1) i) $\int (\sin x - 2\cos x)\,dx$

$= -\cos x - 2\sin x + c$

ii) $\int (3\cos x + 2\sin x)\,dx$

$= 3\sin x - 2\cos x + c$

iii) $\int (5\sin x + 4\cos x)\,dx$

$= -5\cos x + 4\sin x + c$

2) i) $\int \cos 3x\,dx = \frac{1}{3}\sin 3x + c$

ii) $\int \sin(1-x)\,dx$

Let $u = 1-x$

$\Rightarrow \frac{du}{dx} = -1$

$du = -dx$

$-du = dx$

$\int \sin(1-x)\,dx = \int -\sin u\,du$

$= \cos u + c$

$= \cos(1-x) + c$

iii) $\int \sin x \cos^3 x\,dx$

Let $u = \cos x$

$\frac{du}{dx} = -\sin x$

$du = -\sin x\,dx$

$-du = \sin x\,dx$

$\int \sin x \cos^3 x\,dx = \int -u^3\,du$

$= -\frac{u^4}{4} + c$

$= -\frac{\cos^4 x}{4} + c$

iv) $\int \frac{\sin x}{2-\cos x}\,dx$

$= \ln|2-\cos x| + c$

v) $\int \tan x\,dx = \int \frac{\sin x}{\cos x}\,dx$

$= -\int \frac{-\sin x}{\cos x}\,dx$

$= -\ln|\cos x| + c$

vi) $\int \sin 2x(1+\cos 2x)^2\,dx$

Let $u = 1+\cos 2x$

$\Rightarrow \frac{du}{dx} = -2\sin 2x$

$\Rightarrow du = -2\sin 2x\,dx$

$\Rightarrow -\frac{1}{2}du = \sin 2x\,dx$

$\int \sin 2x(1+\cos 2x)^2\,dx = -\frac{1}{2}\int u^2\,du$

$= -\frac{1}{6}u^3 + c$

$= -\frac{1}{6}(1+\cos 2x)^3 + c$

3) i) $\int 2x\sin(x^2)\,dx$

Let $u = x^2$

$\Rightarrow \frac{du}{dx} = 2x$

3 i) (cont) $\Rightarrow du = 2x\,dx$

$$\int 2x\sin(x^2)\,dx = \int \sin u\,du$$

$$= -\cos u + c$$

$$= -\cos(x^2) + c$$

3 ii)

$$\int \cos x\, e^{\sin x}\,dx$$

Let $u = \sin x$

$\Rightarrow \frac{du}{dx} = \cos x$

$\Rightarrow du = \cos x\,dx$

$$\int \cos x\, e^{\sin x}\,dx = \int e^u\,du$$

$$= e^u + c$$

$$= e^{\sin x} + c$$

3 iii)

$$\int \frac{\tan x}{\cos^2 x}\,dx$$

Let $u = \tan x$

$\Rightarrow \frac{du}{dx} = \sec^2 x$

$\Rightarrow \frac{du}{dx} = \frac{1}{\cos^2 x}$

$\Rightarrow du = \frac{dx}{\cos^2 x}$

$$\int \frac{\tan x}{\cos^2 x}\,dx = \int u\,du$$

$$= \frac{u^2}{2} + c$$

$$= \frac{1}{2}\tan^2 x + c$$

3 iv)

$$\int \frac{\cos x}{\sin^2 x}\,dx$$

Let $u = \sin x$

$\Rightarrow \frac{du}{dx} = \cos x$

$du = \cos x\,dx$

$$\int \frac{\cos x}{\sin^2 x}\,dx = \int \frac{1}{u^2}\,du$$

$$= -\frac{1}{u} + c$$

$$= -\frac{1}{\sin x} + c$$

4) i)

$$\int_0^{\frac{\pi}{2}} \cos\left(2x - \frac{\pi}{2}\right)dx$$

Let $u = 2x - \frac{\pi}{2}$

$\Rightarrow \frac{du}{dx} = 2$

$\Rightarrow du = 2dx$

$\Rightarrow \frac{1}{2}du = dx$

when $x = \frac{\pi}{2}$, $u = \pi - \frac{\pi}{2} = \frac{\pi}{2}$

when $x = 0$ $u = 0 - \frac{\pi}{2} = -\frac{\pi}{2}$

$$\int_0^{\frac{\pi}{2}} \cos\left(2x - \frac{\pi}{2}\right)dx = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{1}{2}\cos u\,du$$

$$= \frac{1}{2}\Big[\sin u\Big]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$$

$$= \frac{1}{2}\left[\sin\frac{\pi}{2} - \sin\left(-\frac{\pi}{2}\right)\right]$$

$$= \frac{1}{2}\left[1 - -1\right] = 1$$

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4 ii) $\int_0^{\frac{\pi}{4}} \cos x \sin^3 x \, dx$

Let $u = \sin x$

$\Rightarrow \frac{du}{dx} = \cos x$

$\Rightarrow du = \cos x \, dx$

when $x = 0$, $u = \sin 0 = 0$

when $x = \frac{\pi}{4}$ $u = \sin\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

$$\int_0^{\frac{\pi}{4}} \cos x \sin^3 x \, dx = \int_0^{\frac{1}{\sqrt{2}}} u^3 \, du$$

$$= \left[\frac{u^4}{4}\right]_0^{\frac{1}{\sqrt{2}}}$$

$$= \left(\frac{\frac{1}{4}}{4}\right) - \left(0\right)$$

$$= \frac{1}{16}$$

4 iii) $\int_0^{\sqrt{\pi}} x \sin(x^2) \, dx$

Let $u = x^2$

$\Rightarrow \frac{du}{dx} = 2x$

$\Rightarrow du = 2x \, dx$

$\Rightarrow \frac{1}{2} du = x \, dx$

When $x = 0$, $u = 0$

when $x = \sqrt{\pi}$ $u = \pi$

$$\int_0^{\sqrt{\pi}} x \sin(x^2) \, dx = \int_0^{\pi} \frac{1}{2} \sin u \, du$$

$$= \left[-\frac{1}{2}\cos u\right]_0^{\pi}$$

$$= \left(-\frac{1}{2}\cos\pi\right) - \left(-\frac{1}{2}\cos 0\right)$$

$$= \frac{1}{2} + \frac{1}{2} = 1$$

4 iv) $\int_0^{\frac{\pi}{4}} \frac{e^{\tan x}}{\cos^2 x} \, dx$

Let $u = \tan x$

$\Rightarrow \frac{du}{dx} = \sec^2 x$

$\Rightarrow du = \sec^2 x \, dx$

$\Rightarrow du = \frac{1}{\cos^2 x} dx$

When $x = \frac{\pi}{4}$, $u = \tan\frac{\pi}{4} = 1$

when $x = 0$, $u = \tan 0 = 0$

$$\int_0^{\frac{\pi}{4}} \frac{e^{\tan x}}{\cos^2 x} \, dx = \int_0^1 e^u \, du$$

$$= \left[e^u\right]_0^1$$

$$= e - 1$$

4 v) $\int_0^{\frac{\pi}{4}} \frac{1}{\cos^2 x(1+\tan x)} \, dx$

Let $u = 1 + \tan x$

$\Rightarrow \frac{du}{dx} = \sec^2 x$

$\Rightarrow du = \sec^2 x \, dx$

$\Rightarrow du = \frac{1}{\cos^2 x} dx$

4v) (cont) When $x = \frac{\pi}{4}$, $u = 1 + \tan\frac{\pi}{4} = 2$

when $x = 0$ $u = 1 + \tan 0 = 1$

$$\int_0^{\frac{\pi}{4}} \frac{1}{\cos^2 x(1+\tan x)}\,dx$$

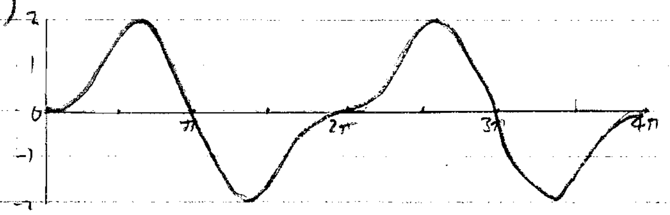
$$= \int_1^2 \frac{1}{u}\,du$$

$$= \Big[\ln u\Big]_1^2$$

$$= \ln 2 - \ln 1$$

$$= \ln 2$$

5 i)



$$y = \sin x(\cos x - 1)^2$$

$$\text{Area} = \int_0^{\pi} \sin x\left(\cos x - 1\right)^2$$

Let $u = \cos x - 1$

$$\Rightarrow \frac{du}{dx} = -\sin x$$

$$\Rightarrow du = -\sin x\,dx$$

$$\Rightarrow -du = \sin x\,dx$$

When $x = \pi$ $u = \cos\pi - 1$
$= -1 - 1 = -2$

when $x = 0$, $u = \cos 0 - 1$
$= 1 - 1 = 0$

$$\int_0^{\pi} \sin x\left(\cos x - 1\right)^2 dx$$

$$= \int_0^{-2} -u^2\,du$$

$$= \left[\frac{-u^3}{3}\right]_0^{-2}$$

$$= \left(-\frac{(-2)^3}{3}\right) - \left(0\right)$$

$$= \frac{8}{3}$$

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