6.6 FURTHER METHODS FOR ADVANCED MATHEMATICS, FP2 (4756) A2

Objectives

To build on and extend students' knowledge of Pure Mathematics and associated techniques.

Assessment

Examination: (72 marks)
1 hour 30 minutes.
The examination paper has two sections.

Section A: All questions are compulsory.
Three questions each worth about 18 marks.
One or two of the questions may be divided into independent parts on different topics in the specification.
Section Total: 54 marks.

Section B: One question worth 18 marks.
Section Total: 18 marks.

Assumed Knowledge

Candidates are expected to know the content for *C1*, *C2*, *C3*, *C4* and *FP1*.

Subject Criteria

Both this unit and *FP1* are required for Advanced GCE Further Mathematics.
The Units *C1, C2, C3* and *C4* are required for Advanced GCE Mathematics.

Calculators

In the MEI Structured Mathematics specification, no calculator is allowed in the examination for *C1*. For all other units, including this one, a graphical calculator is allowed.

SECTION A – All topics in this section are to be studied.

FURTHER METHODS FOR ADVANCED MATHEMATICS, FP2		
Specification	**Ref.**	**Competence Statements**
POLAR COORDINATES		
Polar coordinates in two dimensions.	FP2P1	Understand the meaning of polar coordinates (r,θ) and be able to convert from polar to Cartesian coordinates and vice-versa.
	2	Be able to sketch curves with simple polar equations.
	3	Be able to find the area enclosed by a polar curve.
CALCULUS		
The inverse functions of sine, cosine and tangent.	FP2c1	Understand the definitions of inverse trigonometric functions.
Differentiation of arcsin*x*, arccos*x* and arctan*x*.	2	Be able to differentiate inverse trigonometric functions.
Use of trigonometrical substitutions in integration.	3	Recognise integrals of functions of the form $(a^2 - x^2)^{-\frac{1}{2}}$ and $(a^2 + x^2)^{-1}$ and be able to integrate associated functions by using trigonometrical substitutions.
	4	Use trigonometric identities to integrate functions.
SERIES		
Maclaurin series. Approximate evaluation of a function.	FP2s1	Be able to find the Maclaurin series of a function, including the general term in simple cases.
	2	Appreciate that the series may converge only for a restricted set of values of *x*.
	3	Identify and be able to use the Maclaurin series of standard functions.

FURTHER METHODS FOR ADVANCED MATHEMATICS, FP2			
Ref.	**Notes**	**Notation**	**Exclusions**

POLAR COORDINATES			
FP2P1	Candidates will be expected to know the terms pole and initial line.	$r > 0$: continuous line. $r < 0$: broken line.	
2	e.g. $r = a(1+\cos\theta)$, $r = a\cos 2\theta$.		
3	Area = $\frac{1}{2}\int r^2 \mathrm{d}\theta$.		

CALCULUS			
FP2c1	arcsin: $-\frac{\pi}{2} \le \arcsin \le \frac{\pi}{2}$. arccos: $0 \le \arccos \le \pi$. arctan: $-\frac{\pi}{2} \le \arctan \le \frac{\pi}{2}$.		
2			
3			
4	e.g. $\sin^2 x, \sin^3 x, \sin^4 x, \tan x$.		

SERIES			
FP2s1			
2			
3	Candidates will be expected to identify the series for e^x, $\ln(1+x)$, $\sin x$, $\cos x$ and $(1+x)^n$. The ranges of values for which these series are valid should be understood.		

FURTHER METHODS FOR ADVANCED MATHEMATICS, FP2		
Specification	**Ref.**	**Competence Statements**

COMPLEX NUMBERS

Specification	Ref.	Competence Statements
Modulus- argument form.	FP2j1	Understand the polar (modulus-argument) form of a complex number, and the definition of modulus, argument.
	2	Be able to multiply and divide complex number in polar form.
De Moivre's theorem and simple applications.	3	Understand de Moivre's theorem.
	4	Be able to apply de Moivre's theorem to finding multiple angle formulae and to summing suitable series.
Expression of complex numbers in the form $z = r\mathrm{e}^{\mathrm{j}\theta}$.	5	Understand the definition $\mathrm{e}^{\mathrm{j}\theta} = \cos\theta + \mathrm{j}\sin\theta$ and hence the form $z = r\mathrm{e}^{\mathrm{j}\theta}$.
The *n* n^{th} roots of a complex number.	6	Know that every non-zero complex number has *n* n^{th} roots, and that in the Argand diagram these are the vertices of a regular *n*-gon.
	7	Know that the distinct n^{th} roots of $r\mathrm{e}^{\mathrm{j}\theta}$ are: $r^{\frac{1}{n}}\left[\cos\left(\frac{\theta+2k\pi}{n}\right)+\mathrm{j}\sin\left(\frac{\theta+2k\pi}{n}\right)\right]$ for $k = 0, 1, \ldots, n-1$.
	8	Be able to explain why the sum of all the n^{th} roots is zero.
Applications of complex numbers in Geometry.	9	Appreciate the effect in the Argand diagram of multiplication by a complex number.
	10	Be able to represent complex roots of unity on an Argand diagram.
	11	Be able to apply complex numbers to geometrical problems.

FURTHER METHODS FOR ADVANCED MATHEMATICS, FP2			
Ref.	**Notes**	**Notation**	**Exclusions**

COMPLEX NUMBERS			
FP2j1	$z = r(\cos\theta + j\sin\theta)$ where $r = \lvert z \rvert$ and $\theta = \arg z$.	$\lvert z \rvert$ for modulus, arg z for principal argument, $-\pi < \arg z \le \pi$.	
2	$z_1 z_2 = r_1 r_2[\cos(\theta_1 + \theta_2) + j\sin(\theta_1 + \theta_2)]$ $\frac{z_1}{z_2} = \frac{r_1}{r_2}[\cos(\theta_1 - \theta_2) + j\sin(\theta_1 - \theta_2)]$		
3			
4	e.g. the expression of $\tan 4\theta$ as a rational function of $\tan\theta$. e.g. finding $\sum_{r=0}^{n} \binom{n}{r} \cos r\theta$.		
5			
6			
7			
8			
9	Multiplication by $re^{j\theta}$ corresponds to enlargement with scale factor r with rotation through θ about the origin. e.g. multiplication by j corresponds to a rotation of $\frac{\pi}{2}$ about the origin.		
10			
11	e.g. relating to the geometry of regular polygons.		

FURTHER METHODS FOR ADVANCED MATHEMATICS, FP2		
Specification	**Ref.**	**Competence Statements**

MATRICES		
Determinant and inverse of a 3x3 matrix.	FP2m1	Be able to find the determinant of any 3x3 matrix and the inverse of a non-singular 3x3 matrix.
Eigenvalues and eigenvectors of 2x2 and 3x3 matrices.	2	Understand the meaning of eigenvalue and eigenvector, and be able to find these for 2x2 or 3x3 matrices whenever this is possible.
Diagonalisation and powers of 2x2 and 3x3 matrices	3	Be able to form the matrix of eigenvectors and use this to reduce a matrix to diagonal form.
	4	Be able to find powers of a 2x2 or 3x3 matrix.
Solution of equations.	5	Be able to solve a matrix equation or the equivalent simultaneous equations, and to interpret the solution geometrically.
The use of the Cayley-Hamilton Theorem.	6	Understand the term *characteristic equation* of a 2x2 or 3x3 matrix.
	7	Understand that every 2x2 or 3x3 matrix satisfies its own characteristic equation, and be able to use this.

FURTHER METHODS FOR ADVANCED MATHEMATICS, FP2			
Ref.	**Notes**	**Notation**	**Exclusions**

MATRICES

FP2m1			
2			Repeated eigenvalues. Complex eigenvalues.
3			
4			
5			
6	$\det(\mathbf{M} - \lambda\mathbf{I}) = 0$.		Proof of the Cayley-Hamilton theorem. Knowledge of the Jordan form.
7	e.g. to find relations between the powers of a matrix. e.g. to find the inverse matrix.		

SECTION B **The topic in this section is to be studied. Section B of the examination paper contains one mandatory question.**

SECTION B

FURTHER METHODS FOR ADVANCED MATHEMATICS, FP2		
Specification	**Ref.**	**Competence Statements**

HYPERBOLIC FUNCTIONS		
Hyperbolic functions: definitions, graphs, differentiation and integration.	FP2a4	Understand the definitions of hyperbolic functions and be able to sketch their graphs.
	5	Be able to differentiate and integrate hyperbolic functions.
Inverse hyperbolic functions, including the logarithmic forms. Use in integration.	6	Understand and be able to use the definitions of the inverse hyperbolic functions.
	7	Be able to use the logarithmic forms of the inverse hyperbolic functions.
	8	Be able to integrate $(x^2+a^2)^{-1/2}$ and $(x^2-a^2)^{-1/2}$ and related functions.

FURTHER METHODS FOR ADVANCED MATHEMATICS, FP2			
Ref.	**Notes**	**Notation**	**Exclusions**

HYPERBOLIC FUNCTIONS			
FP2a4	$\sinh x = \frac{1}{2}(e^x - e^{-x})$, $\cosh x = \frac{1}{2}(e^x + e^{-x})$, $\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}$.		
5			
6	arsinh x and artanh x can take any values but arcosh $x \geq 0$.		
7	$\operatorname{arsinh} x = \ln[x + \sqrt{(x^2+1)}]$. $\operatorname{arcosh} x = \ln[x + \sqrt{(x^2-1)}]$, $x \geq 1$. $\operatorname{artanh} x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$, $-1 < x < 1$.		
8			

Option 2 (Investigation of Curves) has been removed from this specification