MEI CORE 4 TRIGONOMETRIC IDENTITIES EXERCISE 8F

1) i

$2\sin 3\theta \cos 3\theta = \sin 6\theta$

ii

$\cos^2 3\theta - \sin^2 3\theta$

$= \cos 6\theta$

iii

$\cos^2 3\theta + \sin^2 3\theta = 1$

iv

$1 - 2\sin^2 \frac{\theta}{2} = \cos\theta$

v

$\sin(\theta - \alpha)\cos\alpha + \cos(\theta - \alpha)\sin\alpha$

$= \sin(\theta - \alpha + \alpha) = \sin\theta$

vi

$3\sin\theta\cos\theta = \frac{3}{2}\sin 2\theta$

vii

$\frac{\sin 2\theta}{2\sin\theta} = \frac{2\sin\theta\cos\theta}{2\sin\theta}$

$= \cos\theta$

viii

$\cos 2\theta - 2\cos^2\theta$

$= 2\cos^2\theta - 1 - 2\cos^2\theta$

$= -1$

2) i

$(\cos x - \sin x)^2$

$= \cos^2 x + \sin^2 x - 2\sin x\cos x$

$= 1 - \sin 2x$

ii

$\cos^4 x - \sin^4 x$

$= (\cos^2 x + \sin^2 x)(\cos^2 x - \sin^2 x)$

using difference of two squares

$= 1 \times \cos 2x = \cos 2x$

iii

$2\cos^2 x - 3\sin^2 x$

$= 2\cos^2 x - 2\sin^2 x - \sin^2 x$

$= 2(\cos^2 x - \sin^2 x) - \sin^2 x$

$= 2\cos 2x - \sin^2 x$

$= 2\cos 2x - \left(\frac{1 - \cos 2x}{2}\right)$

$= 2\cos 2x - \frac{1}{2} + \frac{\cos 2x}{2}$

$= \frac{5}{2}\cos 2x - \frac{1}{2}$

(Note: $\cos 2x = 1 - 2\sin^2 x$

so $2\sin^2 x = 1 - \cos 2x$

$\sin^2 x = \frac{1 - \cos 2x}{2}$)

3) i Prove $\dfrac{1-\cos 2\theta}{1+\cos 2\theta} \equiv \tan^2\theta$

$$\frac{1-\cos 2\theta}{1+\cos 2\theta} \equiv \frac{1-(1-2\sin^2\theta)}{1+(2\cos^2\theta-1)}$$

$$\equiv \frac{2\sin^2\theta}{2\cos^2\theta} \equiv \tan^2\theta$$

ii Prove $\operatorname{cosec} 2\theta + \cot 2\theta \equiv \cot\theta$

$\operatorname{cosec} 2\theta + \cot 2\theta$

$$\equiv \frac{1}{\sin 2\theta} + \frac{\cos 2\theta}{\sin 2\theta}$$

$$\equiv \frac{1+\cos 2\theta}{\sin 2\theta}$$

$$\equiv \frac{1+(2\cos^2\theta-1)}{2\sin\theta\cos\theta}$$

$$\equiv \frac{2\cos^2\theta}{2\sin\theta\cos\theta}$$

$$\equiv \frac{\cos\theta}{\sin\theta} \equiv \cot\theta$$

iii Prove

$$\tan 4\theta \equiv \frac{4t(1-t^2)}{1-6t^2+t^4}$$

where $t = \tan\theta$

$$\tan 4\theta \equiv \frac{\tan 2\theta + \tan 2\theta}{1-\tan^2 2\theta}$$

$$\equiv \frac{2\tan 2\theta}{1-\tan^2 2\theta}$$

$$\equiv \frac{2\left(\frac{2\tan\theta}{1-\tan^2\theta}\right)}{1-\left(\frac{2\tan\theta}{1-\tan^2\theta}\right)^2}$$

Letting $t = \tan\theta$

$$\equiv \frac{2\left(\frac{2t}{1-t^2}\right)}{1-\left(\frac{2t}{1-t^2}\right)^2}$$

Multiply numerator and denominator by $(1-t^2)^2$

$$\equiv \frac{2(2t)(1-t^2)}{(1-t^2)^2-(2t)^2}$$

$$\equiv \frac{4t(1-t^2)}{1+t^4-2t^2-4t^2}$$

$$\equiv \frac{4t(1-t^2)}{1-6t^2+t^4}$$

4) i $\sin(\theta+40^\circ) = 0.7$

$\sin^{-1} 0.7 = 44.4^\circ$

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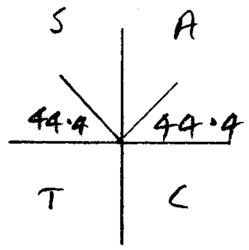
4i) cont



$$\theta + 40^\circ = 44.4^\circ$$

$$\text{or } \theta + 40^\circ = 135.6^\circ$$

$$\Rightarrow \theta = 4.4^\circ, 95.6^\circ$$

4ii) $3\cos^2\theta + 5\sin\theta - 1 = 0$

$$3(1-\sin^2\theta) + 5\sin\theta - 1 = 0$$

$$3 - 3\sin^2\theta + 5\sin\theta - 1 = 0$$

$$3\sin^2\theta - 5\sin\theta - 2 = 0$$

$$(3\sin\theta + 1)(\sin\theta - 2) = 0$$

Either $3\sin\theta + 1 = 0$

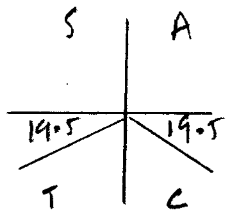
$$3\sin\theta = -1$$

$$\sin\theta = -\frac{1}{3}$$

or $\sin\theta - 2 = 0$

$\sin\theta = 2$ ✗ No solution

$\sin^{-1}\frac{1}{3} = 19.5^\circ$

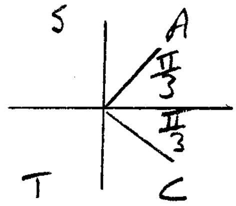


$$\theta = 199.5^\circ, 340.5^\circ$$

4iii) $2\cos\left(\theta - \frac{\pi}{6}\right) = 1$

$$\cos\left(\theta - \frac{\pi}{6}\right) = \frac{1}{2}$$

$\cos^{-1}\frac{1}{2} = \frac{\pi}{3}$



$$\theta - \frac{\pi}{6} = \frac{\pi}{3}, \frac{5\pi}{3}$$

$$\theta = \frac{\pi}{3} + \frac{\pi}{6}, \frac{5\pi}{3} + \frac{\pi}{6}$$

$$\theta = \frac{\pi}{2}, \frac{11\pi}{6}$$

$$\theta = \frac{\pi}{2}, -\frac{\pi}{6} \quad \text{for } -\pi \le \theta \le \pi$$

4iv) $\cos(45^\circ - \theta) = 2\sin(30 + \theta)$

$$\cos 45^\circ \cos\theta + \sin 45 \sin\theta$$

$$= 2\sin 30\cos\theta + 2\cos 30\sin\theta$$

$$\frac{1}{\sqrt{2}}\cos\theta + \frac{1}{\sqrt{2}}\sin\theta = \cos\theta + \sqrt{3}\sin\theta$$

$$\cos\theta + \sin\theta = \sqrt{2}\cos\theta + \sqrt{6}\sin\theta$$

$$\cos\theta - \sqrt{2}\cos\theta = \sqrt{6}\sin\theta - \sin\theta$$

$$\cos\theta(1 - \sqrt{2}) = \sin\theta(\sqrt{6} - 1)$$

$$\frac{1-\sqrt{2}}{\sqrt{6}-1} = \frac{\sin\theta}{\cos\theta} = \tan\theta$$

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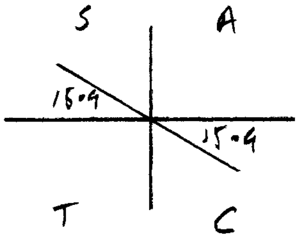
4iv) (cont)

$\theta = \tan^{-1}\left(\frac{1-\sqrt{2}}{\sqrt{6}-1}\right)$

$\theta = -15.9^\circ$



$\theta = 164.1^\circ, -15.9^\circ$

for $-180^\circ \le \theta \le 180^\circ$

4v) $\cos 2\theta + 3\sin\theta = 2$

$1 - 2\sin^2\theta + 3\sin\theta = 2$

$0 = 2 - 1 + 2\sin^2\theta - 3\sin\theta$

$0 = 2\sin^2\theta - 3\sin\theta + 1$

$0 = (2\sin\theta - 1)(\sin\theta - 1)$

$\Rightarrow 2\sin\theta - 1 = 0$

$\sin\theta = \frac{1}{2}$

or $\sin\theta - 1 = 0$

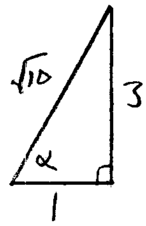
$\sin\theta = 1$

$\sin^{-1}\frac{1}{2} = \frac{\pi}{6}, \frac{5\pi}{6}$

$\sin^{-1} 1 = \frac{\pi}{2}$

$\theta = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}$

4vi) $\cos\theta + 3\sin\theta = 2$



$\sqrt{10}\left(\frac{1}{\sqrt{10}}\cos\theta + \frac{3}{\sqrt{10}}\sin\theta\right) = 2$

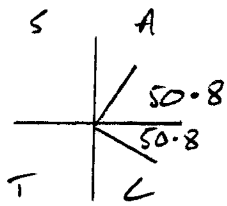
$\sqrt{10}\cos(\theta - \alpha) = 2$

where $\alpha = \tan^{-1}\frac{3}{1} = 71.6^\circ$

$\sqrt{10}\cos(\theta - 71.6^\circ) = 2$

$\cos(\theta - 71.6^\circ) = \frac{2}{\sqrt{10}}$

$\cos^{-1}\frac{2}{\sqrt{10}} = 50.8^\circ$



$\theta - 71.6 = 50.8, 309.2$

$\theta = 50.8 + 71.6, 309.2 + 71.6$

$\theta = 122.4^\circ, 380.8$

$\theta = 122.4^\circ, 20.8$

for $0 \le \theta \le 360^\circ$

4vii) $\tan^2 x - 3\tan x - 4 = 0$

$(\tan x + 1)(\tan x - 4) = 0$

$\Rightarrow \tan x = -1$ or $\tan x = 4$

$\Rightarrow x = 135^\circ$ or $x = \tan^{-1} 4$

$x = 76.0^\circ$