1) i) $y = (x+2)^3$

Let $u = x+2$

$\frac{du}{dx} = 1$

$y = u^3 \qquad \frac{dy}{du} = 3u^2$

$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = 3u^2 \times 1$

$\frac{dy}{dx} = 3(x+2)^2$

ii)

$y = (2x+3)^4$

$\frac{dy}{dx} = 4(2x+3)^3 \times (2)$

$= 8(2x+3)^3$

iii)

$y = (x^2-5)^3$

$\frac{dy}{dx} = 3(x^2-5)^2 \times 2x$

$= 6x(x^2-5)^2$

iv)

$y = (x^3+4)^5$

$\frac{dy}{dx} = 5(x^3+4)^4 \times 3x^2$

$= 15x^2(x^3+4)^4$

v)

$y = (3x+2)^{-1}$

$\frac{dy}{dx} = -1(3x+2)^{-2} \times 3$

$= -3(3x+2)^{-2}$

vi) $y = \frac{1}{(x^2-3)^3}$

$y = (x^2-2)^{-3}$

$\frac{dy}{dx} = -3(x^2-2)^{-4} \times 2x$

$= -6x(x^2-2)^{-4}$

$= \frac{-6x}{(x^2-2)^4}$

vii)

$y = (x^2-1)^{3/2}$

$\frac{dy}{dx} = \frac{3}{2}(x^2-1)^{\frac{1}{2}} \times 2x$

$\frac{dy}{dx} = 3x(x^2-1)^{\frac{1}{2}}$

viii)

$y = \left(\frac{1}{x} + x\right)^3$

$y = (x^{-1} + x)^3$

$\frac{dy}{dx} = 3\left(\frac{1}{x} + x\right)^2 \times (-x^{-2} + 1)$

$= 3\left(\frac{1}{x} + x\right)^2 \left(1 - \frac{1}{x^2}\right)$

ix)

$y = (\sqrt{x} - 1)^4$

$y = (x^{\frac{1}{2}} - 1)^4$

$\frac{dy}{dx} = 4(\sqrt{x} - 1)^3 \left(\frac{1}{2} x^{-\frac{1}{2}}\right)$

$= \frac{2(\sqrt{x}-1)^3}{\sqrt{x}}$

2i) $y = (3x-5)^3$

$\frac{dy}{dx} = 3(3x-5)^2 \times 3$

$= 9(3x-5)^2$

ii) At $(2,1)$ $\frac{dy}{dx} = 9 \times 1^2 = 9$

Using $y - y_1 = m(x - x_1)$

tgt is $y - 1 = 9(x-2)$

$y - 1 = 9x - 18$

$y = 9x - 17$

iii) At $(1,-8)$ $\frac{dy}{dx} = 9(-2)^2 = 36$

$\therefore$ normal has gradient $-\frac{1}{36}$

Using $y - y_1 = m(x - x_1)$

Normal is $y - -8 = -\frac{1}{36}(x-1)$

$y + 8 = -\frac{1}{36}(x-1)$

$36y + 288 = -x + 1$

$36y + x + 287 = 0$

3) i) $y = (2x-1)^4$

$\frac{dy}{dx} = 4(2x-1)^3 \times 2$

$= 8(2x-1)^3$

ii) At st pt $\frac{dy}{dx} = 0$

$\Rightarrow 2x - 1 = 0$

$x = \frac{1}{2}$

When $x = \frac{1}{2}$, $y = (1-1)^4 = 0$

$\frac{d^2y}{dx^2} = 24(2x-1)^2 \times 2$

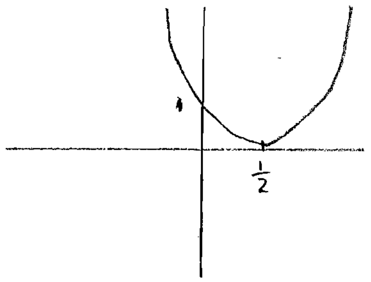
$= 48(2x-1)^2$

When $x = \frac{1}{2}$ $\frac{d^2y}{dx^2} = 0$

This does not confirm nature of stationary point

$y = (2x-1)^4$ is obtained from $y = x^4$ by a stretch parallel to x axis by a factor of $\frac{1}{2}$ and a translation by $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$

Can $\therefore$ deduce st pt is a minimum



4) $y = (x^2-4)^3$

i) $\frac{dy}{dx} = 3(x^2-4)^2 \times 2x$

$= 6x(x^2-4)^2$

ii) At st.pt $\frac{dy}{dx} = 0$

$\Rightarrow x = 0$ or $(x^2-4) = 0$

$\Rightarrow x = 0, 2$ or -2

4 cont) When $x = 0$, $y = -64$

when $x = 2$ $y = 0$

when $x = -2$ $y = 0$

$$\frac{dy}{dx} = 6x(x^2-4)^2$$

when x just < 0 $\frac{dy}{dx}$ is $- \times + = -ve$

x just > 0 $\frac{dy}{dx}$ is $+ \times + = +ve$

$\therefore$ a min at $(0, -64)$

When x just $< +2$ $\frac{dy}{dx}$ is $+ \times + = +ve$

x just $> +2$ $\frac{dy}{dx}$ is $+ \times + = +ve$

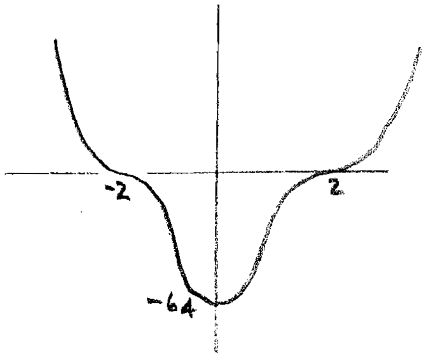
$\therefore$ a point of inflection at $(2, 0)$

When x just < -2 $\frac{dy}{dx}$ is $- \times + = -ve$

x just > -2 $\frac{dy}{dx}$ is $- \times + = -ve$

$\therefore$ a point of inflection at $(-2, 0)$

iii)



5) $y = (x^2 - x - 2)^4$

i) $\frac{dy}{dx} = 4(x^2 - x - 2)^3(2x-1)$

ii) At st.pt $\frac{dy}{dx} = 0$

$\Rightarrow (x^2 - x - 2) = 0$ or $(2x-1) = 0$

$(x-2)(x+1) = 0$ or $(2x-1) = 0$

$\Rightarrow x = 2, -1,$ or $\frac{1}{2}$

when $x = 2$, $y = 0$

when $x = -1$, $y = (1+1-2)^4 = 0$

when $x = \frac{1}{2}$, $y = \left(\frac{1}{4} - \frac{1}{2} - 2\right)^4 = \left(-\frac{9}{4}\right)^4$

≈ 25.6

when x just < 2 $\frac{dy}{dx}$ is $- \times + = -ve$

when x just > 2 $\frac{dy}{dx}$ is $+ \times + = +ve$

$\therefore$ a min at $(2, 0)$

when x just < -1 $\frac{dy}{dx}$ is $+ \times - = -ve$

when x just > -1 $\frac{dy}{dx}$ is $- \times - = +ve$

$\therefore$ a min at $(-1, 0)$

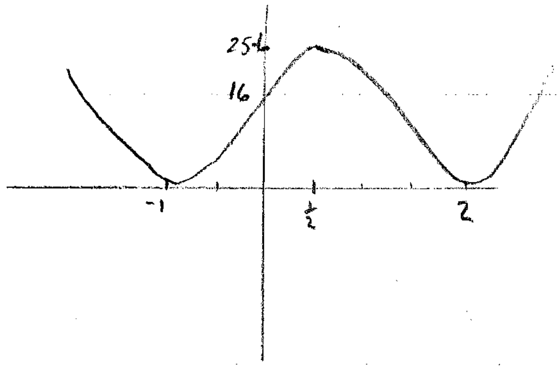
when x just $< \frac{1}{2}$ $\frac{dy}{dx}$ is $- \times - = +ve$

when x just $> \frac{1}{2}$ $\frac{dy}{dx}$ is $- \times + = -ve$

$\therefore$ a max at $\left(\frac{1}{2}, 25.6\right)$

5 cont)

iii)



6)

Let side of square be L

Then $\frac{dL}{dt} = 0.2 \text{ cm s}^{-1}$

Let area of square be A

then $A = L^2$

and $\frac{dA}{dL} = 2L$

Find $\frac{dA}{dt}$ when $L = 10$ cm

$$\frac{dA}{dt} = \frac{dA}{dL} \cdot \frac{dL}{dt}$$

$$\frac{dA}{dt} = 2L \times 0.2 = 0.4L$$

when L = 10 cm

$$\frac{dA}{dt} = 0.4 \times 10 = 4 \text{ cm}^2\text{s}^{-1}$$

7)

$$F = \frac{1}{500r^2}$$

Given $\frac{dr}{dt} = 0.03 \text{ ms}^{-1}$

Find $\frac{dF}{dt}$

$$\frac{dF}{dt} = \frac{dF}{dr} \times \frac{dr}{dt}$$

$$F = \frac{1}{500r^2} = \frac{r^{-2}}{500}$$

$$\frac{dF}{dr} = \frac{-2r^{-3}}{500} = \frac{-1}{250r^3}$$

$$\therefore \frac{dF}{dt} = \frac{-1}{250r^3} \times 0.03$$

$$= -\frac{0.03}{250r^3}$$

When $r = 0.2$

$$\frac{dF}{dt} = \frac{-0.03}{250 \times 0.2^3}$$

$$= -0.015 \text{ Ns}^{-1}$$

8)

$\frac{dr}{dt} = 5$ cm per day

Find $\frac{dA}{dt}$ when $r = 1$m

$$\frac{dA}{dt} = \frac{dA}{dr} \cdot \frac{dr}{dt}$$

$$A = \pi r^2 \Rightarrow \frac{dA}{dr} = 2\pi r$$

$$\therefore \frac{dA}{dt} = 2\pi r \times 5 = 10\pi r \text{ cm}^2/\text{day}$$

When $r = 1$m $= 100$ cm

$$\frac{dA}{dt} = 10\pi \times 100 = 1000\pi \text{ cm}^2/\text{day}$$

9) $y = (x^3 - x^2 + 2)^3$

i) $\frac{dy}{dx} = 3(x^3 - x^2 + 2)^2(3x^2 - 2x)$

When $x = -1$

$\frac{dy}{dx} = 3((-1)^3 - (-1)^2 + 2)^2(3(-1)^2 - 2(-1))$

$= 3(-1 - 1 + 2)^2(3 + 2)$

$= 3 \times 0^2 \times 5 = 0$

$\therefore$ a st pt when $x = -1$

when x just < -1

$\frac{dy}{dx}$ is $+ \times +$ $= +ve$

when x just > -1

$\frac{dy}{dx}$ is $+ \times +$ $= +ve$

$\therefore$ a point of inflection when $x = -1$

When $x = 0$

$\frac{dy}{dx} = 3(2)^2(0) = 0$

$\therefore$ a st pt.

When x just < 0

$\frac{dy}{dx}$ is $+ \times +$ $= +ve$

When x just > 0

$\frac{dy}{dx}$ is $+ \times -$ $= -ve$

$\therefore$ a max when $x = 0$

iii) $\frac{dy}{dx} = 3(x^3 - x^2 + 2)^2(3x^2 - 2x)$

$= 3x(x^3 - x^2 + 2)^2(3x - 2)$

Also a st pt. when $3x - 2 = 0$

$\Rightarrow x = \frac{2}{3}$

When x just $< \frac{2}{3}$

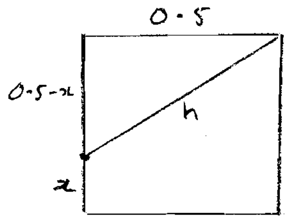
$\frac{dy}{dx}$ is $+ \times + \times -$ $= -ve$

When x just $> \frac{2}{3}$

$\frac{dy}{dx}$ is $+ \times + \times +$ $= +ve$

$\therefore$ a min when $x = \frac{2}{3}$

10)



Walk a distance x before crossing

$h^2 = 0.5^2 + (0.5 - x)^2$

$= 0.25 + 0.25 - x + x^2$

$= 0.5 - x + x^2$

$\therefore h = \sqrt{0.5 - x + x^2}$

Time $T = \frac{x}{5} + \frac{\sqrt{0.5 - x + x^2}}{3}$

For min T $\frac{dT}{dx} = 0$

$T = \frac{x}{5} + \frac{(0.5 - x + x^2)^{\frac{1}{2}}}{3}$

$\frac{dT}{dx} = \frac{1}{5} + \frac{1}{3} \cdot \frac{1}{2}(0.5 - x + x^2)^{-\frac{1}{2}}(2x - 1)$

$= \frac{1}{5} + \frac{1}{6}(0.5 - x + x^2)^{-\frac{1}{2}}(2x - 1)$

10 cont) If $\frac{dT}{dx} = 0$

$$0 = \frac{1}{5} + \frac{1}{6}\frac{(2x-1)}{\sqrt{0.5-x+x^2}}$$

$$-\frac{1}{5} = \frac{(2x-1)}{6\sqrt{0.5-x+x^2}}$$

Squaring $\frac{1}{25} = \frac{(2x-1)^2}{36(0.5-x+x^2)}$

$$36(0.5-x+x^2) = 25(2x-1)^2$$

$$18 - 36x + 36x^2 = 25(4x^2-4x+1)$$

$$18-36x+36x^2 = 100x^2 - 100x + 25$$

$$64x^2 - 64x + 7 = 0$$

$$x = \frac{64 \pm \sqrt{(-64)^2 - 4 \times 64 \times 7}}{128}$$

$$x = \frac{64 \pm \sqrt{2304}}{128}$$

$$x = \frac{64 \pm 48}{128}$$

$x = 0.875$ or $x = 0.125$

$x = 0.875$ not possible since field is only 0.5 Km wide

$\therefore x = 0.125$

Should walk 0.125 Km before heading across field