MEI CORE 3 PROOF REVISION

1) $2t,\ t^2-1,\ t^2+1$

i) $(2t)^2 + (t^2-1)^2$

$= 4t^2 + t^4 - 2t^2 + 1$

$= t^4 + 2t^2 + 1$

$= (t^2+1)^2$

$\therefore$ a Pythagorean triple

ii) $20^2 + 21^2$

$= 400 + 441 = 841$

$\sqrt{841} = 29$

Triple is 20, 21, 29

For $t \geqslant 3$, $2t < t^2-1$

so $2t$ is smallest of

$2t,\ t^2-1,\ t^2+1$

However, if $2t = 20$

$t = 10$

$t^2-1 = 99$, $t^2+1 = 101$

$\therefore$ this formula cannot generate the triple 20, 21, 29

It $\therefore$ does not generate all Pythagorean triples

2) Prove no 1 or 2-digit square ends in 2, 3, 7, 8

Proof by exhaustion

$0^2 = 0$
$1^2 = 1$
$2^2 = 4$
$3^2 = 9$
$4^2 = 16$
$5^2 = 25$
$6^2 = 36$
$7^2 = 49$
$8^2 = 64$
$9^2 = 81$

All the 1 or 2-digit squares have been listed and none end in 2, 3, 7 or 8

Since the last digit of any perfect square is determined by the last digit of the number being squared, the possibilities are limited to those listed above.

So no perfect squares end in 2, 3, 7 or 8

3) Prove the following statement is false

for integers $n > 1$, $n^2 + 3n + 1$ is prime

$1^2 + 3(1) + 1 = 5$

$2^2 + 3(2) + 1 = 11$

$3^2 + 3(3) + 1 = 19$

3 cont)

$4^2 + 3(4) + 1 = 29$

$5^2 + 3(5) + 1 = 41$

$6^2 + 3(6) + 1 = 53$

$7^2 + 3(7) + 1 = 71$

$8^2 + 3(8) + 1 = 89$

$9^2 + 3(9) + 1) = 109$

$10^2 + 3(10) + 1 = 131$

$11^2 + 3(11) + 1 = 155$

155 not prime since divisible by 5

$\therefore$ statement is false

4) 2^{p-1}

i)
$p=2 \quad 2^2 - 1 = 3$ prime

$p=3 \quad 2^3 - 1 = 7$ prime

$p=5 \quad 2^5 - 1 = 31$ prime

$p=7 \quad 2^7 - 1 = 123$ prime

True for all primes $p < 11$

ii) $23 \times 89 = 2047$

$= 2048 - 1$

$= 2^{11} - 1$

so not true when $p = 11$

$\therefore$ not true for all primes p.

5) i) Disprove

If $p > q$ then $\frac{1}{p} < \frac{1}{q}$

Consider $p = +1$, $q = -2$

$1 > -2$ so $p > q$

but $\frac{1}{1} > \frac{1}{-2}$, $1 > -\frac{1}{2}$

so $\frac{1}{p} > \frac{1}{q}$

$\therefore$ statement $\frac{1}{p} < \frac{1}{q}$ not true in this case.

ii) True if p, q both positive or both negative

6) i) A) $(x-y)(x^2+xy+y^2)$

$= x^3 - x^2y + x^2y - xy^2 + xy^2 - y^3$

$= x^3 - y^3$

B) $(x + \frac{1}{2}y)^2 + \frac{3}{4}y^2$

$= x^2 + xy + \frac{1}{4}y^2 + \frac{3}{4}y^2$

$= x^2 + xy + y^2$

6 ii) Combining the results of A and B gives:

6ii) cont

$$(x-y)(x^2+xy+y^2)=x^3-y^3$$

$$(x-y)\left[\left(x+\tfrac{1}{2}y\right)^2+\tfrac{3}{4}y^2\right]=x^3-y^3$$

If $x>y$ then $(x-y)>0$

$$\left[\left(x+\tfrac{1}{2}y\right)^2+\tfrac{3}{4}y^2\right]>0$$

for all values of $x>y$

$\therefore$ since both factors >0

$$x^3-y^3>0$$

$$x^3>y^3$$

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