1) i) $y = \frac{1}{x^4} = x^{-4}$

$\frac{dy}{dx} = -4x^{-5} = -\frac{4}{x^5}$

ii) $y = 4x^{-5}$, $\frac{dy}{dx} = -20x^{-6}$

iii) $y = 7x^{-6}$, $\frac{dy}{dx} = -42x^{-7}$

iv) $y = \frac{3}{x^2} = 3x^{-2}$

$\frac{dy}{dx} = -6x^{-3} = -\frac{6}{x^3}$

v) $y = \frac{3}{x^5} = 3x^{-5}$

$\frac{dy}{dx} = -15x^{-6} = -\frac{15}{x^6}$

vi) $y = \frac{2}{x} + x^3 = 2x^{-1} + x^3$

$\frac{dy}{dx} = -2x^{-2} + 3x^2$

$= -\frac{2}{x^2} + 3x^2$

vii) $y = \frac{5}{x^3} - \frac{2}{x} + 1$

$y = 5x^{-3} - 2x^{-1} + 1$

$\frac{dy}{dx} = -15x^{-4} + 2x^{-2}$

$= -\frac{15}{x^4} + \frac{2}{x^2}$

viii) $y = x^{\frac{1}{4}}$, $\frac{dy}{dx} = \frac{1}{4}x^{-3/4}$

ix) $y = 6x^{\frac{1}{3}}$, $\frac{dy}{dx} = 2x^{-\frac{2}{3}}$

x) $y = x^{3/4}$, $\frac{dy}{dx} = \frac{3}{4}x^{-1/4}$

xi) $y = x^{-\frac{2}{3}}$, $\frac{dy}{dx} = -\frac{2}{3}x^{-5/3}$

xii) $y = \frac{1}{\sqrt{x}} = x^{-\frac{1}{2}}$

$\frac{dy}{dx} = -\frac{1}{2}x^{-3/2}$

xiii) $y = \sqrt[5]{x} = x^{\frac{1}{5}}$

$\frac{dy}{dx} = \frac{1}{5}x^{-4/5}$

xiv) $y = \sqrt{x} + \frac{1}{x^3} + 2x$

$y = x^{\frac{1}{2}} + x^{-3} + 2x$

$\frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}} - 3x^{-4} + 2$

$= \frac{1}{2\sqrt{x}} - \frac{3}{x^4} + 2$

1 xv) $y = (\sqrt[3]{x})^4 = x^{4/3}$

$\frac{dy}{dx} = \frac{4}{3}x^{\frac{1}{3}} = \frac{4\sqrt[3]{x}}{3}$

2)
i) $y = x^{-2}$ $(0.25, 16)$

$\frac{dy}{dx} = -2x^{-3}$

At $(0.25, 16)$ $\frac{dy}{dx} = \frac{-2}{(\frac{1}{4})^3}$

$\frac{dy}{dx} = -\frac{2}{(\frac{1}{64})} = -128$

ii) $y = x^{-1} + x^{-4}$ $(-1, 0)$

$\frac{dy}{dx} = -x^{-2} - 4x^{-5}$

At $(-1,0)$ $\frac{dy}{dx} = -\frac{1}{(-1)^2} - \frac{4}{(-1)^5}$

$= -1 + 4 = 3$

iii) $y = 4x^{-3} + 2x^{-5}$ $(1, 6)$

$\frac{dy}{dx} = -12x^{-4} - 10x^{-6}$

At $(1,6)$ $\frac{dy}{dx} = \frac{-12}{1^4} - \frac{10}{1^6}$

$= -12 - 10 = -22$

iv) $y = 3x^4 - 4 - 8x^{-3}$ $(2, 43)$

$\frac{dy}{dx} = 12x^3 + 24x^{-4}$

At $(2,43)$ $\frac{dy}{dx} = 12 \times 2^3 + \frac{24}{2^4}$

$\frac{dy}{dx} = 96 + \frac{24}{16}$

$= 97.5$

v) $y = \sqrt{x} + 3x$ $(4, 14)$

$\frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}} + 3$

At $(4,14)$ $\frac{dy}{dx} = \frac{1}{2\sqrt{4}} + 3$

$= 3\frac{1}{4}$

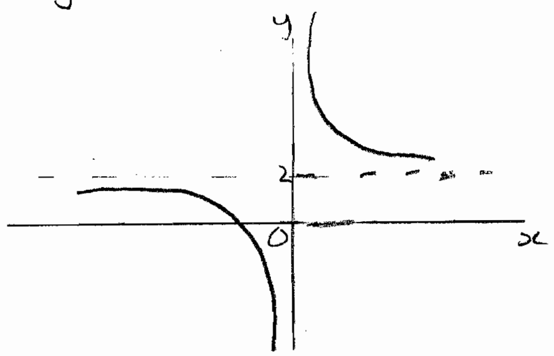
vi) $y = 4x^{-\frac{1}{2}}$ $(9, 1\frac{1}{3})$

$\frac{dy}{dx} = -2x^{-3/2}$

At $(9, 1\frac{1}{3})$ $\frac{dy}{dx} = -2 \times \frac{1}{9^{3/2}}$

$= -2 \times \frac{1}{27} = -\frac{2}{27}$

3)
i) $y = \frac{1}{x} + 2$



3 ii) Crosses x axis at $y=0$

$$\Rightarrow \quad 0 = \frac{1}{x} + 2$$

$$\Rightarrow \quad -2 = \frac{1}{x}$$

$$\Rightarrow \quad x = -\frac{1}{2}$$

Crosses x axis at $\left(-\frac{1}{2}, 0\right)$

3 iii)

$$y = \frac{1}{x} + 2 = x^{-1} + 2$$

$$\frac{dy}{dx} = -1x^{-2} = -\frac{1}{x^2}$$

3 iv) At $\left(-\frac{1}{2}, 0\right)$ $\frac{dy}{dx} = -\frac{1}{\left(-\frac{1}{2}\right)^2}$

$$= -\frac{1}{\frac{1}{4}}$$

$$= -4$$

4)

$$f(x) = 9x + \frac{4}{x} = 9x + 4x^{-1}$$

$$f'(x) = 9 - 4x^{-2}$$

$$f'(x) = 9 - \frac{4}{x^2}$$

Function $f(x)$ decreasing when $f'(x) < 0$

$$9 - \frac{4}{x^2} < 0$$

$$\Rightarrow \quad \frac{4}{x^2} > 9$$

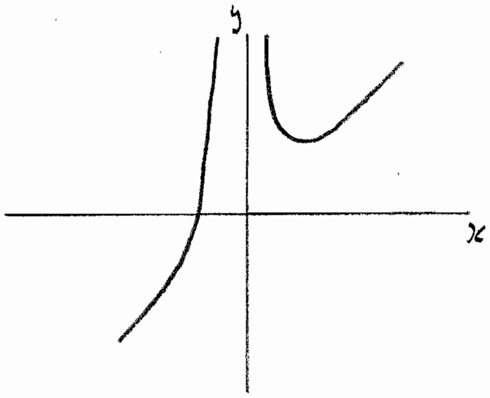
$$\Rightarrow \quad 4 > 9x^2$$

$$\Rightarrow \quad \frac{4}{9} > x^2$$

$$\Rightarrow -\frac{2}{3} < x < \frac{2}{3}$$

5)

$$y = \frac{4}{x^2} + x$$



i) $y = 4x^{-2} + x$

$$\frac{dy}{dx} = -8x^{-3} + 1$$

ii) When $x = -2$,

$$y = \frac{4}{(-2)^2} + (-2)$$

$$= \frac{4}{4} - 2 = -1$$

$\therefore (-2, -1)$ is on curve

iii) At $(-2, -1)$

$$\frac{dy}{dx} = -\frac{8}{(-2)^3} + 1 = 1 + 1 = 2$$

iv) When $x = 2$, $y = \frac{4}{2^2} + 2 = 3$

$\therefore (2,3)$ is on curve

v)

At $(2,3)$ $\frac{dy}{dx} = -\frac{8}{2^3} + 1 = 0$

vi)

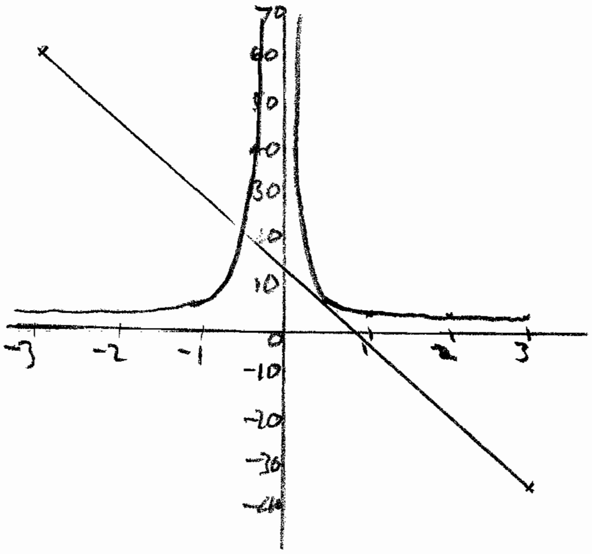
Minimum point at $(2,3)$

6)

i) $y = \frac{1}{x^2} + 1$, $y = -16x + 13$

x	-3	-2	-1	0	1	2	3
y	61	45	29	13	-3	-19	-35

x	-3	-2	-1	0	1	2	3
y	$\frac{10}{9}$	$\frac{5}{4}$	2	∞	2	$\frac{5}{4}$	$\frac{10}{9}$



ii) When $x = 0{\cdot}5$ $y = \frac{1}{\frac{1}{4}} + 1 = 5$

When $x = 0{\cdot}5$ $y = -8 + 3 = 5$

$\therefore (0.5, 5)$ on both graphs

iii) $y = \frac{1}{x^2} + 1 = x^{-2} + 1$

$\frac{dy}{dx} = -2x^{-3} = -\frac{2}{x^3}$

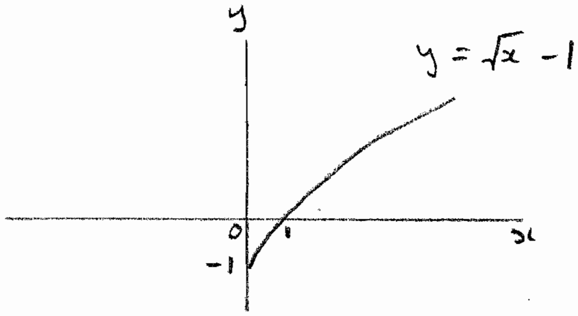
At $(0.5, 5)$ $\frac{dy}{dx} = -\frac{2}{\left(\frac{1}{2}\right)^3} = -16$

iv)

Same gradient at $(0.5, 5)$

$\therefore y = -16x + 13$ is a tgt to $y = \frac{1}{x^2} + 1$ at $(0.5, 5)$

7)



i)

$y = x^{\frac{1}{2}} - 1$

$\frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}}$

ii)

Line parallel to $y = 2x - 1$ has gradient 2

If $\frac{dy}{dx} = 2$ then $\frac{1}{2\sqrt{x}} = 2$

$\Rightarrow \frac{1}{2} = 2\sqrt{x}$

$\Rightarrow \frac{1}{4} = \sqrt{x}$

$\Rightarrow x = \frac{1}{16}$

7ii) cont) when $x = \frac{1}{16}$, $y = \sqrt{\frac{1}{16}} - 1$

$= -\frac{3}{4}$

Point on curve is $\left(\frac{1}{16}, -\frac{3}{4}\right)$

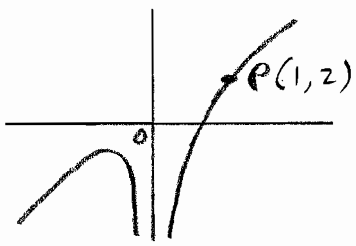
iii) $y = 2x - 1$ is not a tgt to the curve

The only point where $\frac{dy}{dx} = 2$ is $\left(\frac{1}{16}, -\frac{3}{4}\right)$.

When $x = \frac{1}{16}$, $2x - 1 = -\frac{7}{8}$

$\therefore$ $y = 2x - 1$ does not touch the curve at $\left(\frac{1}{16}, -\frac{3}{4}\right)$

8) $y = 3x - \frac{1}{x^2}$



i) $y = 3x - x^{-2}$

$\frac{dy}{dx} = 3 + 2x^{-3} = 3 + \frac{2}{x^3}$

ii) At $(1,2)$ $\frac{dy}{dx} = 3 + \frac{2}{1^3} = 5$

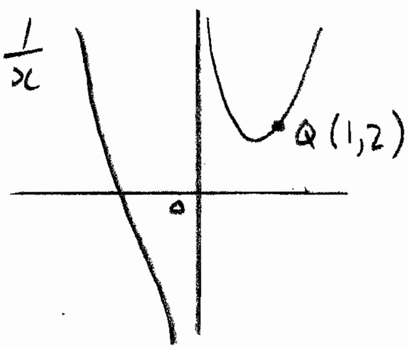
iii) Using $y - y_1 = m(x - x_1)$

Tgt is $y - 2 = 5(x - 1)$

$y - 2 = 5x - 5$

$y = 5x - 3$

9) $y = x^2 + \frac{1}{x}$



i) $y = x^2 + x^{-1}$

$\frac{dy}{dx} = 2x - x^{-2} = 2x - \frac{1}{x^2}$

ii) At $Q(1,2)$ $\frac{dy}{dx} = 2 - \frac{1}{1^2} = 1$

$\therefore$ gradient of tgt $= 1$

iii) Gradient of normal $= -1$

Using $y - y_1 = m(x - x_1)$

Normal is $y - 2 = -1(x - 1)$

$y - 2 = -x + 1$

$x + y = 3$

iv) Solve $\left.\begin{array}{l} x + y = 3 \quad (1) \\ y = x^2 + \frac{1}{x} \quad (2) \end{array}\right\}$

From (1) $y = 3 - x$

Subst for y in (2)

$3 - x = x^2 + \frac{1}{x}$

$3x - x^2 = x^3 + 1$

$x^3 + x^2 - 3x + 1 = 0$

9iv) cont) Curve and line intersect at (1, 2) ∴ $(x-1)$ is a factor of x^3+x^2-3x+1

$$\begin{array}{r|l} & x^2+2x-1 \\ \hline x-1 & x^3+x^2-3x+1 \\ & x^3-x^2 \\ \hline & 2x^2-3x \\ & 2x^2-2x \\ \hline & -x+1 \\ & -x+1 \\ \hline \end{array}$$

$\therefore (x-1)(x^2+2x-1)=0$

$$x = \frac{-2 \pm \sqrt{4+4}}{2}$$

$$x = \frac{-2 \pm 2\sqrt{2}}{2}$$

$$x = -1 \pm \sqrt{2}$$

When $x = -1+\sqrt{2}$
$y = 3-(-1+\sqrt{2}) = 4-\sqrt{2}$

when $x = -1-\sqrt{2}$
$y = 3-(-1-\sqrt{2}) = 4+\sqrt{2}$

Other points of intersection are

$(-1+\sqrt{2},\ 4-\sqrt{2})$

and $(-1-\sqrt{2},\ 4+\sqrt{2})$

10)
i) $y = x^{-2} \Rightarrow \frac{dy}{dx} = -2x^{-3}$

$\Rightarrow \frac{d^2y}{dx^2} = 6x^{-4}$

10ii) $y = x^{3/2} \Rightarrow \frac{dy}{dx} = \frac{3}{2}x^{\frac{1}{2}}$

$\Rightarrow \frac{d^2y}{dx^2} = \frac{3}{4}x^{-\frac{1}{2}}$

10iii) $y = x^4 - \frac{2}{x^3} = x^4 - 2x^{-3}$

$\Rightarrow \frac{dy}{dx} = 4x^3+6x^{-4}$

$= 4x^3 + \frac{6}{x^4}$

$\Rightarrow \frac{d^2y}{dx^2} = 12x^2 - 24x^{-5}$

$= 12x^2 - \frac{24}{x^5}$

11)
i) $y = x + \frac{1}{x} = x + x^{-1}$

$\frac{dy}{dx} = 1 - x^{-2} = 1 - \frac{1}{x^2}$

At st pt $\frac{dy}{dx} = 0$

$1 - \frac{1}{x^2} = 0$

$\Rightarrow x^2 - 1 = 0$

$\Rightarrow x = 1$ or $x = -1$

when $x=1$, $y = 1+\frac{1}{1} = 2$

when $x=-1$, $y = -1+\frac{1}{-1} = -2$

st pts are $(1, 2)$ and $(-1, -2)$

11 i) (cont)

$$\frac{d^2y}{dx^2} = 2x^{-3} = \frac{2}{x^3}$$

When $x = 1$, $\frac{d^2y}{dx^2} = \frac{2}{1^3} > 0$

$\therefore$ a minimum at $(1, 2)$

when $x = -1$, $\frac{d^2y}{dx^2} = \frac{2}{(-1)^3} < 0$

$\therefore$ a maximum at $(-1, -2)$

ii)

$$y = x^3 + \frac{12}{x} = x^3 + 12x^{-1}$$

$$\frac{dy}{dx} = 3x^2 - 12x^{-2} = 3x^2 - \frac{12}{x^2}$$

At st pt $\frac{dy}{dx} = 0$

$$3x^2 - \frac{12}{x^2} = 0$$

$\Rightarrow 3x^4 - 12 = 0$

$\Rightarrow 3x^4 = 12$

$\Rightarrow x^4 = 4$

$\Rightarrow x^2 = 2$

$\Rightarrow x = \pm\sqrt{2}$

When $x = \sqrt{2}$ $y = 2\sqrt{2} + \frac{12}{\sqrt{2}}$

$= 2\sqrt{2} + 6\sqrt{2}$

$= 8\sqrt{2}$

When $x = -\sqrt{2}$, $y = -2\sqrt{2} - 6\sqrt{2}$

$= -8\sqrt{2}$

$\therefore$ st pts at $(\sqrt{2}, 8\sqrt{2})$

and $(-\sqrt{2}, -8\sqrt{2})$

$$\frac{d^2y}{dx^2} = 6x + 24x^{-3}$$

$$= 6x + \frac{24}{x^3}$$

When $x = \sqrt{2}$

$$\frac{d^2y}{dx^2} = 6\sqrt{2} + \frac{24}{2\sqrt{2}} > 0$$

$\therefore$ a min at $(\sqrt{2}, 8\sqrt{2})$

when $x = -\sqrt{2}$

$$\frac{d^2y}{dx^2} = -6\sqrt{2} + \frac{24}{-2\sqrt{2}} < 0$$

$\therefore$ a max at $(-\sqrt{2}, -8\sqrt{2})$

iii)

$$y = 6x - x^{3/2}$$

$$\frac{dy}{dx} = 6 - \frac{3}{2}x^{\frac{1}{2}}$$

At st pt $\frac{dy}{dx} = 0$

$$6 - \frac{3}{2}x^{\frac{1}{2}} = 0$$

$$\Rightarrow 6 = \frac{3}{2}x^{\frac{1}{2}}$$

11 iii) cont)

$6 \times \frac{2}{3} = x^{\frac{1}{2}}$

$4 = x^{\frac{1}{2}}$

$\Rightarrow x = 16$

when $x = 16$, $y = 6 \times 16 - 16^{3/2}$

$= 96 - 64$

$= 32$

st pt at $(16, 32)$

$\frac{d^2y}{dx^2} = -\frac{3}{4}x^{-\frac{1}{2}}$

$= -\frac{3}{4\sqrt{x}}$

when $x = 16$

$\frac{d^2y}{dx^2} = -\frac{3}{16} < 0$

$\therefore$ a max at $(16, 32)$

12)

$y = x - 4\sqrt{x} = x - 4x^{\frac{1}{2}}$

i) $\frac{dy}{dx} = 1 - 2x^{-\frac{1}{2}} = 1 - \frac{2}{\sqrt{x}}$

$\frac{d^2y}{dx^2} = x^{-3/2}$

ii) At t.p $\frac{dy}{dx} = 0$

$\Rightarrow 1 - \frac{2}{dx} = 0$

$\sqrt{x} - 2 = 0$

$\sqrt{x} = 2$

$x = 4$

when $x = 4$, $y = 4 - 4\sqrt{4}$

$= -4$

Turning point is $(4, -4)$

when $x = 4$, $\frac{d^2y}{dx^2} = 4^{-\frac{3}{2}}$

$= \frac{1}{4^{3/2}}$

$= \frac{1}{8} > 0$

$\therefore (4, -4)$ is a minimum

13)

$y = 6\sqrt{x} - x\sqrt{x}$

$y = 6x^{\frac{1}{2}} - x^{3/2}$

$\frac{dy}{dx} = 3x^{-\frac{1}{2}} - \frac{3}{2}x^{\frac{1}{2}}$

$\frac{dy}{dx} = \frac{3}{\sqrt{x}} - \frac{3\sqrt{x}}{2}$

At a t.p. $\frac{dy}{dx} = 0$

$\Rightarrow \frac{3}{\sqrt{x}} - \frac{3\sqrt{x}}{2} = 0$

$\Rightarrow 6 - 3x = 0$

$\Rightarrow x = 2$

13 cont)

$$\frac{d^2y}{dx^2} = -\frac{3}{2}x^{-3/2} - \frac{3}{4}x^{-\frac{1}{2}}$$

$$\frac{d^2y}{dx^2} < 0 \quad \text{for all } x > 0$$

$\therefore$ a maximum at $x = 2$

14)

$$y = x^{5/2} - 10x^{3/2}$$

i) If $0 = x^{5/2} - 10x^{3/2}$

$\Rightarrow 0 = x^{3/2}(x - 10)$

$\Rightarrow x = 0$ or $x = 10$

ii)

$$\frac{dy}{dx} = \frac{5}{2}x^{3/2} - 15x^{1/2}$$

$$\frac{d^2y}{dx^2} = \frac{15}{4}\sqrt{x} - \frac{15}{2}x^{-\frac{1}{2}}$$

when $x = 6$

$$\frac{dy}{dx} = \frac{5}{2} \times 6\sqrt{6} - 15\sqrt{6} = 0$$

$\therefore$ a stationary point

when $x = 6$

$$\frac{d^2y}{dx^2} = \frac{15}{4}\sqrt{6} - \frac{15}{2\sqrt{6}} > 0$$

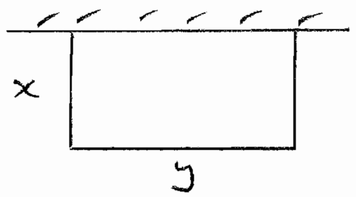
$\therefore$ a minimum

when $x = 6$

$y = 6^{5/2} - 10 \times 6^{3/2}$

$y = -58.8$ to 1 dp

15)



i) Area $= xy$

ii) $T = 2x + y$

iii) Area $= 18 = xy$

$\therefore y = \frac{18}{x}$

Subst for y $\quad T = 2x + \frac{18}{x}$

iv) $T = 2x + 18x^{-1}$

$$\frac{dT}{dx} = 2 - 18x^{-2}$$

$$\frac{d^2T}{dx^2} = 36x^{-3}$$

v) At st pt $\frac{dT}{dx} = 0$

$\Rightarrow 2 - \frac{18}{x^2} = 0$

$\Rightarrow 2x^2 - 18 = 0$

$x^2 = 9$

$x = \pm 3$

Ignore $x = -3$

15v) cont) st pt when $x = 3$

When $x = 3$, $\frac{d^2T}{dx^2} = \frac{36}{3^3} > 0$

$\therefore$ a minimum at $x = 3$

Dimensions $x = 3\,m$

$y = \frac{18}{3} = 6\,m$

16)
i) $V = x^2 y$

ii) Assuming external surfaces only

$A = 4xy + x^2$

iii) $0.5 = x^2 y$

$\Rightarrow y = \frac{0.5}{x^2}$

$\Rightarrow A = 4x\left(\frac{0.5}{x^2}\right) + x^2$

$A = \frac{2}{x} + x^2$

iv) $A = 2x^{-1} + x^2$

$\frac{dA}{dx} = -2x^{-2} + 2x$

$\frac{d^2A}{dx^2} = 4x^{-3} + 2$

v) For st pt $\frac{dA}{dx} = 0$

$\Rightarrow -\frac{2}{x^2} + 2x = 0$

$\Rightarrow -2 + 2x^3 = 0$

$2x^3 = 2$

$x^3 = 1$

$\Rightarrow x = 1$

When $x = 1$, $\frac{d^2A}{dx^2} = \frac{4}{1^3} + 2 > 0$

$\therefore$ a minimum when $x = 1$.

For minimum Area

$x = 1\,m$

$y = \frac{0.5}{1^2} = 0.5\,m$

17)
i) Volume = length × breadth × height

$972 = 3x \times x \times h$

$972 = 3x^2 h$

$\Rightarrow h = \frac{972}{3x^2} = \frac{324}{x^2}$

ii) $A = 2 \times 3x^2 + 2 \times 3xh + 2 \times xh$

$= 6x^2 + 6xh + 2xh$

$= 6x^2 + 8xh$

$= 6x^2 + 8x \times \frac{324}{x^2}$

$= 6x^2 + \frac{2592}{x}$

17iii) $\dfrac{dA}{dx} = 12x - \dfrac{2592}{x^2}$

At st. pt. $\dfrac{dy}{dx} = 0$

$$12x - \frac{2592}{x^2} = 0$$

$$\Rightarrow 12x^3 - 2592 = 0$$

$$\Rightarrow x^3 = \frac{2592}{12} = 216$$

$$\Rightarrow x = 6$$

$$\frac{d^2y}{dx^2} = 12 + \frac{5184}{x^3}$$

When $x = 6$, $\dfrac{d^2y}{dx^2} = 12 + \dfrac{5184}{216}$

$$\frac{d^2y}{dx^2} > 0$$

$\therefore$ A a minimum when $x = 6$

iv) When $x = 6$cm

$$A = 6 \times 6^2 + \frac{2592}{6}$$

$$A = 648 \text{ cm}^2$$

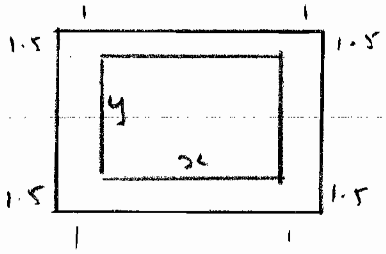
Dimensions

Length $= 3x = 18$ cm

Width $= x = 6$ cm

Height $= \dfrac{324}{x^2} = 9$ cm

18)



i) Area of Lawn $= 24\text{ m}^2$

$$\therefore xy = 24$$

$$\Rightarrow y = \frac{24}{x}$$

ii) $A = (x+2)(y+3)$

$$= (x+2)\left(\frac{24}{x} + 3\right)$$

$$= 24 + \frac{48}{x} + 3x + 6$$

$$A = 30 + 3x + \frac{48}{x}$$

iii) $\dfrac{dA}{dx} = 3 - \dfrac{48}{x^2}$

For min Area $\dfrac{dA}{dx} = 0$

$$3 - \frac{48}{x^2} = 0$$

$$3x^2 = 48$$

$$x^2 = 16$$

$$x = 4 \quad (\text{ignore } -4)$$

$\dfrac{d^2A}{dx^2} = +\dfrac{96}{x^3} > 0$ when $x = 4$

$\therefore$ a minimum

18iii) cont) when $x = 4$

$$A = 30 + 3\times 4 + \frac{48}{4}$$

$$A = 30 + 12 + 12$$

$$A = 54 \text{ m}^2$$

19) i) $V = \pi r^2 h$

$$\Rightarrow \frac{\pi}{8} = \pi r^2 h$$

$$\Rightarrow h = \frac{\pi}{8\pi r^2} = \frac{1}{8r^2}$$

ii)

$$A = \pi r^2 + 2\pi r h$$

$$A = \pi r^2 + 2\pi r \times \frac{1}{8r^2}$$

$$A = \pi r^2 + \frac{\pi}{4r}$$

iii)

$$\frac{dA}{dr} = 2\pi r - \frac{\pi}{4r^2}$$

$$= 2\pi r - \frac{\pi}{4} r^{-2}$$

$$\frac{d^2A}{dr^2} = 2\pi + \frac{\pi r^{-3}}{2}$$

$$= 2\pi + \frac{\pi}{2r^3}$$

For min A, $\frac{dA}{dr} = 0$

$$\Rightarrow 2\pi r - \frac{\pi}{4r^2} = 0$$

$$8\pi r^3 - \pi = 0$$

$$8\pi r^3 = \pi$$

$$r^3 = \frac{\pi}{8\pi} = \frac{1}{8}$$

$$\Rightarrow r = \frac{1}{2}$$

when $r = \frac{1}{2}$, $\frac{d^2A}{dr^2} = 2\pi + \frac{\pi}{(\frac{1}{4})}$

$\frac{d^2A}{dr^2} > 0 \therefore$ a min

Dimensions for minimum Area

$$r = \frac{1}{2}, \quad h = \frac{1}{8(\frac{1}{2})^2}$$

$$h = \frac{1}{2}$$

$$h = r = \frac{1}{2} \text{ metre}$$

iv)

$$A = \pi\left(\frac{1}{2}\right)^2 + \frac{\pi}{4\times\frac{1}{2}}$$

$$A = \frac{\pi}{4} + \frac{\pi}{2}$$

$$A = \frac{3\pi}{4} \text{ m}^2$$

20) Cost

$$= 0.8v^2 + \frac{2000}{v}$$

per hour

20 i) cont)

Time $= \dfrac{100}{V}$

ii)

$$C = \left(0.8V^2 + \frac{2000}{V}\right) \times \frac{100}{V}$$

$$C = 80V + \frac{200000}{V^2}$$

iii)

$$\frac{dC}{dV} = 80 - \frac{400000}{V^3}$$

For min C, $\dfrac{dC}{dV} = 0$

$$\Rightarrow 80 - \frac{400000}{V^3} = 0$$

$$\Rightarrow 80V^3 - 400000 = 0$$

$$\Rightarrow V^3 = \frac{400000}{80}$$

$$\Rightarrow V^3 = 5000$$

$$\Rightarrow V = 17.1 \text{ km h}^{-1}$$

Check $\dfrac{d^2C}{dV^2} = +\dfrac{1200000}{V^4}$

$\dfrac{d^2C}{dV^2} > 0$ when $V = 17.1$

$\therefore$ C is a minimum

iv)

$$C \approx 80 \times 17.1 + \frac{200000}{17.1^2}$$

$$C \approx £2052$$

21)

$$V = \pi r^2 h$$

i)

$V = 1$, $\therefore$ $1 = \pi r^2 h$

$$h = \frac{1}{\pi r^2}$$

ii)

$$S = 2\pi r h + \pi r^2$$

$$= 2\pi r \times \frac{1}{\pi r^2} + \pi r^2$$

$$S = \frac{2}{r} + \pi r^2$$

iii)

For S min, $\dfrac{dS}{dr} = 0$

$$\frac{dS}{dr} = -\frac{2}{r^2} + 2\pi r$$

$$2\pi r - \frac{2}{r^2} = 0$$

$$\Rightarrow 2\pi r^3 - 2 = 0$$

$$2\pi r^3 = 2$$

$$r^3 = \frac{2}{2\pi} = \frac{1}{\pi}$$

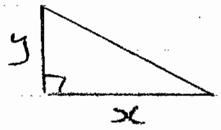
$$r \approx 0.683 \text{ m}$$

Check $\dfrac{d^2S}{dr^2} = 2\pi + \dfrac{4}{r^3}$

> 0 when $r = 0.683$

$\therefore$ S is a minimum for $r = 0.683$ m

22)



Area $= 8\text{ cm}^2$

i)

Area $= \frac{1}{2} \times \text{base} \times \text{height}$

$8 = \frac{1}{2} \times x \times y$

$8 = \frac{xy}{2}$

$\Rightarrow \quad y = \frac{16}{x}$

ii)

$S = x^2 + \left(\frac{16}{x}\right)^2$

$S = x^2 + \frac{256}{x^2}$

iii

$\frac{dS}{dx} = 2x - \frac{512}{x^3}$

For S min, $\frac{dS}{dx} = 0$

$\Rightarrow \quad 2x - \frac{512}{x^3} = 0$

$\Rightarrow \quad 2x^4 - 512 = 0$

$\Rightarrow \quad x^4 = 256$

$\Rightarrow \quad x = 4$ (ignore -4)

Check $\frac{d^2S}{dx^2} = 2 + \frac{1536}{x^4}$

> 0 for $x = 4$

$\therefore$ S a min when $x = 4$

When $x = 4$

$S = 4^2 + \frac{256}{4^2}$

$S = 16 + 16$

$S = 32$

iv)

Shortest length for hypotenuse $= \sqrt{32}$

$= 4\sqrt{2}$ cm