1 i) $\int 10x^{-4}\,dx = \frac{10x^{-3}}{-3} + c$

$= -\frac{10}{3}x^{-3} + c$

ii) $\int (2x - 3x^{-4})\,dx$

$= \frac{2x^2}{2} - \frac{3x^{-3}}{-3} + c$

$= x^2 + x^{-3} + c$

iii) $\int (2 + x^3 + 5x^{-3})\,dx$

$= 2x + \frac{x^4}{4} + \frac{5x^{-2}}{-2} + c$

$= 2x + \frac{x^4}{4} - \frac{5x^{-2}}{2} + c$

iv) $\int (6x^2 - 7x^{-2})\,dx$

$= \frac{6x^3}{3} - \frac{7x^{-1}}{-1} + c$

$= 2x^3 + 7x^{-1} + c$

v) $\int 5x^{\frac{1}{4}}\,dx$

$= \frac{5x^{5/4}}{\frac{5}{4}} + c$

$= 4x^{5/4} + c$

vi) $\int \frac{1}{x^4}\,dx$

$= \int x^{-4}\,dx$

$= \frac{x^{-3}}{-3} + c$

$= -\frac{1}{3x^3} + c$

vii) $\int \sqrt{x}\,dx = \int x^{\frac{1}{2}}\,dx$

$= \frac{x^{3/2}}{3/2} + c$

$= \frac{2}{3}x^{3/2} + c$

viii) $\int \left(2x^4 - \frac{4}{x^2}\right)dx$

$= \int (2x^4 - 4x^{-2})\,dx$

$= \frac{2x^5}{5} - \frac{4x^{-1}}{-1} + c$

$= \frac{2x^5}{5} + \frac{4}{x} + c$

ix) $\int 6x^{-3/2}\,dx$

$= \frac{6x^{-\frac{1}{2}}}{-\frac{1}{2}} + c = -12x^{-\frac{1}{2}} + c$

1x) $\int x\sqrt{x}\,dx = \int x^{3/2}\,dx$

$= \dfrac{x^{5/2}}{\frac{5}{2}} + c$

$= \dfrac{2}{5}x^{5/2} + c$

2) i) $\int_1^4 3x^{-2}\,dx$

$= \left[\dfrac{3x^{-1}}{-1}\right]_1^4$

$= \left[-\dfrac{3}{x}\right]_1^4$

$= \left(-\dfrac{3}{4}\right) - \left(-\dfrac{3}{1}\right)$

$= -\dfrac{3}{4} + 3 \quad = 2\dfrac{1}{4}$

ii) $\int_2^4 8x^{-3}\,dx$

$= \left[\dfrac{8x^{-2}}{-2}\right]_2^4$

$= \left[-\dfrac{4}{x^2}\right]_2^4$

$= \left(-\dfrac{4}{16}\right) - \left(-\dfrac{4}{4}\right)$

$= -\dfrac{1}{4} + 1 \quad = \dfrac{3}{4}$

iii) $\int_2^4 12x^{\frac{1}{2}}\,dx$

$= \left[\dfrac{12x^{3/2}}{3/2}\right]_2^4$

$= \left[8x^{3/2}\right]_2^4$

$= (8 \times 8) - (8 \times 2^{3/2})$

$= 64 - 22.6$

$= 41.4$ to 3 sig fig

iv) $\int_1^4 x^{-5/2}\,dx$

$= \left[\dfrac{x^{-3/2}}{-\frac{3}{2}}\right]_1^4$

$= \left[-\dfrac{2}{3}x^{-3/2}\right]_1^4$

$= \left[-\dfrac{2}{3x^{3/2}}\right]_1^4$

$= \left(-\dfrac{2}{3 \times 8}\right) - \left(-\dfrac{2}{3 \times 1}\right)$

$= -\dfrac{2}{24} + \dfrac{2}{3} \quad = \dfrac{14}{24}$

$= \dfrac{7}{12}$

2) v) $\int_{-3}^{-1} \frac{6}{x^3}\,dx$

$= \int_{-3}^{-1} 6x^{-3}\,dx$

$= \left[\frac{6x^{-2}}{-2}\right]_{-3}^{-1}$

$= \left[-\frac{3}{x^2}\right]_{-3}^{-1}$

$= \left(-\frac{3}{1}\right) - \left(-\frac{3}{9}\right)$

$= -3 + \frac{1}{3} = -2\frac{2}{3}$

---

vi) $\int_{1}^{9} \frac{1}{x^{3/2}}\,dx$

$= \int_{1}^{9} x^{-3/2}\,dx$

$= \left[\frac{x^{-\frac{1}{2}}}{-\frac{1}{2}}\right]_{1}^{9}$

$= \left[-\frac{2}{x^{1/2}}\right]_{1}^{9}$

$= \left(-\frac{2}{3}\right) - \left(-\frac{2}{1}\right)$

$= -\frac{2}{3} + 2 = 1\frac{1}{3}$

---

vii) $\int_{1}^{8} \left(\frac{x^2 + 3x + 4}{x^4}\right) dx$

$= \int_{1}^{8} \left(\frac{1}{x^2} + \frac{3}{x^3} + \frac{4}{x^4}\right) dx$

$= \int_{1}^{8} \left(x^{-2} + 3x^{-3} + 4x^{-4}\right) dx$

$= \left[\frac{x^{-1}}{-1} + \frac{3x^{-2}}{-2} + \frac{4x^{-3}}{-3}\right]_{1}^{8}$

$= \left[-\frac{1}{x} - \frac{3}{2x^2} - \frac{4}{3x^3}\right]_{1}^{8}$

$= \left(-\frac{1}{8} - \frac{3}{128} - \frac{1}{384}\right) - \left(-1 - \frac{3}{2} - \frac{4}{3}\right)$

$= 3.682$ to 3 dp

---

viii) $\int_{4}^{9} \left(x - \frac{1}{\sqrt{x}}\right) dx$

$= \int_{4}^{9} \left(x - x^{-\frac{1}{2}}\right) dx$

$= \left[\frac{x^2}{2} - \frac{x^{1/2}}{\frac{1}{2}}\right]_{4}^{9}$

$= \left[\frac{x^2}{2} - 2\sqrt{x}\right]_{4}^{9}$

$= \left(\frac{81}{2} - 6\right) - \left(\frac{16}{2} - 4\right)$

$= 30.5$

3) $y = \sqrt{x} + \frac{1}{\sqrt{x}}$ for $x > 0$

$$\text{Area} = \int_1^9 y\,dx = \int_1^9 \left(x^{\frac{1}{2}} + x^{-\frac{1}{2}}\right)dx$$

$$= \left[\frac{x^{3/2}}{3/2} + \frac{x^{1/2}}{\frac{1}{2}}\right]_1^9$$

$$= \left[\frac{2}{3}x^{3/2} + 2x^{1/2}\right]_1^9$$

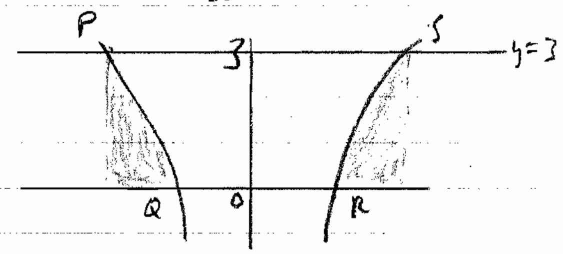
$$(18+6) - \left(\frac{2}{3} + 2\right)$$

$$= 24 - 2\tfrac{2}{3}$$

$$= 21\tfrac{1}{3} \text{ units}^2$$

4) $y = 4 - \frac{16}{x^2}$, $y = 3$

i)



When $y = 3$

$$3 = 4 - \frac{16}{x^2}$$

$$\frac{16}{x^2} = 4 - 3 = 1$$

$$\Rightarrow x = \pm 4$$

$\therefore$ $P(-4, 3)$ and $S(4, 3)$

When $y = 0$

$$4 - \frac{16}{x^2} = 0$$

$$\Rightarrow x^2 = 4$$

$$\Rightarrow x = \pm 2$$

$\therefore$ $Q(-2, 0)$ and $R(2, 0)$

ii)

$$\text{Shaded Area} = 2\int_2^4 \left(4 - \frac{16}{x^2}\right)dx$$

$$= 2\int_2^4 \left(4 - 16x^{-2}\right)dx$$

$$= 2\left[4x - \frac{16x^{-1}}{-1}\right]_2^4$$

$$= 2\left[4x + \frac{16}{x}\right]_2^4$$

$$= 2\left[(16+4) - (8+8)\right]$$

$$= 8 \text{ units}^2$$

5) $y = \sqrt{x}$, $y = \frac{128}{x^3}$

Intersect when $\sqrt{x} = \frac{128}{x^3}$

$$x^{7/2} = 128$$

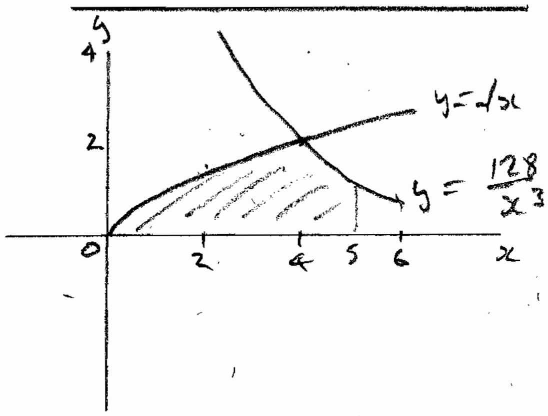
$$x = 128^{\frac{2}{7}}$$

$$x = 4$$

5 i) cont When $x = 4$ $y = \sqrt{4} = 2$

Intersect at $(4, 2)$

ii)



iii)

iv) Area $= \int_0^4 \sqrt{x}\,dx + \int_4^5 \frac{128}{x^3}\,dx$

$= \left[\frac{2}{3}x^{3/2}\right]_0^4 + \left[-\frac{64}{x^2}\right]_4^5$

$= \left[\left(\frac{16}{3}\right) - (0)\right] + \left[\left(-\frac{64}{25}\right) - \left(-\frac{64}{16}\right)\right]$

$= \frac{16}{3} - \frac{64}{25} + 4$

$= 6.773$ units^2 to 3 dp.

6) $y = 1 - \frac{3}{x^2}$ $\quad y = -\frac{2}{x^3}$

Meet when

$1 - \frac{3}{x^2} = -\frac{2}{x^3}$

$\Rightarrow x^3 - 3x = -2$

$\Rightarrow x^3 - 3x + 2 = 0$

as required

$(-2)^3 - 3(-2) + 2$

$= -8 + 6 + 2 = 0$

$\therefore x = -2$ is a root

When $x = -2$ $y = -\frac{2}{(-2)^3} = \frac{1}{4}$

$\therefore$ A is point $(-2, \frac{1}{4})$

$$\begin{array}{r|l} & x^2 - 2x + 1 \\ \hline x+2 & x^3 \qquad - 3x + 2 \\ & x^3 + 2x^2 \\ \hline & -2x^2 - 3x \\ & -2x^2 - 4x \\ \hline & x + 2 \\ & x + 2 \\ \hline \end{array}$$

$(x+2)(x^2 - 2x + 1) = 0$

$(x+2)(x-1)^2 = 0$

$\Rightarrow x = -2$ or $x = 1$

When $x = 1$, $y = -\frac{2}{1^3} = -2$

$\therefore$ B is point $(1, -2)$

iii Area $= \int_{-2}^{-1} \left(-\frac{2}{x^3} - \left(1 - \frac{3}{x^2}\right)\right) dx$

$= \int_{-2}^{-1} \left(-\frac{2}{x^3} - 1 + \frac{3}{x^2}\right) dx$

6iii) cont
$$= \int_{-2}^{-1} \left(-2x^{-3} - 1 + 3x^{-2}\right) dx$$

$$= \left[\frac{-2x^{-2}}{-2} - x + \frac{3x^{-1}}{-1} \right]_{-2}^{-1}$$

$$= \left[\frac{1}{x^2} - x - \frac{3}{x} \right]_{-2}^{-1}$$

$$= (1 + 1 + 3) - \left(\frac{1}{4} + 2 + \frac{3}{2}\right)$$

$$= 5 - 3\tfrac{3}{4} = 1\tfrac{1}{4} \text{ units}^2$$

7)
$$\frac{dy}{dx} = \frac{2}{x^2} - 3$$

i) $$\frac{dy}{dx} = 2x^{-2} - 3$$

$$y = \frac{2x^{-1}}{-1} - 3x + c$$

$$y = -\frac{2}{x} - 3x + c$$

ii Through $(2, 10)$

$$\Rightarrow 10 = -\frac{2}{2} - 3(2) + c$$

$$10 = -1 - 6 + c$$

$$\Rightarrow c = 17$$

$$y = -\frac{2}{x} - 3x + 17$$

8)
$$\frac{dy}{dx} = \sqrt{x} = x^{\frac{1}{2}}$$

$$\Rightarrow y = \frac{x^{3/2}}{3/2} + c$$

$$y = \frac{2}{3}x^{3/2} + c$$

Through $(9, 20)$

$$\Rightarrow 20 = \frac{2}{3} \times 9^{3/2} + c$$

$$20 = 18 + c$$

$$\Rightarrow c = 2$$

$$y = \frac{2}{3}x^{3/2} + 2$$

9)
$$\text{Area} = \int_1^8 x \, dy$$

$$y = x^3 \Rightarrow x = \sqrt[3]{y}$$

$$\text{Area} = \int_1^8 y^{\frac{1}{3}} \, dy$$

$$= \left[\frac{y^{4/3}}{4/3} \right]_1^8$$

9 cont) $= \left[\frac{3}{4} y^{4/3} \right]_1^8$

$= \left(\frac{3}{4} \times 16 \right) - \left(\frac{3}{4} \right)$

$= 11\frac{1}{4}$ units^2

---

10) $\int_1^4 x \, dy$ when $y = \frac{1}{\sqrt{x}}$

$y = \frac{1}{\sqrt{x}} \Rightarrow y^2 = \frac{1}{x}$

$\Rightarrow x = \frac{1}{y^2}$

$\therefore \int_1^4 x \, dy = \int_1^4 \frac{1}{y^2} \, dy$

$= \int_1^4 y^{-2} \, dy$

$= \left[\frac{y^{-1}}{-1} \right]_1^4 = \left[-\frac{1}{y} \right]_1^4$

$= \left(-\frac{1}{4} \right) - \left(-\frac{1}{1} \right)$

$= \frac{3}{4}$

---

11) $\frac{dv}{dt} = \frac{2}{t^{\frac{1}{2}}}$ for $t > 0$

$v = \int \frac{2}{t^{\frac{1}{2}}} \, dt$

$v = \int 2t^{-\frac{1}{2}} \, dt$

$v = \frac{2t^{\frac{1}{2}}}{\frac{1}{2}} + c$

$v = 4t^{\frac{1}{2}} + c$

Given $v = 5$ when $t = 0$

$\therefore 5 = 0 + c$

$\Rightarrow c = 5$

$v = 4t^{\frac{1}{2}} + 5$

---

12) $v = 20\sqrt{t} - 5t \quad 0 \le t \le 16$

$v = 20t^{\frac{1}{2}} - 5t$

i) $\frac{dv}{dt} = 10t^{-\frac{1}{2}} - 5$

At max $\frac{dv}{dt} = 0$

$\frac{10}{\sqrt{t}} - 5 = 0$

$10 - 5\sqrt{t} = 0$

$10 = 5\sqrt{t}$

$2 = \sqrt{t}$

$\Rightarrow t = 4$ seconds

check $\frac{d^2v}{dt^2} = -5t^{-3/2}$

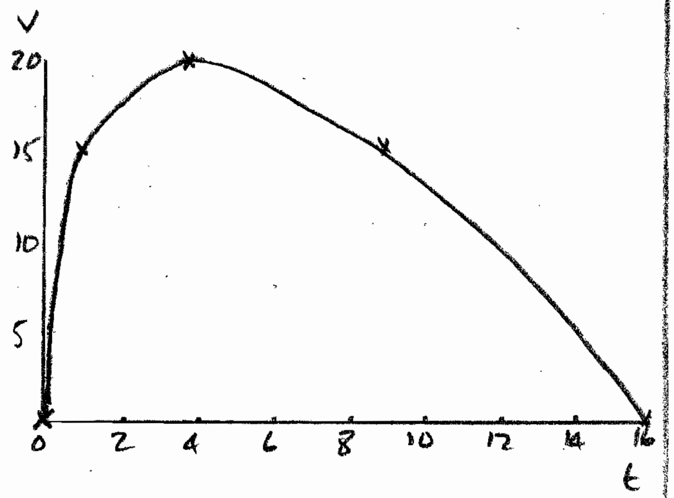
12i) cont when $t = 4$ $\frac{d^2v}{dt^2} = -5 \times 8$

$= -40 < 0$

$\therefore$ a max when $t = 4$

12ii)

t	0	1	4	9	16
v	0	15	20	15	0



12iii)

$$\text{Area} = \int_0^{16} v \, dt$$

$$= \int_0^{16} (20\sqrt{t} - 5t) \, dt$$

$$= \int_0^{16} (20t^{\frac{1}{2}} - 5t) \, dt$$

$$= \left[\frac{20t^{3/2}}{3/2} - \frac{5t^2}{2} \right]_0^{16}$$

$$= \left[\frac{40t^{3/2}}{3} - \frac{5t^2}{2} \right]_0^{16}$$

$$= \left(\frac{40 \times 64}{3} - \frac{5 \times 256}{2} \right) - (0 - 0)$$

$= 213\frac{1}{3}$ units^2

or $213\frac{1}{3}$ m travelled