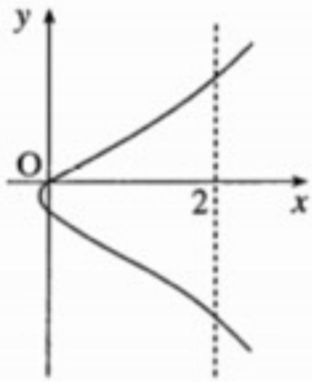
MEI Core 3 Implicit Differentiation Questions Jan 05 - May 09

1 Fig. 7 shows the curve defined implicitly by the equation

$$y^2 + y = x^3 + 2x,$$

together with the line $x = 2$.



Not to scale

Fig. 7

Find the coordinates of the points of intersection of the line and the curve.

Find $\frac{dy}{dx}$ in terms of x and y. Hence find the gradient of the curve at each of these two points. [8]

2 A curve is defined implicitly by the equation

$$y^3 = 2xy + x^2.$$

(i) Show that $\frac{dy}{dx} = \frac{2(x+y)}{3y^2 - 2x}$. [4]

(ii) Hence write down $\frac{dx}{dy}$ in terms of x and y. [1]

3 Fig. 6 shows the triangle OAP, where O is the origin and A is the point $(0, 3)$. The point $P(x, 0)$ moves on the positive x-axis. The point $Q(0, y)$ moves between O and A in such a way that $AQ + AP = 6$.

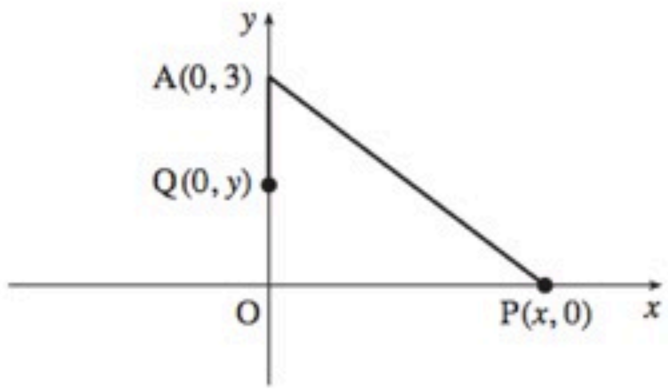


Fig. 6

(i) Write down the length AQ in terms of y. Hence find AP in terms of y, and show that

$$(y+3)^2 = x^2 + 9.$$ [3]

(ii) Use this result to show that $\dfrac{dy}{dx} = \dfrac{x}{y+3}$. [2]

(iii) When $x = 4$ and $y = 2$, $\dfrac{dx}{dt} = 2$. Calculate $\dfrac{dy}{dt}$ at this time. [3]

4 A curve has equation $2y^2 + y = 9x^2 + 1$.

(i) Find $\dfrac{dy}{dx}$ in terms of x and y. Hence find the gradient of the curve at the point $A(1, 2)$. [4]

(ii) Find the coordinates of the points on the curve at which $\dfrac{dy}{dx} = 0$. [4]

5 Fig. 6 shows the curve $e^{2y} = x^2 + y$.

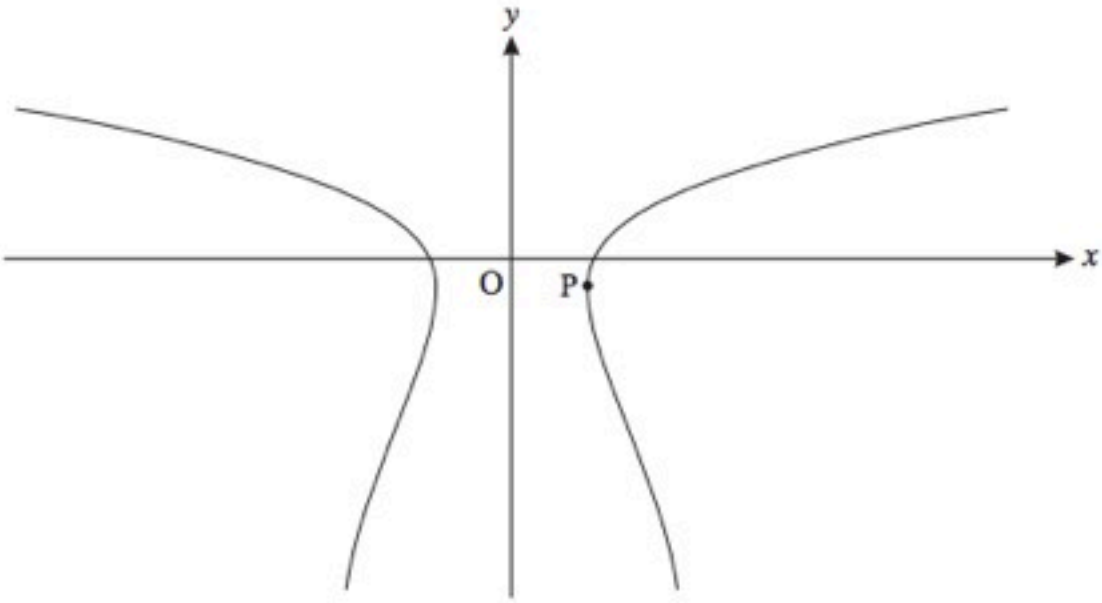


Fig. 6

(i) Show that $\dfrac{dy}{dx} = \dfrac{2x}{2e^{2y} - 1}$. [4]

(ii) Hence find to 3 significant figures the coordinates of the point P, shown in Fig. 6, where the curve has infinite gradient. [4]

6 Given that $x^2 + xy + y^2 = 12$, find $\dfrac{dy}{dx}$ in terms of x and y. [5]