MEI core 3 In TEGRAGTOM of trigonometric functions ExERCISES
1)

$$
\begin{aligned}
& \int(\sin x-2 \cos x) d x \\
& =-\cos x-2 \sin x+c
\end{aligned}
$$

ii)

$$
\begin{aligned}
& \int(3 \cos x+2 \sin x) d x \\
= & 3 \sin x-2 \cos x+c
\end{aligned}
$$

iii) $\int \overline{(5 \sin x+4 \cos x) d x}$

$$
=-5 \cos x+4 \sin x+c
$$

2) i) $\int \cos 3 x d x=\frac{1}{3} \sin 3 x+c$
ii).

$$
\int \sin (1-x) d x
$$

Let $u=1-x$

$$
\begin{aligned}
\Rightarrow \frac{d x}{d x} & =-1 \\
d x & =-d x \\
-d x & =d x \\
\int \sin (1-x) d x & =\int-\sin u d x \\
& =\cos u+c \\
& =\cos (1-x)+c
\end{aligned}
$$

iii) $\int \sin x \cos ^{3} x d x$

$$
\begin{aligned}
\text { Let } u & =\cos x \\
\frac{d u}{d x} & =-\sin x \\
d u & =-\sin x d x \\
-d u & =\sin x d x \\
\int \sin x \cos ^{3} x d x & =\int-u^{3} d u
\end{aligned}
$$

$$
\begin{aligned}
& =-\frac{u^{4}}{4}+c \\
& =-\frac{\cos ^{4} x}{4}+c
\end{aligned}
$$

iv)

$$
\begin{aligned}
& \int \frac{\sin x}{2-\cos x} d x \\
& =\ln |2-\cos x|+c
\end{aligned}
$$

v)

$$
\begin{aligned}
& \int \tan x d x=\int \frac{\sin x}{\cos x} d x \\
& =-\int \frac{-\sin x}{\cos x} d x \\
& =-\ln |\cos x|+c
\end{aligned}
$$

vi)

$$
\int \sin 2 x(1+\cos 2 x)^{2} d x
$$

Let $u=1+\cos 2 x$

$$
\Rightarrow \frac{d x}{d x}=-2 \sin 2 x
$$

$$
\Rightarrow \quad d x=-2 \sin 2 x d x
$$

$$
\Rightarrow-\frac{1}{2} d u=\sin 2 x d x
$$

$$
\int \sin 2 x(1+\cos 2 x)^{2} d x=-\frac{1}{2} \int u^{2} d u
$$

$$
\begin{aligned}
& =-\frac{1}{6} u^{3}+c \\
& =-\frac{1}{6}(1+\cos 2 x)^{3}+c
\end{aligned}
$$

3) 

$$
\int 2 x \sin \left(x^{2}\right) d x
$$

$$
\text { Let } u=x^{2}
$$

$$
\Rightarrow \frac{d u}{d x}=2 x
$$



Bi) $\Rightarrow d u=2 x d x$
cont)

$$
\begin{gathered}
\int 2 x \sin \left(x^{2}\right) d x=\int \sin u d u \\
=-\cos u+c \\
=-\cos \left(x^{2}\right)+c
\end{gathered}
$$

Bi)

$$
\int \cos x e^{\sin x} d x
$$

Let $u=\sin x$

$$
\begin{aligned}
& \Rightarrow \frac{d x}{d x}=\cos x \\
& \Rightarrow d u=\cos x d x
\end{aligned}
$$

$$
\int \cos x e^{\sin x} d x=\int e^{u} d u
$$

$$
=e^{u}+c
$$

$$
=e^{\sin x}+c
$$

3iii)

$$
\int \frac{\tan x}{\cos ^{2} x} d x
$$

Let $x=\tan x$

$$
\Rightarrow \frac{d u}{d x}=\sec ^{2} x
$$

$$
\Rightarrow \frac{d u}{d x}=\frac{1}{\cos ^{2} x}
$$

$$
\Rightarrow d u=\frac{d x}{6 x^{2}}
$$

$$
\begin{aligned}
\int \frac{\tan x}{\cos ^{2} x} d x & =u d u \\
& =\frac{u^{2}}{2}+c \\
& =\frac{1}{2} \tan ^{2} x+c
\end{aligned}
$$

Sis) $\int \frac{\cos x}{\sin ^{2} x} d x$
Let $u=\sin x$

$$
\begin{aligned}
\Rightarrow \frac{d y}{d x} & =\cos x \\
d x & \cos x d x
\end{aligned}
$$

$$
\begin{array}{r}
\int \frac{\cos x}{\sin ^{2} x} d x=\int \frac{1}{u^{2}} d u \\
\quad=-\frac{1}{u}+c \\
\quad=-\frac{1}{\sin x}+c
\end{array}
$$

4) i)

$$
\int_{0}^{\frac{\pi}{2}} \cos \left(2 x-\frac{\pi}{2}\right) d x
$$

Let $u=2 x-\frac{\pi}{2}$

$$
\Rightarrow \frac{d u}{d x}=2
$$

$$
\Rightarrow \quad d u=2 d x
$$

$$
7 \frac{1}{2} d x=d x
$$

when $x=\frac{\pi}{2}, a^{2}=\pi-\frac{\pi}{2}=\frac{\pi}{2}$
when $x=0$ u $=0-\frac{r}{r}=-\frac{\pi}{4}$

$$
\begin{aligned}
& \int_{0}^{\frac{\pi}{2}} \cos \left(2 x-\frac{\pi}{2}\right) d x=\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \frac{1}{2} \cos u d x \\
& \quad=\frac{1}{2}[\sin u]_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \\
& \quad=\frac{1}{2}\left[\sin \pi-\sin \left(\frac{\pi}{2}\right]\right. \\
& =\frac{1}{2}[1--1]=1
\end{aligned}
$$



Ai)

$$
\begin{aligned}
& \int_{0}^{\frac{\pi}{4}} \cos x \sin ^{3} x d x \\
& \text { Let } u=\sin x \\
& \Rightarrow \frac{d u}{d x}=\cos x \\
& \Rightarrow d x=\cos x d x
\end{aligned}
$$

$$
\text { When } x=0, u=\sin 0=0
$$

$$
\text { When } x=\frac{\pi}{4} \quad u=\sin \frac{\pi}{4}=\frac{1}{12}
$$

$$
\int_{0}^{\frac{\pi}{4}} \cos x \sin ^{3} x d x=\int_{0}^{\frac{1}{2}} u^{3} d x
$$

$$
=\left[\frac{u^{1}}{4}\right]_{0}^{\frac{1}{\sqrt{2}}}
$$

$$
=\left(\frac{1}{4}\right) \cdot(0)
$$

4.iii)

$$
\begin{aligned}
& \int_{0}^{x} \sin \left(x^{2}\right) d x \\
& \operatorname{Let} u=x^{2} \\
& \Rightarrow d x=2 x \\
& \Rightarrow d x=2 x d x \\
& \Rightarrow \frac{1}{2} d x=x d x
\end{aligned}
$$

when $x=0 ; u=0$
when $x=\sqrt{\pi} y=\pi$

$$
\int_{0}^{\pi} x \sin \left(x^{2}\right) d x=\int_{0}^{\pi} \frac{1}{2} \sin u d x
$$

$$
\begin{aligned}
& =\left[-\frac{1}{2} \cos u\right]_{0}^{\pi} \\
& =\left(-\frac{1}{2} \cos \pi\right)-\left(-\frac{1}{2} \cos \theta\right) \\
& =\frac{1}{2}+\frac{1}{2}=1
\end{aligned}
$$

4iv)

$$
\begin{aligned}
& \int_{0}^{\frac{\pi}{4}} \frac{e^{\tan x}}{\cos ^{2} x} d x \\
& \text { Let } u=\tan x \\
& \Rightarrow \frac{d u}{d x}=\sec ^{2} x \\
& \Rightarrow d x=\sec ^{2} x d x \\
& \Rightarrow d u=\frac{1}{\cos ^{2} x} d x
\end{aligned}
$$

When $x=\frac{\pi}{4}, u=\tan \frac{\pi}{4}=1$ when $x=0, u=\tan 0=0$

$$
\begin{gathered}
\int_{0}^{\frac{\pi}{4}} \frac{e^{\tan x}}{\cos ^{2} x} d x=\int_{0}^{1} e^{u} d x \\
\quad=\left[e^{u}\right]_{0}^{1} \\
=e-1
\end{gathered}
$$

$\int_{0}^{\frac{\pi}{4}} \frac{1}{\cos ^{2} x(1+\tan x)} d x$
Let $u=1+\tan x$

$$
\begin{aligned}
& \Rightarrow \frac{d u}{d x}=\sec ^{2} x \\
& \Rightarrow d u=\sec ^{2} x d x \\
& \Rightarrow d x=\frac{1}{\cos ^{2} x} d x
\end{aligned}
$$

MEI CORES integration of trigonambizic functions EXERCNSSC
$\begin{aligned}4 v) \text { When } x=\frac{\pi}{4}, u=1+\tan \frac{\pi}{4}=2 \text { when } x=0, u & =\cos 0-1 \\ \text { cont) } & =1 \cdots 1=\end{aligned}$
when $x=0 \quad u=1+\tan 0=1$

$$
\begin{aligned}
& \int_{0}^{\frac{\pi}{4}} \frac{1}{\cos ^{2} x(1+\tan x)} d x \\
&=\int_{1}^{2} \frac{1}{u} d u \\
&=[\ln u]_{1}^{2} \\
&=\ln 2-\ln 1 \\
&=\ln 2
\end{aligned}
$$



$$
y=\sin x(\cos x-1)^{2}
$$

Area $=\int_{0}^{\pi} \sin x(\cos x-1)^{2}$
Let $u=\cos x-1$

$$
\begin{aligned}
& \Rightarrow \frac{d x}{d x}=-\sin x \\
& \Rightarrow d x=-\sin x d x \\
& \Rightarrow-d x=\sin x d x
\end{aligned}
$$

when $x=\pi \quad u=\cos \pi-1$

$$
=\quad 1-1:-2
$$

