-1}$$

4) i) $x^{2/3} + y^{2/3} = 5$

diff x

$$\frac{2}{3} x^{-1/3} + \frac{2}{3} y^{-1/3} \frac{dy}{dx} = 0$$

$$\Rightarrow x^{-1/3} + y^{-1/3} \frac{dy}{dx} = 0$$

$$y^{-1/3} \frac{dy}{dx} = -x^{-1/3}$$

$$\frac{dy}{dx} = -\frac{x^{-1/3}}{y^{-1/3}} = -\frac{y^{1/3}}{x^{1/3}}$$

$$\frac{dy}{dx} = -\left(\frac{y}{x}\right)^{1/3}$$

4 ii)

Find $\frac{dy}{dt}$ when $x=1$
 $y=8$
 $\frac{dx}{dt}=6$

$$\frac{dy}{dt} = \frac{dy}{dx} \times \frac{dx}{dt}$$

$$\frac{dy}{dt} = -\left(\frac{y}{x}\right)^{\frac{1}{3}} \times \frac{dx}{dt}$$

Subst values gives

$$\frac{dy}{dt} = -\left(\frac{8}{1}\right)^{\frac{1}{3}} \times 6$$

$$\frac{dy}{dt} = -2 \times 6$$

$$\frac{dy}{dt} = -12$$