MEI CORE 3 IMPLICIT DIFFERENTIATION REVISION

1) $$y^2 + y = x^3 + 2x$$

line $x = 2$

Solve simultaneously to find points of intersection

$$y^2 + y = 2^3 + 2(2)$$
$$y^2 + y = 8 + 4$$
$$y^2 + y - 12 = 0$$
$$(y+4)(y-3) = 0$$
$$\Rightarrow \quad y = -4 \quad \text{or} \quad y = 3$$

Points of intersection are

$(2,-4)$ and $(2,3)$

$$y^2 + y = x^3 + 2x$$
$$\Rightarrow \quad 2y\frac{dy}{dx} + \frac{dy}{dx} = 3x^2 + 2$$
$$\frac{dy}{dx}(2y+1) = 3x^2 + 2$$
$$\frac{dy}{dx} = \frac{3x^2+2}{2y+1}$$

At $(2,-4)$

$$\frac{dy}{dx} = \frac{3(2)^2+2}{2(-4)+1}$$

$$\frac{dy}{dx} = \frac{14}{-7} = -2$$

At $(2,3)$

$$\frac{dy}{dx} = \frac{3(2)^2+2}{2(3)+1} = \frac{14}{7} = 2$$

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2) $$y^3 = 2xy + x^2$$

i) $$3y^2\frac{dy}{dx} = 2x\frac{dy}{dx} + 2y + 2x$$
$$3y^2\frac{dy}{dx} - 2x\frac{dy}{dx} = 2(x+y)$$
$$(3y^2 - 2x)\frac{dy}{dx} = 2(x+y)$$
$$\frac{dy}{dx} = \frac{2(x+y)}{3y^2-2x}$$

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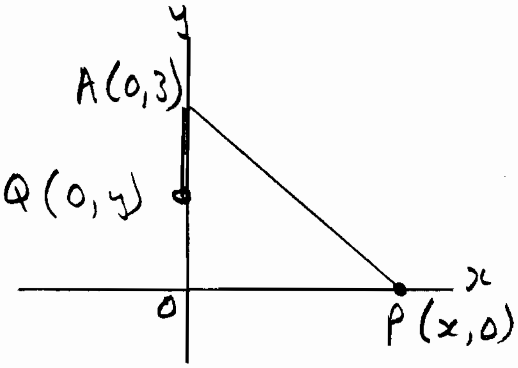
ii) $$\frac{dx}{dy} = \frac{1}{\frac{dy}{dx}}$$

$$\Rightarrow \quad \frac{dx}{dy} = \frac{3y^2-2x}{2(x+y)}$$

3) 

Given $AQ + AP = 6$

i) $AQ = 3 - y$

$\therefore AP = 6 - (3-y) = 3+y$

By Pythagoras $OP^2 + OA^2 = AP^2$

$\therefore x^2 + 3^2 = (y+3)^2$

$x^2 + 9 = (y+3)^2$

ii) $(y+3)^2 = x^2 + 9$

$2(y+3) \times 1 \dfrac{dy}{dx} = 2x$

$\dfrac{dy}{dx} = \dfrac{2x}{2(y+3)}$

$\dfrac{dy}{dx} = \dfrac{x}{y+3}$

iii) When $x = 4$ and $y = 2$, $\dfrac{dx}{dt} = 2$

Find $\dfrac{dy}{dt}$

$\dfrac{dy}{dt} = \dfrac{dy}{dx} \cdot \dfrac{dx}{dt}$

$\Rightarrow \dfrac{dy}{dt} = \dfrac{x}{y+3} \times \dfrac{dx}{dt}$

when $x = 4$, $y = 2$, $\dfrac{dx}{dt} = 2$

$\dfrac{dy}{dt} = \dfrac{4}{2+3} \times 2 = \dfrac{8}{5}$

4) $2y^2 + y = 9x^2 + 1$

i) $4y\dfrac{dy}{dx} + \dfrac{dy}{dx} = 18x$

$(4y+1)\dfrac{dy}{dx} = 18x$

$\dfrac{dy}{dx} = \dfrac{18x}{4y+1}$

At $(1, 2)$

$\dfrac{dy}{dx} = \dfrac{18(1)}{4(2)+1} = \dfrac{18}{9} = 2$

ii) When $\dfrac{dy}{dx} = 0$, $18x = 0$

$\Rightarrow x = 0$

$\Rightarrow 2y^2 + y = 0 + 1$

$2y^2 + y - 1 = 0$

$(2y-1)(y+1) = 0$

$\Rightarrow y = \frac{1}{2}$ or $y = -1$

Coords of points where $\dfrac{dy}{dx} = 0$

are $(0, \frac{1}{2})$ and $(0, -1)$

5) $e^{2y} = x^2 + y$

i) $2e^{2y}\frac{dy}{dx} = 2x + \frac{dy}{dx}$

$2e^{2y}\frac{dy}{dx} - \frac{dy}{dx} = 2x$

$(2e^{2y} - 1)\frac{dy}{dx} = 2x$

$\frac{dy}{dx} = \frac{2x}{2e^{2y} - 1}$

ii) If $\frac{dy}{dx}$ is infinite

then $2e^{2y} - 1 = 0$

$2e^{2y} = 1$

$e^{2y} = \frac{1}{2}$

$2y = \ln\left(\frac{1}{2}\right)$

$y = \frac{1}{2}\ln\left(\frac{1}{2}\right)$

$y = -0.34657$

$\Rightarrow e^{2(-0.34657)} = x^2 - 0.34657$

$e^{2(-0.34657)} + 0.34657 = x^2$

$x = \pm\sqrt{e^{2(-0.34657)} + 0.34657}$

$x = \pm 0.92009$

At P, $x > 0$

so $P(0.920, -0.347)$

to 3 sig fig

6) $x^2 + xy + y^2 = 12$

$\Rightarrow 2x + x\frac{dy}{dx} + y + 2y\frac{dy}{dx} = 0$

$x\frac{dy}{dx} + 2y\frac{dy}{dx} = -2x - y$

$(x+2y)\frac{dy}{dx} = -(2x+y)$

$\frac{dy}{dx} = -\left(\frac{2x+y}{x+2y}\right)$

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