1 i) $x = 3t^2 \qquad y = 2t^3$

$\frac{dx}{dt} = 6t \qquad \frac{dy}{dt} = 6t^2$

$\frac{dy}{dx} = \frac{dy}{dt} \Big/ \frac{dx}{dt} = \frac{6t^2}{6t} = t$

1 ii)

$x = \theta - \cos\theta \qquad y = \theta + \sin\theta$

$\frac{dx}{d\theta} = 1 + \sin\theta \qquad \frac{dy}{d\theta} = 1 + \cos\theta$

$\frac{dy}{dx} = \frac{dy}{d\theta} \Big/ \frac{dx}{d\theta} = \frac{1+\cos\theta}{1+\sin\theta}$

1 iii)

$x = t + \frac{1}{t} \qquad y = t - \frac{1}{t}$

$\frac{dx}{dt} = 1 - \frac{1}{t^2} \qquad \frac{dy}{dt} = 1 + \frac{1}{t^2}$

$\frac{dy}{dx} = \frac{dy}{dt} \Big/ \frac{dx}{dt}$

$= \frac{1 + \frac{1}{t^2}}{1 - \frac{1}{t^2}}$

$= \frac{t^2+1}{t^2-1}$

1 iv) $x = 3\cos\theta \qquad y = 2\sin\theta$

$\frac{dx}{d\theta} = -3\sin\theta \qquad \frac{dy}{d\theta} = 2\cos\theta$

$\frac{dy}{dx} = \frac{dy}{d\theta} \Big/ \frac{dx}{d\theta} = \frac{2\cos\theta}{-3\sin\theta}$

$= -\frac{2}{3}\cot\theta$

1 v)

$x = (t+1)^2 \qquad y = (t-1)^2$

$\frac{dx}{dt} = 2(t+1) \qquad \frac{dy}{dt} = 2(t-1)$

$\frac{dy}{dx} = \frac{dy}{dt} \Big/ \frac{dx}{dt} = \frac{2(t-1)}{2(t+1)}$

$= \frac{t-1}{t+1}$

1 vi)

$x = \theta\sin\theta + \cos\theta$

$\frac{dx}{d\theta} = \theta\cos\theta + \sin\theta - \sin\theta = \theta\cos\theta$

$y = \theta\cos\theta - \sin\theta$

$\frac{dy}{d\theta} = -\theta\sin\theta + \cos\theta - \cos\theta$

$= -\theta\sin\theta$

$\frac{dy}{dx} = \frac{dy}{d\theta} \Big/ \frac{dx}{d\theta} = \frac{-\theta\sin\theta}{\theta\cos\theta}$

$= -\tan\theta$

1 vii)

$x = e^{2t} + 1 \qquad y = e^t$

$\frac{dx}{dt} = 2e^{2t} \qquad \frac{dy}{dt} = e^t$

$\frac{dy}{dx} = \frac{dy}{dt} \Big/ \frac{dx}{dt} = \frac{e^t}{2e^{2t}}$

$= \frac{1}{2e^t}$

1 viii)
$$x = \frac{t}{1+t}$$
$$\frac{dx}{dt} = \frac{(1+t)1 - t(1)}{(1+t)^2}$$
$$y = \frac{t}{1-t}$$
$$\frac{dy}{dt} = \frac{(1-t)1 - t(-1)}{(1-t)^2}$$
$$\frac{dy}{dt} = \frac{1}{(1-t)^2}$$
$$\frac{dy}{dx} = \frac{dy}{dt} \Big/ \frac{dx}{dt}$$
$$= \frac{\frac{1}{(1-t)^2}}{\frac{1}{(1+t)^2}}$$
$$= \frac{(1+t)^2}{(1-t)^2}$$

2)
$$x = \tan\theta \qquad y = \tan 2\theta$$
$$\frac{dx}{d\theta} = \sec^2\theta \qquad \frac{dy}{d\theta} = 2\sec^2 2\theta$$
$$\frac{dy}{dx} = \frac{dy}{d\theta} \Big/ \frac{dx}{d\theta} = \frac{2\sec^2 2\theta}{\sec^2\theta}$$
$$\frac{dy}{dx} = \frac{2\cos^2\theta}{\cos^2 2\theta}$$
When $\theta = \frac{\pi}{6}$
$$\frac{dy}{dx} = \frac{2\cos^2(\frac{\pi}{6})}{\cos^2(\frac{\pi}{3})}$$
$$= \frac{2\left(\frac{\sqrt{3}}{2}\right)^2}{\left(\frac{1}{2}\right)^2}$$
$$\frac{dy}{dx} = \frac{\frac{6}{4}}{\frac{1}{4}} = 6$$

2 ii)
when $\theta = \frac{\pi}{6}$
$$x = \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$$
$$y = \tan\frac{\pi}{3} = \sqrt{3}$$
Using $y - y_1 = m(x - x_1)$

Eqn of tgt
$$y - \sqrt{3} = 6\left(x - \frac{1}{\sqrt{3}}\right)$$
$$y - \sqrt{3} = 6x - \frac{6}{\sqrt{3}}$$
$$y - \sqrt{3} = 6x - 2\sqrt{3}$$
$$y = 6x - \sqrt{3}$$

2 iii)
Normal has gradient $= -\frac{1}{6}$

Using $y - y_1 = m(x - x_1)$
$$y - \sqrt{3} = -\frac{1}{6}\left(x - \frac{1}{\sqrt{3}}\right)$$
$$y - \sqrt{3} = -\frac{1}{6}x + \frac{1}{6\sqrt{3}}$$
$$y - \sqrt{3} = -\frac{1}{6}x + \frac{\sqrt{3}}{18}$$
$$18y + 3x = \sqrt{3} + 18\sqrt{3}$$
$$18y + 3x = 19\sqrt{3}$$

3) $x = t^2$ $y = 1 - \frac{1}{2t}$

i) Cuts x axis when $y = 0$

$0 = 1 - \frac{1}{2t}$

$\Rightarrow 2t = 1$
$t = \frac{1}{2}$

when $t = \frac{1}{2}$, $x = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

P is point $\left(\frac{1}{4}, 0\right)$

3 ii)

$\frac{dx}{dt} = 2t$ $\frac{dy}{dt} = \frac{1}{2t^2}$

$\frac{dy}{dx} = \frac{dy}{dt} \Big/ \frac{dx}{dt} = \frac{\frac{1}{2t^2}}{2t}$

$\frac{dy}{dx} = \frac{1}{4t^3}$

when $t = \frac{1}{2}$,

$\frac{dy}{dx} = \frac{1}{4(\frac{1}{2})^3} = \frac{1}{\frac{1}{2}} = 2$

3 iii)

Using $y - y_1 = m(x - x_1)$

$y - 0 = 2(x - \frac{1}{4})$

Tgt $y = 2x - \frac{1}{2}$

3 iv)

Cuts y-axis at $(0, -\frac{1}{2})$

4) $x = at^2$ $y = 2at$

$\frac{dx}{dt} = 2at$ $\frac{dy}{dt} = 2a$

$\frac{dy}{dx} = \frac{dy}{dt} \Big/ \frac{dx}{dt} = \frac{2a}{2at} = \frac{1}{t}$

i) tgt at point $(at^2, 2at)$

using $y - y_1 = m(x - x_1)$

$y - 2at = \frac{1}{t}(x - at^2)$

$y - 2at = \frac{1}{t}x - at$

$y = \frac{1}{t}x + at$

4 ii)

Normal has gradient $-t$.

Using $y - y_1 = m(x - x_1)$

$y - 2at = -t(x - at^2)$

$y = -tx + at^3 + 2at$

4 iii)

Normal cuts x axis when $y = 0$

$\Rightarrow 0 = -tx + at^3 + 2at$

$tx = at^3 + 2at$

$x = at^2 + 2a$

Cuts x axis at $(at^2 + 2a, 0)$

Cuts y axis when $x = 0$

4 iii) cont) $\Rightarrow y = at^3 + 2at$

Cuts y axis at $(0, at^3 + 2at)$

5)

$x = \cos\theta$, $y = \cos 2\theta$

i) $\frac{dx}{d\theta} = -\sin\theta$ $\quad \frac{dy}{d\theta} = -2\sin 2\theta$

$\frac{dy}{dx} = \frac{dy}{d\theta} \Big/ \frac{dx}{d\theta} = \frac{-2\sin 2\theta}{-\sin\theta}$

$= \frac{2\sin 2\theta}{\sin\theta}$

$= \frac{2 \times 2\sin\theta\cos\theta}{\sin\theta}$

$= 4\cos\theta$

ii)

$\therefore \frac{dy}{dx} = 4x$

$\Rightarrow \frac{d^2y}{dx^2} = 4$

$\therefore \frac{d^2y}{dx^2} - 4 = 0$

6)

$x = at$ $\quad y = \frac{b}{t}$

i) $\frac{dx}{dt} = a$ $\quad \frac{dy}{dt} = -\frac{b}{t^2}$

$\frac{dy}{dx} = \frac{dy}{dt} \Big/ \frac{dx}{dt} = \frac{-\frac{b}{t^2}}{a}$

$\frac{dy}{dx} = -\frac{b}{at^2}$

6ii) tgt at $\left(at, \frac{b}{t}\right)$

Using $y - y_1 = m(x - x_1)$

$y - \frac{b}{t} = -\frac{b}{at^2}(x - at)$

$y - \frac{b}{t} = -\frac{b}{at^2}x + \frac{b}{t}$

$y = -\frac{b}{at^2}x + \frac{2b}{t}$

6iii)

Cuts y axis when $x = 0$

$\therefore Y$ at point $\left(0, \frac{2b}{t}\right)$

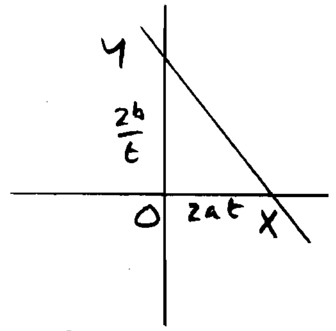
Cuts x axis when $y = 0$

$0 = -\frac{b}{at^2}x + \frac{2b}{t}$

$\frac{b}{at^2}x = \frac{2b}{t}$

$x = \frac{2bat^2}{bt} = 2at$

$\therefore X$ at point $(2at, 0)$



Area of Δ OXY $= \frac{1}{2} \times 2at \times \frac{2b}{t}$

$= 2ab$

which is constant

7) $x = 4t \qquad y = 2t^2$

i) $\frac{dx}{dt} = 4 \qquad \frac{dy}{dt} = 4t$

$$\frac{dy}{dx} = \frac{dy}{dt} \Big/ \frac{dx}{dt} = \frac{4t}{4} = t$$

$\therefore$ At $P(4t, 2t^2)$

gradient $= t$

7ii) Using $y - y_1 = m(x - x_1)$

$$y - 2t^2 = t(x - 4t)$$

$$y - 2t^2 = tx - 4t^2$$

$$y = tx - 2t^2$$

7iii)

Solve $\begin{cases} y = t_1 x - 2t_1^2 & (1) \\ y = t_2 x - 2t_2^2 & (2) \end{cases}$

(1) - (2)

$$0 = (t_1 - t_2)x + 2t_2^2 - 2t_1^2$$

$$2(t_1^2 - t_2^2) = (t_1 - t_2)x$$

$$\frac{2(t_1 + t_2)\cancel{(t_1 - t_2)}}{\cancel{(t_1 - t_2)}} = x$$

$$x = 2(t_1 + t_2)$$

Subst for x in (1)

$$y = t_1 \times 2(t_1 + t_2) - 2t_1^2$$

$$y = 2t_1^2 + 2t_1t_2 - 2t_1^2$$

$$y = 2t_1t_2$$

S is point $\left(2(t_1+t_2), 2t_1t_2\right)$

7iv) If $t_1 + t_2 = 2$

S is point $\left(2 \times 2, 2t_1(t_1 - 2)\right)$

$= \left(4, 2t_1^2 - 4t_1\right)$

Lies on straight line

$$x = 4$$

8) $x = 1 - t^2 \qquad y = 2t + 1$

i) $Q(0, 3)$ lies on curve when $t = 1$

Since $1 - 1^2 = 0$, $2(1) + 1 = 3$

8ii) $\frac{dx}{dt} = -2t \qquad \frac{dy}{dt} = 2$

$$\frac{dy}{dx} = \frac{dy}{dt} \Big/ \frac{dx}{dt} = \frac{2}{-2t} = -\frac{1}{t}$$

8iii) Using $y - y_1 = m(x - x_1)$

for $Q(0, 3)$ with parameter $t = 1$

$$y - 3 = -\frac{1}{1}(x - 0)$$

$$y - 3 = -x$$

$$y = -x + 3$$

iv) When $x = 4$, $y = -4 + 3 = -1$

$\therefore R(4, -1)$ is on tgt.

8v) Other tgt $3y - x + 7 = 0$

At point where tgt meets curve

$3(2t+1) - (1-t^2) + 7 = 0$

$6t + 3 - 1 + t^2 + 7 = 0$

$t^2 + 6t + 9 = 0$

$(t+3)^2 = 0$

$\Rightarrow t = -3$

Point $S = (1-(-3)^2, 2(-3)+1)$

$= (-8, -5)$

9) i) $x = 1 - 2t$, $y = t^2$

$P(5,4)$ on curve since

when $t = -2$

$1 - 2(-2) = 5$, $(-2)^2 = 4$

9ii)

$\frac{dx}{dt} = -2$, $\frac{dy}{dt} = 2t$

$\frac{dy}{dx} = \frac{dy}{dt} \Big/ \frac{dx}{dt} = \frac{2t}{-2} = -t$

9iii)

Using $y - y_1 = m(x - x_1)$

at point $(1-2t, t^2)$

$y - t^2 = -t(x - (1-2t))$

$y - t^2 = -t(x - 1 + 2t)$

$y - t^2 = -tx + t - 2t^2$

$y = -tx + t - t^2$

At P, $t = -2$

$\therefore y = 2x + (-2) - (-2)^2$

$y = 2x - 2 - 4$

$y = 2x - 6$

9iv)

Normal has gradient $\frac{1}{t}$

At P, normal has gradient $-\frac{1}{2}$

Using $y - y_1 = m(x - x_1)$,

Normal is $y - 4 = -\frac{1}{2}(x - 5)$

$y - 4 = -\frac{1}{2}x + \frac{5}{2}$

$y = -\frac{1}{2}x + \frac{13}{2}$

At Q $t^2 = -\frac{1}{2}(1-2t) + \frac{13}{2}$

$2t^2 = -1 + 2t + 13$

$2t^2 - 2t - 12 = 0$

$t^2 - t - 6 = 0$

$(t-3)(t+2) = 0$

$\Rightarrow t = 3$ or $t = -2$

At Q, $t = 3$

and at P, $t = -2$

Q is point $(1-2(3), 3^2) = (-5, 9)$

10) $x = 4\cos t$, $y = 3\sin t$

i) $\frac{dx}{dt} = -4\sin t$ $\frac{dy}{dt} = 3\cos t$

$$\frac{dy}{dx} = \frac{dy}{dt} \Big/ \frac{dx}{dt} = \frac{3\cos t}{-4\sin t}$$

$$= -\frac{3}{4}\cot t$$

10ii) Position at time t is $(4\cos t, 3\sin t)$

Using $y - y_1 = m(x - x_1)$

tgt is $y - 3\sin t = -\frac{3}{4}\cot t(x - 4\cos t)$

$$y - 3\sin t = -\frac{3}{4}\left(\frac{\cos t}{\sin t}\right)x + \frac{3\cos^2 t}{\sin t}$$

$$(\sin t)y - 3\sin^2 t = -\frac{3}{4}(\cos t)x + 3\cos^2 t$$

$$(\sin t)y = -\frac{3}{4}(\cos t)x + 3(\sin^2 t + \cos^2 t)$$

$$(\sin t)y = -\frac{3}{4}(\cos t)x + 3$$

$$4y\sin t = -3x\cos t + 12$$

$$4y\sin t + 3x\cos t = 12$$

10iii) Line $x + y = 0$
has gradient $= -1$

Travelling parallel to this

when $-\frac{3}{4}\cot t = -1$

$$\Rightarrow \cot t = \frac{4}{3}$$

$$\Rightarrow \tan t = \frac{3}{4}$$

$= 0.6435 + n\pi$ radians

for $n = 0, 1, 2, 3, \ldots$

11) $x = 3 + 2\cos\theta$, $y = 3 + 2\sin\theta$

i) $\frac{dx}{d\theta} = -2\sin\theta$ $\frac{dy}{d\theta} = 2\cos\theta$

$$\frac{dy}{dx} = \frac{dy}{d\theta} \Big/ \frac{dx}{d\theta} = \frac{2\cos\theta}{-2\sin\theta} = -\cot\theta$$

Find tgt at $(3 + 2\cos\theta, 3 + 2\sin\theta)$

using $y - y_1 = m(x - x_1)$

$$y - 3 - 2\sin\theta = -\frac{\cos\theta}{\sin\theta}(x - 3 - 2\cos\theta)$$

$$y\sin\theta - 3\sin\theta - 2\sin^2\theta = -\cos\theta(x - 3 - 2\cos\theta)$$

$$y\sin\theta - 3\sin\theta - 2\sin^2\theta = -x\cos\theta + 3\cos\theta + 2\cos^2\theta$$

$$y\sin\theta + x\cos\theta = 3\sin\theta + 3\cos\theta + 2$$

11ii) If $\sin\theta + \cos\theta = -\frac{2}{3}$

then
$$y\sin\theta + x\cos\theta = 3\left(-\frac{2}{3}\right) + 2 = 0$$

which is a straight line passing through origin

11iii)
$$\sin\theta + \cos\theta$$

$$= \sqrt{2}\left(\frac{1}{\sqrt{2}}\sin\theta + \frac{1}{\sqrt{2}}\cos\theta\right)$$

11 iii) (cont) $= \sqrt{2}\sin\left(\theta + \frac{\pi}{4}\right)$

Solve

$$\sin\theta + \cos\theta = -\frac{2}{3}$$

$$\sqrt{2}\sin\left(\theta + \frac{\pi}{4}\right) = -\frac{2}{3}$$

$$\sin\left(\theta + \frac{\pi}{4}\right) = -\frac{2}{3\sqrt{2}}$$

$$\theta + \frac{\pi}{4} = \sin^{-1}\left(-\frac{\sqrt{2}}{3}\right)$$

$$\theta + \frac{\pi}{4} = \pi + 0.4909,\ 2\pi - 0.4909$$

$$\theta = \pi + 0.4909 - \frac{\pi}{4},\ 2\pi - 0.4909 - \frac{\pi}{4}$$

$$\theta = 2.8471,\ 5.0069 \text{ rads.}$$

11 iv)