MEI CORE1 COORDINATE GEOMETRY EXERCISE 2F

1) $y = 3x + 1$ (1)
$y = x^2 + 3$ (2)

In (1) when $x = 1$, $y = 3\times1 + 1 = 4$

In (2) when $x = 1$, $y = 1^2 + 3 = 4$

$\therefore$ (1,4) is a point of intersection

Subst for y in (2)

$3x + 1 = x^2 + 3$

$0 = x^2 - 3x + 3 - 1$

$x^2 - 3x + 2 = 0$
$(x - 2)(x - 1) = 0$

$\Rightarrow x = 2$ or $x = 1$

When $x = 2$, $y = 3\times2 + 1 = 7$

(2,7) is other point of intersection

2)
i) Show $x + y = 6$ is a tgt to $x^2 + y^2 = 18$

$y = 6 - x$ (1)
$x^2 + y^2 = 18$ (2)

Subst for y in (2)

$x^2 + (6 - x)^2 = 18$

$x^2 + 36 - 12x + x^2 = 18$

$2x^2 - 12x + 36 - 18 = 0$
$2x^2 - 12x + 18 = 0$
$x^2 - 6x + 9 = 0$

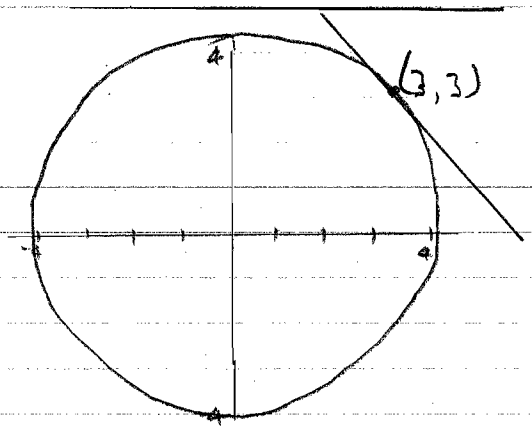
$(x - 3)(x - 3) = 0$

$\Rightarrow x = 3$

when $x = 3$ $y = 6 - 3 = 3$

(3,3) is only point of intersection, $\therefore$ line is a tgt to the circle.

ii)



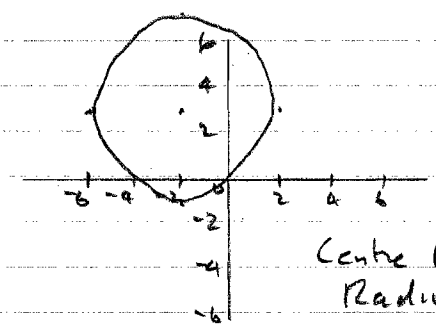
By symmetry line is of form
$x + y = k$

and passes through $(-3,-3)$

$-3 + -3 = k$
$\therefore k = -6$

Line is $x + y = -6$

3)



Centre $(-2, 3)$
Radius 4

Tgts parallel to axes

$x = -6$, $x = 2$

$y = 7$, $y = -1$

4) $y = 2x - 1$ ①

$y = x^2 - 4$ ②

Subst for y in ②

$2x - 1 = x^2 - 4$

$0 = x^2 - 2x - 4 + 1$

$x^2 - 2x - 3 = 0$

$(x - 3)(x + 1) = 0$

$\Rightarrow x = 3$ or $x = -1$

When $x = 3$, $y = 2 \times 3 - 1 = 5$

When $x = -1$ $y = 2x - 1 - 1 = -3$

Points of intersection are

$(3, 5)$ and $(-1, -3)$

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5) i) $y = 2x$ ①

$y = x^2 + 6x - 5$ ②

Subst for y in ②

$2x = x^2 + 6x - 5$

$0 = x^2 + 6x - 2x - 5$

$x^2 + 4x - 5 = 0$

$(x + 5)(x - 1) = 0$

$\Rightarrow x = -5$ or $x = 1$

When $x = -5$, $y = 2x - 5 = -10$

When $x = 1$, $y = 2 \times 1 = 2$

Points of intersection are

$(-5, -10)$ and $(1, 2)$

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ii) $y = 2x$ ①

$y = x^2 + 6x + 5$ ②

Subst for y in ②

$2x = x^2 + 6x + 5$

$0 = x^2 + 6x - 2x + 5$

$x^2 + 4x + 5 = 0$

$b^2 - 4ac = 16 - 20 = -4$

Discriminant < 0 $\therefore$ no solution

Line and curve do not cross

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6) $x - 3y + 15 = 0$ ①

$x^2 + y^2 + 2x - 6y + 5 = 0$ ②

from ① $x = 3y - 15$

Subst for x in ②

$(3y - 15)^2 + y^2 + 2(3y - 15) - 6y + 5 = 0$

$9y^2 - 90y + 225 + y^2 + 6y - 30 - 6y + 5 = 0$

$10y^2 - 90y + 200 = 0$

$y^2 - 9y + 20 = 0$

$(y - 5)(y - 4) = 0$

$\Rightarrow y = 5$ or $y = 4$

6 cont) When $y = 5$:

$x = 3 \times 5 - 15 = 0$

When $y = 4$

$x = 3 \times 4 - 15 = -3$

Points of intersection

$(0, 5)$ and $(-3, 4)$

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7)

$y = x + 1$ ①

$y = x^3 - 3x^2 + 3x + 1$ ②

Subst for y in ②

$x + 1 = x^3 - 3x^2 + 3x + 1$

$0 = x^3 - 3x^2 + 2x$

$0 = x(x^2 - 3x + 2)$

$0 = x(x-2)(x-1)$

$\Rightarrow x = 0$ or $x = 2$ or $x = 1$

When $x = 0$, $y = 0 + 1 = 1$

When $x = 1$ $y = 1 + 1 = 2$

When $x = 2$ $y = 2 + 1 = 3$

Points of intersection are

$(0, 1)$, $(1, 2)$ and $(2, 3)$

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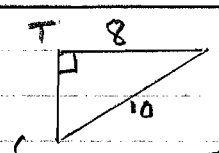
8)

i)



T is Top

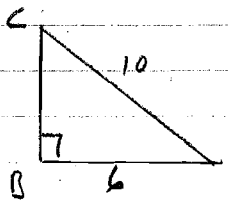
By Pythagoras

$|CT|^2 = 10^2 - 8^2$

$= 100 - 64$

$= 36$

$|CT| = 6$ cm



B is bottom

By Pythagoras

$|BC|^2 = 10^2 - 6^2$

$= 100 - 36$

$= 64$

$|BC| = 8$ cm

Height $= |BC| + |CT|$

$= 8 + 6 = 14$ cm

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ii)

Centre $(0, 8)$ radius 10

$(x - 0)^2 + (y - 8)^2 = 10^2$

$x^2 + (y - 8)^2 = 10^2$

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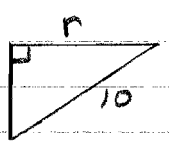
iii)

Depth 12 cm

This is 4 cm above centre



r = radius of surface

By Pythagoras $r^2 = 10^2 - 4^2$

$= 100 - 16$

$= 84$

Area $= \pi r^2$

$= 84\pi$ cm^2 (or 264 cm^2)