

MEI Core 3 Logs and Exponentials Questions Jan 05 - May 09

- 1** The temperature $T^{\circ}\text{C}$ of a liquid at time t minutes is given by the equation

$$T = 30 + 20e^{-0.05t}, \quad \text{for } t \geq 0.$$

Write down the initial temperature of the liquid, and find the initial rate of change of temperature.

Find the time at which the temperature is 40°C . [6]

- 2** A population is P million at time t years. P is modelled by the equation

$$P = 5 + ae^{-bt},$$

where a and b are constants.

The population is initially 8 million, and declines to 6 million after 1 year.

- (i) Use this information to calculate the values of a and b , giving b correct to 3 significant figures. [5]
- (ii) What is the long-term population predicted by the model? [1]

- 3** The mass M kg of a radioactive material is modelled by the equation

$$M = M_0e^{-kt},$$

where M_0 is the initial mass, t is the time in years, and k is a constant which measures the rate of radioactive decay.

- (i) Sketch the graph of M against t . [2]
- (ii) For Carbon 14, $k = 0.000121$. Verify that after 5730 years the mass M has reduced to approximately half the initial mass. [2]

The half-life of a radioactive material is the time taken for its mass to reduce to exactly half the initial mass.

- (iii) Show that, in general, the half-life T is given by $T = \frac{\ln 2}{k}$. [3]
- (iv) Hence find the half-life of Plutonium 239, given that for this material $k = 2.88 \times 10^{-5}$. [1]

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4 The value £ V of a car is modelled by the equation $V = Ae^{-kt}$, where t is the age of the car in years and A and k are constants. Its value when new is £10 000, and after 3 years its value is £6000.

(i) Find the values of A and k . [5]

(ii) Find the age of the car when its value is £2000. [2]

5 A cup of water is cooling. Its initial temperature is 100°C . After 3 minutes, its temperature is 80°C .

(i) Given that $T = 25 + ae^{-kt}$, where T is the temperature in $^{\circ}\text{C}$, t is the time in minutes and a and k are constants, find the values of a and k . [5]

(ii) What is the temperature of the water

(A) after 5 minutes,

(B) in the long term? [3]

6 The profit £ P made by a company in its n th year is modelled by the exponential function

$$P = Ae^{bn}.$$

In the first year (when $n = 1$), the profit was £10 000. In the second year, the profit was £16 000.

(i) Show that $e^b = 1.6$, and find b and A . [6]

(ii) What does this model predict the profit to be in the 20th year? [2]