MEI Core 3 Proof Questions Jan 05 - May 09

1 Positive integers a, b and c are said to form a Pythagorean triple if $a^2 + b^2 = c^2$.

(i) Given that t is an integer greater than 1, show that $2t$, $t^2 - 1$ and $t^2 + 1$ form a Pythagorean triple. [3]

(ii) The two smallest integers of a Pythagorean triple are 20 and 21. Find the third integer.

Use this triple to show that not all Pythagorean triples can be expressed in the form $2t$, $t^2 - 1$ and $t^2 + 1$. [3]

2 Use the method of exhaustion to prove the following result.

No 1- or 2-digit perfect square ends in 2, 3, 7 or 8

State a generalisation of this result. [3]

3 Prove that the following statement is false.

For all integers n greater than or equal to 1, $n^2 + 3n + 1$ is a prime number. [2]

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4 **(i)** Verify the following statement:

'$2^p - 1$ is a prime number for all prime numbers p less than 11'. [2]

(ii) Calculate 23×89, and hence disprove this statement:

'$2^p - 1$ is a prime number for all prime numbers p'. [2]

5 **(i)** Disprove the following statement.

'If $p > q$, then $\frac{1}{p} < \frac{1}{q}$.' **[2]**

(ii) State a condition on p and q so that the statement is true. **[1]**

6 **(i)** Show that

(*A*) $(x - y)(x^2 + xy + y^2) = x^3 - y^3$,

(*B*) $(x + \frac{1}{2}y)^2 + \frac{3}{4}y^2 = x^2 + xy + y^2$. **[4]**

(ii) Hence prove that, for all real numbers x and y, if $x > y$ then $x^3 > y^3$. **[3]**