MEI CORE 3 INTEGRATION BY SUBSTITUTION EXERCISE 5A

1 i) $\int (x+1)^3\,dx$

Let $u = x+1$

$\Rightarrow \frac{du}{dx} = 1$

$\Rightarrow du = dx$

$\int (x+1)^3\,dx = \int u^3\,du$

$= \frac{u^4}{4} + c$

$= \frac{(x+1)^4}{4} + c$

1 ii) $\int 2\sqrt{2x-1}\,dx$

Let $u = 2x-1$

$\Rightarrow \frac{du}{dx} = 2$

$\Rightarrow du = 2dx$

$\int 2\sqrt{2x-1}\,dx = \int u^{\frac{1}{2}}\,du$

$= \frac{u^{\frac{3}{2}}}{3/2} + c$

$= \frac{2}{3}u^{3/2} + c$

$= \frac{2}{3}(2x-1)^{\frac{3}{2}} + c$

1 iii) $\int 3x^2(x^3+1)^7\,dx$

Let $u = x^3+1$

$\Rightarrow \frac{du}{dx} = 3x^2$

$\Rightarrow du = 3x^2\,dx$

$\int 3x^2(x^3+1)^7\,dx = \int u^7\,du$

$= \frac{u^8}{8} + c$

$= \frac{(x^3+1)^8}{8} + c$

1 iv) $\int 2x(x^2+1)^5\,dx$

Let $u = x^2+1$

$\Rightarrow \frac{du}{dx} = 2x$

$\Rightarrow du = 2x\,dx$

$\int 2x(x^2+1)^5\,dx = \int u^5\,du$

$= \frac{u^6}{6} + c$

$= \frac{(x^2+1)^6}{6} + c$

1 v) $\int 3x^2(x^3-2)^4\,dx$

Let $u = x^3-2$

$\Rightarrow \frac{du}{dx} = 3x^2$

$\Rightarrow du = 3x^2\,dx$

$\int 3x^2(x^3-2)^4\,dx = \int u^4\,du$

$= \frac{u^5}{5} + c = \frac{(x^3-2)^5}{5} + c$

1vi) $\int x\sqrt{2x^2-5}\,dx$

Let $u = 2x^2-5$

$\Rightarrow \frac{du}{dx} = 4x$

$\Rightarrow du = 4x\,dx$

$\frac{1}{4}du = x\,dx$

$\int x\sqrt{2x^2-5}\,dx = \int \frac{1}{4}u^{\frac{1}{2}}\,du$

$= \frac{1}{4}\frac{u^{3/2}}{\frac{3}{2}} + c$

$= \frac{1}{4}\cdot\frac{2}{3}u^{3/2} + c$

$= \frac{u^{3/2}}{6} + c$

$= \frac{(2x^2-5)^{3/2}}{6} + c$

1vii) $\int x\sqrt{2x+1}\,dx$

Let $u = 2x+1$

$\Rightarrow \frac{du}{dx} = 2$

$\Rightarrow du = 2dx$

$\Rightarrow \frac{1}{2}du = dx$

Also since $u = 2x+1$

$u-1 = 2x$

$\frac{u-1}{2} = x$

$\int x\sqrt{2x+1}\,dx = \int \left(\frac{u-1}{2}\right)u^{\frac{1}{2}}\frac{1}{2}\,du$

$= \int \frac{1}{4}(u-1)u^{\frac{1}{2}}\,du$

$= \int \left(\frac{1}{4}u^{3/2} - \frac{1}{4}u^{\frac{1}{2}}\right)du$

$= \frac{1}{4}\frac{u^{5/2}}{5/2} - \frac{1}{4}\frac{u^{3/2}}{\frac{3}{2}} + c$

$= \frac{1}{4}\cdot\frac{2}{5}u^{5/2} - \frac{1}{4}\cdot\frac{2}{3}u^{3/2} + c$

$= \frac{u^{5/2}}{10} - \frac{u^{3/2}}{6} + c$

$= \frac{(2x+1)^{5/2}}{10} - \frac{(2x+1)^{3/2}}{6} + c$

1viii) $\int \frac{x}{\sqrt{x+9}}\,dx$

$= \int x(x+9)^{-\frac{1}{2}}\,dx$

Let $u = x+9$

$\Rightarrow \frac{du}{dx} = 1$

$\Rightarrow du = dx$

Also since $u = x+9$

$u-9 = x$

$\int x(x+9)^{-\frac{1}{2}}\,dx = \int (u-9)u^{-\frac{1}{2}}\,du$

$= \int \left(u^{\frac{1}{2}} - 9u^{-\frac{1}{2}}\right)du$

$= \frac{u^{3/2}}{\frac{3}{2}} - \frac{9u^{\frac{1}{2}}}{\frac{1}{2}} + c$

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1 viii cont)

$$= \frac{2}{3}u^{3/2} - 18u^{\frac{1}{2}} + c$$

$$= \frac{2}{3}(x+9)^{3/2} - 18(x+9)^{\frac{1}{2}} + c$$

2) i) $\int_{-1}^{4} (x-3)^4 \, dx$

Let $u = x - 3$

$\Rightarrow \frac{du}{dx} = 1$

$\Rightarrow du = dx$

when $x = 4$, $u = 4 - 3 = 1$
when $x = -1$, $u = -1 - 3 = -4$

$$\int_{-1}^{4} (x-3)^4 \, dx = \int_{-4}^{1} u^4 \, du$$

$$= \left[\frac{u^5}{5}\right]_{-4}^{1}$$

$$= \frac{1}{5} - \frac{(-1024)}{5}$$

$$= \frac{1025}{5} = 205$$

2 ii) $\int_{0}^{3} (3x+2)^6 \, dx$

Let $u = 3x + 2$

$\Rightarrow \frac{du}{dx} = 3$

$\Rightarrow du = 3dx$

$\Rightarrow \frac{1}{3}du = dx$

When $x = 3$, $u = 3 \times 3 + 2 = 11$

When $x = 0$, $u = 3 \times 0 + 2 = 2$

$$\int_{0}^{3} (3x+2)^6 \, dx = \int_{2}^{11} \frac{1}{3}u^6 \, du$$

$$= \left[\frac{1}{3}\frac{u^7}{7}\right]_{2}^{11}$$

$$= \left[\frac{u^7}{21}\right]_{2}^{11}$$

$$= \frac{11^7}{21} - \frac{2^7}{21} = 928{,}000$$

to 3 sig fig

2 iii) $\int_{5}^{9} \sqrt{x-5} \, dx$

Let $u = x - 5$

$\Rightarrow \frac{du}{dx} = 1$

$\Rightarrow du = dx$

when $x = 9$, $u = 9 - 5 = 4$
when $x = 5$, $u = 5 - 5 = 0$

$$\int_{5}^{9} \sqrt{x-5} \, dx = \int_{0}^{4} u^{\frac{1}{2}} \, du$$

$$= \left[\frac{u^{3/2}}{\frac{3}{2}}\right]_{0}^{4}$$

$$= \left[\frac{2}{3}u^{3/2}\right]_{0}^{4} = \frac{2}{3} \times 8 - \frac{2}{3} \times 0$$

$$= \frac{16}{3} = 5\frac{1}{3}$$

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2 iv) $\displaystyle\int_2^{15} \sqrt[3]{2x-3}\,dx$

Let $u = 2x-3$

$\Rightarrow \dfrac{du}{dx} = 2$

$\Rightarrow du = 2dx$

$\Rightarrow \frac{1}{2}du = dx$

when $x = 15$, $u = 2\times15-3 = 27$
when $x = 2$, $u = 2\times2-1 = 1$

$\displaystyle\int_2^{15} \sqrt[3]{2x-3}\,dx = \int_1^{27} \frac{1}{2}u^{\frac{1}{3}}\,du$

$= \left[\dfrac{1}{2}\dfrac{u^{4/3}}{\frac{4}{3}}\right]_1^{27}$

$= \left[\dfrac{3}{8}u^{4/3}\right]_1^{27}$

$= \dfrac{3}{8}\left(81-1\right) = \dfrac{3}{8}\times 80$

$= 30$

2 v) $\displaystyle\int_1^{5} x^2(x^3+1)^2\,dx$

Let $u = x^3+1$

$\Rightarrow \dfrac{du}{dx} = 3x^2$

$\Rightarrow du = 3x^2dx$

$\Rightarrow \frac{1}{3}du = x^2dx$

When $x = 5$, $u = 5^3+1 = 126$
when $x = 1$, $u = 1^3+1 = 2$

$\displaystyle\int_1^5 x^2(x^3+1)^2dx = \int_1^{126}\frac{1}{3}u^2\,du$

$= \left[\dfrac{u^3}{9}\right]_1^{126}$

$= \dfrac{126^3}{9} - \dfrac{1}{9} = 222{,}000$

to 3 sig fig

2 vi) $\displaystyle\int_{-1}^{2} 2x(x-3)^5\,dx$

Let $u = x-3$

$\Rightarrow \dfrac{du}{dx} = 1$

$\Rightarrow du = dx$

Also since $u = x-3$
$u+3 = x$

when $x = 2$, $u = 2-3 = -1$
when $x = -1$, $u = -1-3 = -4$

$\displaystyle\int_{-1}^{2} 2x(x-3)^5dx = \int_{-4}^{-1} 2(u+3)u^5\,du$

$= \displaystyle\int_{-4}^{-1}(2u^6+6u^5)\,du$

$= \left[\dfrac{2u^7}{7} + u^6\right]_{-4}^{-1}$

$= \left(\dfrac{-2}{7}+1\right) - \left(\dfrac{2(-4)^7}{7} + (-4)^6\right)$

2vi) cont)

$$= -\frac{2}{7} + 1 + \frac{32768}{7} - 4096$$

$$= 586 \text{ to 3 sig fig}$$

2vii)

$$\int_1^5 x\sqrt{x-1}\,dx$$

Let $u = x-1$

$$\Rightarrow \frac{du}{dx} = 1$$

$$\Rightarrow du = dx$$

Also $u + 1 = x$

When $x = 5$, $u = 5-1 = 4$
when $x = 1$ $u = 1-1 = 0$

$$\int_1^5 x\sqrt{x-1}\,dx = \int_0^4 (u+1)u^{\frac{1}{2}}\,du$$

$$= \int_0^4 \left(u^{3/2} + u^{1/2}\right) du$$

$$= \left[\frac{u^{5/2}}{5/2} + \frac{u^{3/2}}{3/2}\right]_0^4$$

$$= \left[\frac{2u^{5/2}}{5} + \frac{2u^{3/2}}{3}\right]_0^4$$

$$= \left(\frac{2}{5}\times 32 + \frac{2}{3}\times 8\right) - (0+0)$$

$$= \frac{64}{5} + \frac{16}{3} = 18\tfrac{2}{15}$$

3) i)

$$\int_2^4 (x-2)^3\,dx$$

Let $u = x - 2$

$$\Rightarrow \frac{du}{dx} = 1$$

$$\Rightarrow du = dx$$

When $x = 4$, $u = 4-2 = 2$
when $x = 2$, $u = 2-2 = 0$

$$\int_2^4 (x-2)^3\,dx = \int_0^2 u^3\,du$$

$$= \left[\frac{u^4}{4}\right]_0^2$$

$$= \frac{16}{4} - 0 = 4$$

ii)

$$\int_0^2 (x-2)^3 = -4$$

since $y = (x-2)^3$ is obtained by translating $y = x^3$ by $\begin{pmatrix}2\\0\end{pmatrix}$

Rotational symmetry about $(0,0)$ of $y = x^3$ means that

$$\int_{-2}^0 x^3\,dx = -\int_0^2 x^3\,dx$$

$\int_0^2 (x-2)^3\,dx$ is the corresponding area of the translated graph.

iii)

$$\int_0^2 (x-2)^3\,dx$$

Use previous substitution

when $x = 2$, $u = 2-2 = 0$
when $x = 0$, $u = 0-2 = -2$

3 iii) cont)
$$\int_0^2 (x-2)^3\,dx = \int_{-2}^{0} u^3\,du$$

$$= \left[\frac{u^4}{4}\right]_{-2}^{0}$$

$$= 0 - \frac{(-2)^4}{4} = -\frac{16}{4}$$

$$= -4$$

4 i)
$$\int_{-1}^{0} \left((x-1)^4 - 1\right) dx$$

Let $u = x-1$

$\Rightarrow \frac{du}{dx} = 1$

$\Rightarrow du = dx$

When $x = 0$, $u = 0-1 = -1$

when $x = -1$, $u = -1-1 = -2$

$$\int_{-1}^{0} \left(\left((x-1)^4 - 1\right)\right) dx$$

$$= \int_{-2}^{-1} \left(u^4 - 1\right) du$$

$$= \left[\frac{u^5}{5} - u\right]_{-2}^{-1}$$

$$= \left(\frac{(-1)^5}{5} - (-1)\right) - \left(\frac{(-2)^5}{5} - (-2)\right)$$

$$= \left(-\frac{1}{5} + 1\right) - \left(\frac{-32}{5} + 2\right)$$

$$= -\frac{1}{5} + 1 + \frac{32}{5} - 2$$

$$= \frac{31}{5} - 1 = 5\tfrac{1}{5}$$

4 ii) Same substitution

when $x = 2$, $u = 2-1 = 1$

when $x = 0$ $u = 0-1 = -1$

$$\int_0^2 \left((x-1)^4 - 1\right) dx = \left[\frac{u^5}{5} - u\right]_{-1}^{1}$$

$$= \left(\frac{1}{5} - 1\right) - \left(\frac{-1}{5} - -1\right)$$

$$= \frac{1}{5} - 1 + \frac{1}{5} - 1$$

$$= \frac{2}{5} - 2 = -\frac{8}{5}$$

Area $= \frac{8}{5}$ since $-$ sign indicates area below x-axis

iii) Total shaded area

$$= 5\tfrac{1}{5} + 1\tfrac{3}{5}$$

$$= 6\tfrac{4}{5}$$

iv) Evaluating $\int_{-1}^{2} \left((x-1)^4 - 1\right) dx$

would give area of $A - B$

since $-$ve area below x-axis would be offset against $+$ve area above x-axis.

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5) i) $\int_3^5 (x-3)^3\,dx$

Let $u = x-3$

$\Rightarrow \frac{du}{dx} = 1$

$\Rightarrow du = dx$

when $x=5$, $u = 5-3 = 2$

when $x=3$, $u = 3-3 = 0$

$$\int_3^5 (x-3)^3\,dx = \int_0^2 u^3\,du$$

$$= \left[\frac{u^4}{4}\right]_0^2$$

$$= \frac{16}{4} - 0 = 4$$

5 ii) $\int_2^4 (x-4)^2\,dx$

Let $u = x-4$

$\Rightarrow \frac{du}{dx} = 1$

$\Rightarrow du = dx$

when $x=4$, $u = 4-4 = 0$

when $x=2$, $u = 2-4 = -2$

$$\int_2^4 (x-4)^2\,dx = \int_{-2}^0 u^2\,du$$

$$= \left[\frac{u^3}{3}\right]_{-2}^0$$

$$= 0 - \frac{(-2)^3}{3}$$

$$= \frac{8}{3}$$

5 iii) $f(x) = 6x(x^2+1)^3$

$f(-x) = 6(-x)((-x)^2+1)^3$

$= -6x(x^2+1)^3$

$= -f(x)$

$\therefore$ $f(x)$ is odd function with rotational symmetry about $(0,0)$

Shaded Area $\therefore$ given by

$$2\int_0^1 6x(x^2+1)^3\,dx$$

Let $u = x^2+1$

$\Rightarrow \frac{du}{dx} = 2x$

$\Rightarrow du = 2x\,dx$

when $x=1$, $u = 1^2+1 = 2$

when $x=0$ $u = 0^2+1 = 1$

$$2\int_0^1 6x(x^2+1)^3\,dx = 2\int_1^2 3u^3\,du$$

$$= 6\left[\frac{u^4}{4}\right]_1^2$$

$$= 6\left[\frac{16}{4} - \frac{1}{4}\right]$$

$$= 6 \times \frac{15}{4} = \frac{45}{2}$$

$$= 22\tfrac{1}{2}$$

5iv) $\displaystyle\int_2^4 \frac{x}{(x-1)^3}\,dx$

$= \displaystyle\int_2^4 x(x-1)^{-3}\,dx$

Let $u = x-1$

$\Rightarrow \frac{du}{dx} = 1$

$\Rightarrow du = dx$

Also $u+1 = x$

When $x=4$, $u = 4-1 = 3$

when $x=2$, $u = 2-1 = 1$

$\displaystyle\int_2^4 x(x-1)^{-3}\,dx = \int_1^3 (u+1)u^{-3}\,du$

$= \displaystyle\int_1^3 (u^{-2}+u^{-3})\,du$

$= \left[\frac{u^{-1}}{-1} + \frac{u^{-2}}{-2}\right]_1^3$

$= \left[-\frac{1}{u} - \frac{1}{2u^2}\right]_1^3$

$= \left(-\frac{1}{3} - \frac{1}{18}\right) - \left(-\frac{1}{1} - \frac{1}{2}\right)$

$= -\frac{6}{18} - \frac{1}{18} + 1 + \frac{1}{2}$

$= 1\frac{1}{2} - \frac{7}{18}$

$= 1\frac{9}{18} - \frac{7}{18}$

$= 1\frac{2}{18} = 1\frac{1}{9}$

6)i) $y = x\sqrt{1+x}$

At A, $y=0$

$0 = x\sqrt{1+x}$

$\Rightarrow x = 0$ or $\sqrt{1+x} = 0$

$\Rightarrow x = -1$

$\therefore$ A is point $(-1, 0)$

Function defined for $x \geq -1$

ii) Area $= \left|\displaystyle\int_{-1}^0 x\sqrt{1+x}\,dx\right|$

$\displaystyle\int_{-1}^0 x\sqrt{1+x}\,dx$

Let $u = 1+x$

$\Rightarrow \frac{du}{dx} = 1$

$\Rightarrow du = dx$

Also $u-1 = x$

when $x=0$ $u = 1+0 = 1$

when $x=-1$ $u = 1+-1 = 0$

$\displaystyle\int_{-1}^0 x\sqrt{1+x}\,dx = \int_0^1 (u-1)u^{\frac{1}{2}}\,du$

$= \displaystyle\int_0^1 (u^{3/2} - u^{\frac{1}{2}})\,du$

$= \left[\frac{u^{5/2}}{5/2} - \frac{u^{3/2}}{3/2}\right]_0^1$

$= \left[\frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2}\right]_0^1$

$= \left(\frac{2}{5} - \frac{2}{3}\right) - (0-0)$

6 ii) (cont)

$$= \frac{6}{15} - \frac{10}{15} = -\frac{4}{15}$$

$$\text{Area} = \left| -\frac{4}{15} \right| = \frac{4}{15}$$

7) i) a)

$$\int (1+x)^3 \, dx$$

Let $u = 1+x$

$$\Rightarrow \frac{du}{dx} = 1$$

$$\Rightarrow du = dx$$

$$\int (1+x)^3 dx = \int u^3 du$$

$$= \frac{u^4}{4} + c$$

$$= \frac{(1+x)^4}{4} + c$$

b)

$$\int_{-1}^{1} x(1+x)^3 dx$$

Let $u = 1+x$

$$\Rightarrow \frac{du}{dx} = 1$$

$$\Rightarrow du = dx$$

Also $u - 1 = x$

When $x = 1$, $u = 1+1 = 2$
when $x = -1$ $u = 1+(-1) = 0$

$$\int_{-1}^{1} x(1+x)^3 dx = \int_{0}^{2} (u-1)u^3 du$$

$$= \int_0^2 (u^4 - u^3) du$$

$$= \left[\frac{u^5}{5} - \frac{u^4}{4} \right]_0^2$$

$$= \left(\frac{32}{5} - \frac{16}{4} \right) - (0 - 0)$$

$$= 2\tfrac{2}{5}$$

7 ii)

$$\int_0^1 x\sqrt{1+x^2}\, dx$$

Let $t = 1+x^2$

$$\Rightarrow \frac{dt}{dx} = 2x$$

$$\Rightarrow dt = 2x\,dx$$

$$\Rightarrow \tfrac{1}{2} dt = x\,dx$$

When $x = 1$, $t = 1+1^2 = 2$
when $x = 0$, $t = 1+0^2 = 1$

$$\int_0^1 x\sqrt{1+x^2}\,dx = \int_1^2 \tfrac{1}{2} t^{\frac{1}{2}} dt$$

$$= \left[\frac{1}{2} \frac{t^{3/2}}{\frac{3}{2}} \right]_1^2$$

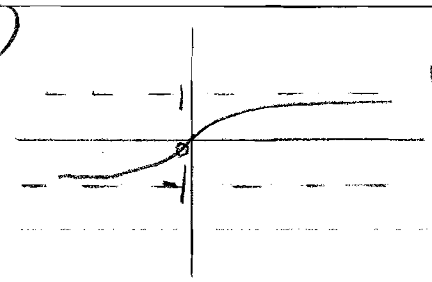
$$= \left[\frac{1}{2} \cdot \frac{2}{3} t^{3/2} \right]_1^2$$

$$= \left[\frac{t^{3/2}}{3} \right]_1^2$$

$$= \frac{2\sqrt{2}}{3} - \frac{1}{3}$$

$$= \frac{2\sqrt{2} - 1}{3}$$

8)



$$y = \frac{x}{\sqrt{x^2+4}}$$

8 cont)

ii) Area $= \displaystyle\int_0^2 \frac{x}{\sqrt{4+x^2}}\,dx$

$= \displaystyle\int_0^2 x(4+x^2)^{-\frac{1}{2}}\,dx$

Let $u = 4+x^2$

$\Rightarrow \frac{du}{dx} = 2x$

$\Rightarrow du = 2x\,dx$

$\Rightarrow \frac{1}{2}du = x\,dx$

when $x = 2$, $u = 4+2^2 = 8$

when $x = 0$, $u = 4+0^2 = 4$

$\displaystyle\int_0^2 x(4+x^2)\,dx = \int_4^8 \frac{1}{2}u^{-\frac{1}{2}}\,du$

$= \left[\frac{1}{2}\frac{u^{\frac{1}{2}}}{\frac{1}{2}}\right]_4^8$

$= \left[u^{\frac{1}{2}}\right]_4^8$

$= \sqrt{8} - \sqrt{4}$

$= 2\sqrt{2} - 2$

$= 2(\sqrt{2}-1)$

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9) i) $\frac{d}{dx}(2x-1)^7 = 7(2x-1)^6 \times 2$

$= 14(2x-1)^6$

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ii

$y = 4x(2x-1)^7$

$\frac{dy}{dx} = 4x \times 14(2x-1)^6 + (2x-1)^7 \times 4$

$= 56x(2x-1)^6 + 4(2x-1)(2x-1)^6$

$= 56x(2x-1)^6 + (8x-4)(2x-1)^6$

$= (64x-4)(2x-1)^6$

$= 4(16x-1)(2x-1)^6$

At st.pt. $\frac{dy}{dx} = 0$

$\Rightarrow (2x-1) = 0 \Rightarrow x = \frac{1}{2}$

or $(16x-1) = 0 \Rightarrow x = \frac{1}{16}$

From graph min point when $x = \frac{1}{16}$

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iii)

Area $= \displaystyle\int_0^{\frac{1}{2}} 4x(2x-1)^7\,dx$

Let $u = 2x-1$

$\Rightarrow \frac{du}{dx} = 2$

$\Rightarrow du = 2dx \Rightarrow \frac{1}{2}du = dx$

Also $u+1 = 2x$

When $x = \frac{1}{2}$, $u = 2\times\frac{1}{2}-1 = 0$

when $x = 0$, $u = 2\times 0 - 1 = -1$

$\displaystyle\int_0^{\frac{1}{2}} 4x(2x-1)^7\,dx$

$= \displaystyle\int_{-1}^0 2(u+1)u^7\frac{1}{2}\,du$

$= \displaystyle\int_{-1}^0 (u^8+u^7)\,du$

9 cont) iii

$$= \left[\frac{u^9}{9} + \frac{u^8}{8} \right]_{-1}^{0}$$

$$= (0+0) - \left(\frac{-1}{9} + \frac{1}{8} \right)$$

$$= +\frac{1}{9} - \frac{1}{8} = \frac{8}{72} - \frac{9}{72}$$

$$= -\frac{1}{72}$$

Area $= \frac{1}{72}$ — sign indicates area below x-axis

9 iv)

$f(x) = 4x(2x-1)^7$ domain $x \geqslant \frac{1}{2}$

Given $f(1) = 4$

find $\frac{d}{dx} f^{-1}(x)$ when $x = 4$

$$\frac{df(x)}{dx} = 4(2x-1)^6(16x-1)$$

When $x = 1$ $\frac{d f(x)}{dx} = 4(1)^6(15)$

$$= 60$$

Corresponding point on $f^{-1}(x)$ is $(4, 1)$

and gradient $= \frac{1}{60}$

10) i) a)

$$\int \left(\frac{4}{\sqrt{x}} + \frac{3}{x^3} \right) dx$$

$$= \int \left(4x^{-\frac{1}{2}} + 3x^{-3} \right) dx$$

$$= \frac{4x^{\frac{1}{2}}}{\frac{1}{2}} + \frac{3x^{-2}}{-2} + c$$

$$= 8x^{\frac{1}{2}} - \frac{3}{2}x^{-2} + c$$

$$= 8\sqrt{x} - \frac{3}{2x^2} + c$$

b)

$$\int 6x(1+x^2)^{\frac{1}{2}} dx$$

Let $u = 1 + x^2$

$\Rightarrow \frac{du}{dx} = 2x$

$\Rightarrow du = 2x\,dx$

$$\int 6x(1+x^2)^{\frac{1}{2}} dx = \int 3u^{\frac{1}{2}} du$$

$$= \frac{3u^{3/2}}{\frac{3}{2}} + c$$

$$= 2u^{3/2} + c = 2(1+x^2)^{3/2} + c$$

ii)

$$\int_1^4 \frac{(1+\sqrt{x})^3}{\sqrt{x}} dx$$

Let $x = u^2$

$\Rightarrow u = x^{\frac{1}{2}}$

$\frac{du}{dx} = \frac{1}{2}x^{-\frac{1}{2}}$

$du = \frac{1}{2}x^{-\frac{1}{2}} dx$

$2\,du = \frac{dx}{\sqrt{x}}$

When $x = 4$, $u = 2$

when $x = 1$, $u = 1$

$$\int_1^4 \frac{(1+\sqrt{x})^3}{\sqrt{x}} dx = \int_1^2 (1+u)^3 2\,du$$

$$= \int_a^b k(1+u)^3 du$$

where $a = 1$, $b = 2$, $k = 2$

10 cont)

$$\int_1^2 2(1+u)^3\,du$$

Let $w = 1+u$

$\Rightarrow \frac{dw}{du} = 1$

$dw = du$

when $u = 2,\ w = 3$

when $u = 1,\ w = 2$

$$\int_1^2 2(1+u)^3\,du = \int_2^3 2w^3\,dw$$

$$= \left[\frac{2w^4}{4}\right]_2^3$$

$$= \left[\frac{w^4}{2}\right]_2^3$$

$$= \frac{81}{2} - \frac{16}{2}$$

$$= \frac{65}{2} = 32\tfrac{1}{2}$$

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