MEI CORE 3 LOGS AND EXPONENTIALS REVISION

1) $T = 30 + 20e^{-0.05t}$

Initial temp when $t = 0$

$T = 30 + 20e^{0}$

$T = 30 + 20 = 50$

$T = 50\,°C$

$\frac{dT}{dt} = -0.05 \times 20e^{-0.05t}$

$= -e^{-0.05t}$

Initial rate of change when $t = 0$

$\frac{dT}{dt} = -e^{0} = -1 \quad °C\,t^{-1}$

Find time when $T = 40°C$

$40 = 30 + 20e^{-0.05t}$

$40 - 30 = 20e^{-0.05t}$

$\frac{10}{20} = e^{-0.05t}$

$\ln\left(\frac{10}{20}\right) = -0.05t$

$\frac{\ln\left(\frac{1}{2}\right)}{-0.05} = t$

$t = 13.86$ minutes

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2) $P = 5 + ae^{-bt}$

when $t = 0$, $P = 8$

when $t = 1$, $P = 6$

i) $t=0, P=8$ gives

$8 = 5 + ae^{0}$

$8 = 5 + a \Rightarrow a = 3$

$t = 1$, $P = 6$ gives

$6 = 5 + 3e^{-b \times 1}$

$1 = 3e^{-b}$

$\frac{1}{3} = e^{-b}$

$\ln\left(\frac{1}{3}\right) = -b \Rightarrow b = +1.0986$

$b = +1.10$ to 3 s.f.

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ii) As $t \to \infty$

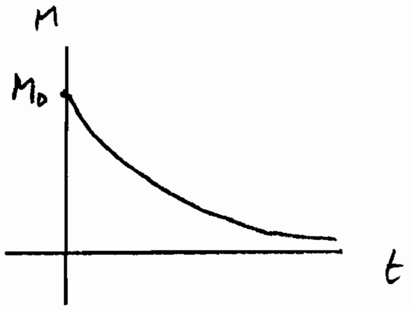
$P \to 5 + 3e^{-\infty}$

$P \to 5 + 0 = 5$

long term population 5 million

3) $M = M_0 e^{-kt}$

i)



ii) $k = 0.000121$

When $t = 5730$

$M = M_0 \times e^{-0.000121 \times 5370}$

$M = 0.522 M_0 \approx \frac{1}{2} M_0$

iii) If $M = \frac{1}{2} M_0$

$\frac{1}{2} M_0 = M_0 e^{-kt}$

$\Rightarrow \frac{1}{2} = e^{-kt}$

$\Rightarrow \ln\left(\frac{1}{2}\right) = -kt$

$\Rightarrow \ln\left(\frac{2}{1}\right)^{-1} = -kt$

$-\ln 2 = -kt$

$\ln 2 = kt$

$\frac{\ln 2}{k} = t$

If T is half-life

$T = \frac{\ln 2}{k}$

iv) If $k = 2.88 \times 10^{-5}$

$T = \frac{\ln 2}{2.88 \times 10^{-5}} = 24{,}068$

$T = 24{,}068$ years

4) $V = Ae^{-kt}$

$V = 10000$ when $t = 0$
$V = 6000$ when $t = 3$

i)

$10000 = Ae^{0}$

$\Rightarrow 10000 = A$

$6000 = 10000 e^{-3k}$

$\frac{6000}{10000} = e^{-3k}$

$\ln 0.6 = -3k$

$\frac{\ln 0.6}{-3} = k$

$k = 0.170$ to 3 s.f

$A = 10000$

ii

$2000 = 10000 e^{-0.170t}$

$\frac{2000}{10000} = e^{-0.170t}$

$\ln 0.2 = -0.170t$

$\frac{\ln 0.2}{-0.170} = t$

$t = 9.47$ yrs

5) $T = 100$ when $t = 0$

$T = 80$ when $t = 3$

i) $$T = 25 + ae^{-kt}$$

$T = 100$, $t = 0$ gives

$$100 = 25 + ae^{0}$$

$$100 = 25 + a$$

$$\Rightarrow \underline{a = 75}$$

$T = 80$, $t = 3$ gives

$$80 = 25 + 75e^{-3k}$$

$$80 - 25 = 75e^{-3k}$$

$$\frac{55}{75} = e^{-3k}$$

$$\ln\left(\frac{55}{75}\right) = -3k$$

$$\frac{\ln\left(\frac{55}{75}\right)}{-3} = k$$

$$k = 0.103 \quad \text{to 3 s.f}$$

ii) When $t = 5$

A) $$T = 25 + 75e^{-0.103 \times 5}$$

$$T = 69.8\,^\circ c$$

B) As $t \to \infty$ $T \to 25 + 0$

$$T \to 25\,^\circ c$$

6) $$P = Ae^{bn}$$

$n = 1$, $P = 10000$
$n = 2$, $P = 16000$

Sub $n = 1$, $P = 10000$

$$10000 = Ae^{b} \quad (1)$$

sub $n = 2$, $P = 16000$

$$16000 = Ae^{2b} \quad (2)$$

(2) ÷ (1) $$\frac{16000}{10000} = \frac{Ae^{2b}}{Ae^{b}}$$

$$1.6 = e^{b}$$

$$\Rightarrow \ln 1.6 = b$$

$$\Rightarrow b = 0.470 \quad \text{to 3 s.f}$$

Sub for b in (1)

$$10000 = Ae^{b}$$

$$\frac{10000}{e^{0.470}} = A$$

$$A = 6250$$

ii) $$P = 6250 \times e^{20 \times 0.470}$$

$$P = £75,552,380$$