MEI CORE 2 GEOMETRIC PROGRESSIONS EXERCISE 7C

1) i) 5, 10, 20, 40, ...

GP $r = 2$ $a = 5$

7^{th} term $= ar^6 = 5 \times 2^6 = 320$

ii) 2, 4, 6, 8, ... AP not GP

iii) 1, −1, 1, −1, ...

GP $r = -1$ $a = 1$

7^{th} term $= ar^6 = 1 \times (-1)^6 = 1$

iv) 5, 5, 5, 5, ...

GP $r = 1$, $a = 5$

7^{th} term $= 5$

v) 6, 3, 0, −3, ... AP not GP

vi) 6, 3, $1\frac{1}{2}$, $\frac{3}{4}$, ...

GP $r = \frac{1}{2}$, $a = 6$

7^{th} term $= ar^6 = 6 \times (\frac{1}{2})^6 = \frac{3}{32}$

or 0.09375

vii) 1, 1.1, 1.11, 1.111, ...

Not a GP

2) i) GP $a = 3$, $r = 2$

8^{th} term $= ar^7 = 3 \times 2^7 = 384$

ii) $S_n = \dfrac{a(r^n - 1)}{r - 1}$

$S_8 = \dfrac{3(2^8 - 1)}{2 - 1}$

$S_8 = 765$

3) GP

$a = 5$, 5^{th} term $ar^4 = 1280$

i) $a = 5$ ①

$ar^4 = 1280$ ②

② ÷ ① $r^4 = \frac{1280}{5} = 256$

$\Rightarrow r = \pm 4$

$r = 4$ since all terms +ve

ii) 8^{th} term $= ar^7 = 5 \times 4^7$

$= 81,920$

4) GP

$a = \frac{1}{9}$, $r = 3$

i) 5^{th} term $= ar^4 = \frac{1}{9} \times 3^4$

$= 9$

ii) If n^{th} term exceeds 1000

$ar^{n-1} > 1000$

$\frac{1}{9} \times 3^{n-1} > 1000$

$3^{n-1} > 9000$

$3^8 = 6561$, $3^9 = 19,683$

4ii) cont)

$\Rightarrow n-1=9$

$\Rightarrow n=10$

10^{th} term is first to exceed 1000

5) GP 8, 16, ..., 2048

i) $a=8,\ r=2$

Let 2048 be n^{th} term

$\Rightarrow ar^{n-1}=2048$

$\Rightarrow 8\times 2^{n-1}=2048$

$\Rightarrow 2^{n-1}=256$

$2^8=256$ so $n=9$

9 terms in sequence

ii) $S_n=\dfrac{a(r^n-1)}{r-1}$

$S_9=\dfrac{8(2^9-1)}{2-1}$

$S_9=4088$

6) GP

$200, 50, \ldots\ 0.1953125$

i) $a=200,\ r=0.25$

Let 0.1953125 be n^{th} term

$ar^{n-1}=0.1953125$

$200\times 0.25^{n-1}=0.1953125$

$0.25^{n-1}=0.0009765625$

By trial and error from calc

$0.25^5=0.0009765625$

$\Rightarrow n=6\quad \therefore$ 6 terms

ii) $S_n=\dfrac{a(1-r^n)}{1-r}$

$S_6=200\left(\dfrac{1-0.25^6}{1-0.25}\right)$

$S_6=266.6$ to 4 s.f.

7) GP

i) 5^{th} term $ar^4=48$ ①

9^{th} term $ar^8=768$ ②

② ÷ ① $r^4=16$

$\Rightarrow r=\pm 2$

$r=2$ since all terms +ve

ii) $a\times 2^4=48$

$16a=48$

$a=3$

iii) $S_n=\dfrac{a(r^n-1)}{r-1}$

$S_{10}=\dfrac{3(2^{10}-1)}{2-1}$

$S_{10}=3069$

8) GP $4, 2, 1, \ldots$

i) $r = \frac{1}{2}$

ii) $S_\infty = \frac{a}{1-r} = \frac{4}{1-\frac{1}{2}}$

$S_\infty = 8$

9) GP

$0.7, 0.07, 0.007, \ldots$

i) $r = 0.1$

ii) $S_\infty = \frac{a}{1-r} = \frac{0.7}{1-0.1}$

$= \frac{0.7}{0.9} = \frac{7}{9}$

iii) GP

$0.7, -0.07, +0.007, \ldots$

$a = 0.7, \quad r = -0.1$

$S_\infty = \frac{a}{1-r} = \frac{0.7}{1+0.1}$

$= \frac{0.7}{1.1} = \frac{7}{11}$

Also $0.7 - 0.07 = 0.63$

$0.63 + 0.007 = 0.637$

$0.637 - 0.0007 = 0.6363$

Continuing gives $0.\dot{6}\dot{3}$

10) GP $100, 90, 81, \ldots$

i) $r = 0.9$

ii) Let n^{th} term be first < 1

$ar^{n-1} < 1$

$100 \times 0.9^{n-1} < 1$

$0.9^{n-1} < 0.01$

By calc $0.9^{25} = 0.07\ldots$

$0.9^{28} = 0.05\ldots$

$0.9^{32} = 0.03\ldots$

$0.9^{40} = 0.0147\ldots$

$0.9^{43} = 0.0107\ldots$

$0.9^{44} = 0.00969\ldots$

$n - 1 = 44$

$\Rightarrow n = 45$

45^{th} term is first < 1

iii) $S_\infty = \frac{a}{1-r} = \frac{100}{1-0.9}$

$S_\infty = 1000$

iv) Want $S_n > 990$

$S_n = \frac{a(1-r^n)}{1-r} = \frac{100(1-0.9^n)}{1-0.9}$

$S_n = 1000(1-0.9^n)$

We want $1000(1-0.9^n) > 990$

$1 - 0.9^n > \frac{990}{1000}$

$1 - 0.99 > 0.9^n$

$0.01 > 0.9^n$

10iv) cont) From part ii we found

$0.99^{43} > 0.01$

$0.99^{44} < 0.01$

$\therefore \quad n = 44$

Exceeds 99% after 44 terms

11) GP

$a = 4 \qquad S_\infty = 5$

i) $S_\infty = \dfrac{a}{1-r} = 5$

$\Rightarrow \dfrac{4}{1-r} = 5$

$4 = 5(1-r)$

$4 = 5 - 5r$

$5r = 1$

$r = \frac{1}{5}$

ii) $S_\infty = 5 - 4 = 1$

12) GP

i) 3^{rd} term $= ar^2 = 16$ ①

4^{th} term $= ar^3 = 12.8$ ②

② ÷ ① $\quad r = \dfrac{12.8}{16} = 0.8$

Subst in ① $\quad a \times 0.8^2 = 16$

$\Rightarrow a = \dfrac{16}{0.64}$

$a = 25$

$r = 0.8$

12ii) $S_n = 2^{(2n+1)} - 2$

$a = S_1 = 2^3 - 2 = 6$

$S_2 = a + ar = 2^5 - 2 = 30$

$\Rightarrow \quad 6 + 6r = 30$

$6r = 24$

$r = 4$

$\therefore \quad a = 6, \quad r = 4$

13)

i) $a = 3$

$ar = 4$

$\Rightarrow r = \frac{4}{3}$

3rd term $= ar^2$

$= 3 \times \left(\frac{4}{3}\right)^2$

$= \dfrac{16}{3}$ or $5\frac{1}{3}$

ii) a) $x,\ 4,\ x+6$

are consecutive terms of GP

$\Rightarrow \dfrac{4}{x} = r = \dfrac{x+6}{4}$

Cross multiplying gives

$16 = x(x+6)$

$16 = x^2 + 6x$

$x^2 + 6x - 16 = 0$

13ii) cont

$$(x+8)(x-2)=0$$

$\Rightarrow x=-8$

or $x=2$

b) when $x=-8$

$$r=\frac{4}{-8}=-\frac{1}{2}$$

when $x=2$

$$r=\frac{4}{2}=2$$

13iii)

a) $x,\ 4,\ x+6$

are 6^{th}, 7^{th} and 8^{th} terms of GP

For S_∞ to exist, $r=-\frac{1}{2}$

7^{th} term $=ar^6=4$

$$a\left(-\tfrac{1}{2}\right)^6=4$$

$$\frac{a}{64}=4$$

$$a=256$$

b)

$$S_\infty=\frac{a}{1-r}=\frac{256}{1--\frac{1}{2}}$$

$$=\frac{256}{3/2}$$

$$=\frac{512}{3}=170\tfrac{2}{3}$$

14) GP $54, 18, 6, 2$

i)

$$r=\frac{18}{54}=\frac{1}{3}$$

ii)

n^{th} term $=ar^{n-1}$

$$=54\times\left(\frac{1}{3}\right)^{n-1}$$

iii)

$$S_n=\frac{a(1-r^n)}{1-r}$$

$$=\frac{54\left(1-\left(\frac{1}{3}\right)^n\right)}{1-\frac{1}{3}}$$

$$=\frac{54\left(1-\left(\frac{1}{3}\right)^n\right)}{\frac{2}{3}}$$

$$=81\left(1-\left(\tfrac{1}{3}\right)^n\right)$$

iv)

$$S_\infty=\frac{a}{1-r}$$

$$=\frac{54}{1-\frac{1}{3}}$$

$$=\frac{54}{3/2}$$

$$=81$$

v)

$$S_n=81\left(1-\left(\tfrac{1}{3}\right)^n\right)>80.999$$

$$1-\left(\tfrac{1}{3}\right)^n>\frac{80.999}{81}$$

$$1-\frac{80.999}{81}>\left(\tfrac{1}{3}\right)^n$$

14v) cont)

$$\frac{81-80.999}{81} > \left(\frac{1}{3}\right)^n$$

$$\frac{0.001}{81} > \left(\frac{1}{3}\right)^n$$

$$\frac{81}{0.001} < 3^n$$

$$81000 < 3^n$$

By calc $3^{10} = 59049$

$3^{11} = 177147$

$\therefore$ 11 terms are required

15) GP

i)

$$S_3 = \frac{a(1-r^3)}{1-r} = 4.88$$

$$S_\infty = \frac{a}{1-r} = 10$$

Subst for $\frac{a}{1-r}$

$$10(1-r^3) = 4.88$$

$$\Rightarrow 1-r^3 = 0.488$$

$$1-0.488 = r^3$$

$$0.512 = r^3$$

$$\Rightarrow r = 0.8$$

Find a

$$S_\infty = \frac{a}{1-r} = 10$$

$$\frac{a}{1-0.8} = 10$$

$$\frac{a}{0.2} = 10$$

$$\Rightarrow a = 10 \times 0.2 = 2$$

$$a = 2$$

$u_k = 2, 2\times0.8, 2\times0.8^2, \ldots$

$u_k = 2, 1.6, 1.28, \ldots$

for $k = 1, 2, 3, \ldots$

$(u_k)^2 = 4, 2.56, 1.6384, \ldots$

This is a GP with

$a = 4, \quad r = 0.64$

$$\therefore \sum_{k=1}^{\infty}(u_k)^2 = \frac{a}{1-r}$$

$$= \frac{4}{1-0.64}$$

$$= \frac{4}{0.36}$$

$$= 11.\dot{1}$$

$$= 11\tfrac{1}{9}$$

16)

i) 20, 10, 5, 2.5, 1.25 litres

ii) 0, 10, 15, 17.5, 18.75 litres

16iii) 1st sequence is a GP

$a = 20, \quad r = \frac{1}{2}$

2nd sequence is not a GP

since $\frac{15}{10} \neq \frac{17.5}{15}$

so no constant ratio between consecutive terms

17)

i) GP $a = 30^\circ, \quad r = 0.95$

n^{th} term $= ar^{n-1}$

We require $ar^{n-1} < 1$

$30 \times 0.95^{n-1} < 1$

$0.95^{n-1} < \frac{1}{30}$

$0.95^{n-1} < 0.0333\dot{3}$

By calc

$0.95^{66} = 0.03386\ldots\ldots$

$0.95^{67} = 0.03217\ldots\ldots$

$\Rightarrow n - 1 = 67$

$n = 68$

After 68 swings angle is $< 1^\circ$

ii) $S_n = \frac{a(1-r^n)}{1-r}$

$S_n = \frac{30(1-0.95^n)}{1-0.95}$

$S_n = 600(1-0.95^n)$

$S_{10} = 600(1-0.95^{10})$

$= 240.76^\circ$ to 2 dp.

18)

i) Bounces $10 \times \frac{2}{3}$ after 1st impact

Bounces $10 \times \left(\frac{2}{3}\right)^2$ after 2nd impact

Bounces $10 \times \left(\frac{2}{3}\right)^n$ after n^{th} impact

ii) Distance $= 20$ m to 1st impact

Another $20 \times \frac{2}{3}$ to 2nd impact

Another $20 \times \left(\frac{2}{3}\right)^2$ to 3rd impact

GP $a = 20, \quad r = \frac{2}{3}$

Find S_{10}

$S_n = \frac{a(1-r^n)}{1-r}$

$S_{10} = \frac{20\left(1-\left(\frac{2}{3}\right)^{10}\right)}{1-\frac{2}{3}}$

$S_{10} = 60\left(1-\left(\frac{2}{3}\right)^{10}\right)$

$S_{10} = 58.9595$

$= 59.0$ m to 3 s.f.

19)

AP $a, \quad a+d, \quad a+2d$

GP $a, \quad ar, \quad ar^2$

Since $\frac{ar}{a} = \frac{ar^2}{ar} = r$

$\Rightarrow \frac{a+d}{a} = \frac{a+2d}{a+d}$

$\Rightarrow (a+d)^2 = a(a+2d)$

$\Rightarrow a^2 + 2ad + d^2 = a^2 + 2ad$

$\Rightarrow d^2 = 0$

$\Rightarrow d = 0$

Since $a + d = ar$

$a = ar$

$\Rightarrow r = 1$

20)

i) $15000, \ 15000 \times 1.08, \ 15000 \times 1.08^2$

GP $a = 15000, \quad r = 1.08$

ii) 10^{th} term $= ar^9$

$= 15000 \times 1.08^9$

$=$ £29,985 to n'est £

iii) $S_n = \frac{a(r^n - 1)}{r - 1}$

$S_{10} = \frac{15000(1.08^{10} - 1)}{1.08 - 1}$

$=$ £217,298

$\approx$ £217,500

iv) AP, $a = 15000$

$S_n = \frac{n}{2}(2a + (n-1)d)$

$S_{10} = \frac{10}{2}(30000 + 9d)$

Require $5(30000 + 9d) = 217,298$

$30000 + 9d = 43,459.6$

$9d = 13459.6$

$d = 1495.51$

$d =$ £1496 to n'est £

21)

i) After 3 months

a) $1 + 3 + 9 + 27 = 40$

After 6 months

b) $40 + 81 + 243 + 729$

$= 1093$

ii) $1093 \times$ £20

$=$ £21,860

iii) £21,860 $-$ £200

$=$ £21,660

21 iii) cont

My profit $= £200 - £20$

$= £180$

21 iv)

After 1 month recruit 3

2nd month recruit 9

3rd mont recruit 27

GP $a = 3$, $r = 3$

Total recruits after n months

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

$$S_n = \frac{3(3^n - 1)}{3 - 1}$$

$$S_n = 1.5(3^n - 1)$$

Require $1.5(3^n - 1) > 60$ million

$$3^n - 1 > 40{,}000{,}000$$

$$3^n > 40{,}000{,}001$$

By calc $3^{15} = 14{,}348{,}907$

$3^{16} = 43{,}046{,}721$

By 16th month all UK residents in the pyramid

22) i) AP $a = 5$ kg, $d = 2$ kg

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$S_n = \frac{n}{2}(10 + 2(n-1))$$

$$S_n = 5n + n(n-1)$$

$$S_n = 5n + n^2 - n$$

$$S_n = n^2 + 4n$$

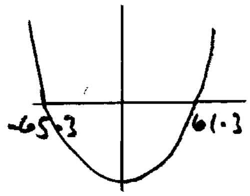
Require $n^2 + 4n > 4000$

$$n^2 + 4n - 4000 > 0$$

$$n = \frac{-4 \pm \sqrt{16 + 16000}}{2}$$

$$n = \frac{-4 \pm 126.55}{2}$$

$n \approx -65.3$ or $n \approx 61.3$



$$n > 61.3$$

Will take 62 days

22 ii) a) GP $a = 200$, $r = 0.95$

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

$$S_n = \frac{200(1 - 0.95^n)}{1 - 0.05}$$

$$S_n = 4000(1 - 0.95^n)$$

22 ii) b) Require $4000(1-0.95^n) > 3900$

$$1-0.95^n > \frac{3900}{4000}$$

$$1-\frac{3900}{4000} > 0.95^n$$

$$\frac{4000-3900}{4000} > 0.95^n$$

$$0.025 > 0.95^n$$

By calc $0.95^{71} = 0.0262\ldots$

$0.95^{72} = 0.0248\ldots$

Takes 72 days to spread 3900kg

c) Will never spread 4000 kg

Since $S_\infty = \dfrac{200}{1-0.05}$

$= 4000$ kg

Would need an infinite number of days!!

23) i)a) GP 6, 2.4, 0.96

$a = 6$, $r = \dfrac{2.4}{6} = 0.4$

n^{th} term $= ar^{n-1}$

$= 6 \times 0.4^{n-1}$

b) $S_n = \dfrac{a(1-r^n)}{1-r}$

$$S_8 = \frac{6(1-0.4^8)}{1-0.4}$$

$$S_8 = 9.9934464$$

23 ii) After 1st 15×0.4

After 2nd 15×0.4^2

After n^{th} 15×0.4^n

Require $15 \times 0.4^n < 0.01$

$$0.4^n < \frac{0.01}{15}$$

$$0.4^n < 0.000\dot{6}$$

By calc $0.4^7 = 0.00163\ldots$

$0.4^8 = 0.00065536\ldots$

After 8 impacts

23 iii) To 1st impact 15 m

To 2nd impact $2 \times 15 \times 0.4$

To 3rd impact $2 \times 15 \times 0.4^2$

Sum to n^{th} impact

$$15 + 30 \times 0.4$$
$$+ 30 \times 0.4^2$$
$$+ \ldots + 30 \times 0.4^n$$

23iii) cont)

$= 15 + 12 + 12\times0.4 + 12\times0.4^2 + \dots + 12\times0.4^{n-1}$

$= 15 + S_n$

where S_n is sum of first n terms of GP with $a=12$, $r=0.4$

$= 15 + \frac{a(1-r^n)}{1-r}$

$= 15 + \frac{12(1-0.4^n)}{1-0.4}$

$= 15 + 20(1-0.4^n)$

$= 35 - 20\times0.4^n$ m

As $n\to\infty$, $0.4^n \to 0$

Distance $\to$ 35m

Travels 35m before it comes to rest.

23iv)

$15 + 12 + 12\times0.4 + 12\times0.4^2 + +$

$= 27 + 12\sum_{n=1}^{\infty} 0.4^n$

This is equivalent to the answer given in the back of the textbook.

You can consider $\sum_{n=1}^{\infty} 0.4^n$

as $0.4 + 0.4\times0.4 + 0.4\times0.4^2 + +$

GP $a=0.4$, $r=0.4$

$S_\infty = \frac{a}{1-r} = \frac{0.4}{0.6} = \frac{2}{3}$

So $27 + 12\sum_{n=1}^{\infty} 0.4^n$

$= 27 + 12\times\frac{2}{3}$

$= 27 + 8 = 35$ m

Answer in text book is

$15\left[1 + 2\sum_{n=1}^{\infty} 0.4^n\right]$

$= 15\left[1 + 2\times\frac{2}{3}\right]$

$= 15\left[1 + \frac{4}{3}\right]$

$= 15\left[\frac{7}{3}\right] = 35$ m

24) i) After 1yr they owe

$(50000\times1.08) - P$

After 2yrs they owe

$(50000\times1.08 - P)\times1.08 - P$

$= 50000\times1.08^2 - 1.08P - P$

After 3 yrs they owe

$(50000\times1.08^2 - 1.08P - P)\times1.08 - P$

24i) (cont)

$= 50000 \times 1.08^3 - 1.08^2 P - 1.08P - P$

$= 50000 \times 1.08^3 - P(1 + 1.08 + 1.08^2)$

24ii)

After n years they owe

$50000 \times 1.08^n - P(1 + 1.08 + \ldots + 1.08^{n-1})$

$= 50000 \times 1.08^n - P\left(\dfrac{a(r^n - 1)}{r-1}\right)$

where $a = 1$, $r = 1.08$

$= 50000 \times 1.08^n - \dfrac{P(1.08^n - 1)}{0.08}$

$= 50000 \times 1.08^n - \dfrac{25P}{2}(1.08^n - 1)$

We require this to be 0 when $n = 30$

$50000 \times 1.08^{30} - \dfrac{25P}{2}(1.08^{30} - 1) = 0$

$\Rightarrow 100000 \times 1.08^{30} = 25P(1.08^{30} - 1)$

$\dfrac{100000 \times 1.08^{30}}{25(1.08^{30} - 1)} = P$

$P = £4441.37$

24iii)

Paid $£4441.37 \times 30$

$= £133,241.10p$

$50000 \times M^{30} = 133,241.10$

$M^{30} = \dfrac{133241.10}{50000}$

$M = \sqrt[30]{\dfrac{133241.10}{50000}}$

$M = \sqrt[30]{2.664822}$

$M = (2.664822)^{\frac{1}{30}}$

$M = 1.03321$

M is the multiplier each year

$k = 3.321\ \%$

$k = 3.32\ \%$ to 3 s.f.

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