1 i) $\displaystyle\int_0^1 x e^{3x}\,dx$

Let $u = x$ Let $\dfrac{dv}{dx} = e^{3x}$

$\Rightarrow \dfrac{du}{dx} = 1$ $\Rightarrow v = \dfrac{1}{3}e^{3x}$

Using $\displaystyle\int u\frac{dv}{dx} = uv - \int v\frac{du}{dx}$

$$\int_0^1 x e^{3x}\,dx = \left[\tfrac{1}{3}x e^{3x}\right]_0^1 - \int_0^1 \tfrac{1}{3}e^{3x}\,dx$$

$$= \left[\tfrac{1}{3}x e^{3x}\right]_0^1 - \left[\tfrac{1}{9}e^{3x}\right]_0^1$$

$$= \left[\tfrac{1}{3}e^{3} - 0\right] - \left[\tfrac{1}{9}e^{3} - \tfrac{1}{9}\right]$$

$$= \frac{2}{9}e^{3} + \frac{1}{9}$$

(ii) $\displaystyle\int_0^{\pi} (x-1)\cos x\,dx$

Let $u = x-1$ Let $\dfrac{dv}{dx} = \cos x$

$\Rightarrow \dfrac{du}{dx} = 1$ $\Rightarrow v = \sin x$

Using $\displaystyle\int u\frac{dv}{dx} = uv - \int v\frac{du}{dx}$

$$\int_0^{\pi} (x-1)\cos x = \left[(x-1)\sin x\right]_0^{\pi} - \int_0^{\pi} \sin x\,dx$$

$$= \left[(x-1)\sin x\right]_0^{\pi} - \left[-\cos x\right]_0^{\pi}$$

$$= \left[(\pi-1)\sin\pi - (0-1)\sin 0\right] - \left[-\cos\pi - -\cos 0\right]$$

$$= [0 - 0] - [1 + 1] = -2$$

1 iii) $\displaystyle\int_0^2 (x+1)e^{x}\,dx$

Let $u = x+1$ Let $\dfrac{dv}{dx} = e^{x}$

$\Rightarrow \dfrac{du}{dx} = 1$ $\Rightarrow v = e^{x}$

Using $\displaystyle\int u\frac{dv}{dx} = uv - \int v\frac{du}{dx}$

$$\int_0^2 (x+1)e^{x}\,dx = \left[(x+1)e^{x}\right]_0^2 - \int_0^2 e^{x}\,dx$$

$$= \left[(x+1)e^{x}\right]_0^2 - \left[e^{x}\right]_0^2$$

$$= \left[3e^{2} - 1e^{0}\right] - \left[e^{2} - e^{0}\right]$$

$$= 3e^{2} - 1 - e^{2} + 1$$

$$= 2e^{2}$$

1 iv) $\displaystyle\int_1^2 \ln 2x\,dx = \int_1^2 1\ln 2x\,dx$

Let $u = \ln 2x$ Let $\dfrac{dv}{dx} = 1$

$\Rightarrow \dfrac{du}{dx} = \dfrac{2}{2x}$ $\Rightarrow v = x$

$\Rightarrow \dfrac{du}{dx} = \dfrac{1}{x}$

Using $\displaystyle\int u\frac{dv}{dx} = uv - \int v\frac{du}{dx}$

$$\int_1^2 \ln 2x\,dx = \left[x\ln 2x\right]_1^2 - \int_1^2 x\frac{1}{x}\,dx$$

$$= \left[x\ln 2x\right]_1^2 - \int_1^2 1\,dx$$

1(iv) cont

$$= \left[x \ln 2x\right]_1^2 - \left[x\right]_1^2$$

$$= \left[2\ln 4 - 1\ln 2\right] - \left[2 - 1\right]$$

$$= 2\ln 4 - \ln 2 - 2 + 1$$

$$= 4\ln 2 - \ln 2 - 1$$

$$= 3\ln 2 - 1$$

1(v) $\displaystyle\int_0^{\frac{\pi}{2}} x \sin 2x \, dx$

Let $u = x$ $\Rightarrow \frac{du}{dx} = 1$

Let $\frac{dv}{dx} = \sin 2x$ $\Rightarrow v = -\frac{1}{2}\cos 2x$

Using $\displaystyle\int u \frac{dv}{dx} = uv - \int v \frac{du}{dx}$

$$\int_0^{\frac{\pi}{2}} x \sin 2x \, dx$$

$$= \left[-\frac{1}{2} x \cos 2x\right]_0^{\frac{\pi}{2}} + \int_0^{\frac{\pi}{2}} \frac{1}{2}\cos 2x \, dx$$

$$= \left[-\frac{1}{2} x \cos 2x\right]_0^{\frac{\pi}{2}} + \left[\frac{1}{4}\sin 2x\right]_0^{\frac{\pi}{2}}$$

$$= \left[-\frac{\pi}{4}\cos\pi - -\frac{1}{2}\times 0\cos 0\right]$$

$$+ \left[\frac{1}{4}\sin\pi - \frac{1}{4}\sin 0\right]$$

$$= \left[\frac{\pi}{4} + 0\right] + \left[0 - 0\right]$$

$$= \frac{\pi}{4}$$

1(vi) $\displaystyle\int_1^4 x^2 \ln x \, dx$

Let $u = \ln x$ $\Rightarrow \frac{du}{dx} = \frac{1}{x}$

Let $\frac{dv}{dx} = x^2$ $\Rightarrow v = \frac{x^3}{3}$

Using $\displaystyle\int u \frac{dv}{dx} = uv - \int v \frac{du}{dx}$

$$\int_1^4 x^2 \ln x \, dx$$

$$= \left[\frac{x^3}{3}\ln x\right]_1^4 - \int_1^4 \frac{x^3}{3}\frac{1}{x} \, dx$$

$$= \left[\frac{x^3}{3}\ln x\right]_1^4 - \int_1^4 \frac{x^2}{3} \, dx$$

$$= \left[\frac{x^3}{3}\ln x\right]_1^4 - \left[\frac{x^3}{9}\right]_1^4$$

$$= \left[\frac{64}{3}\ln 4 - \frac{1}{3}\ln 1\right] - \left[\frac{64}{9} - \frac{1}{9}\right]$$

$$= \frac{64}{3}\ln 4 - \frac{63}{9}$$

$$= \frac{64}{3}\ln 4 - 7$$

2 i) $y = (2 - x)e^{-x}$

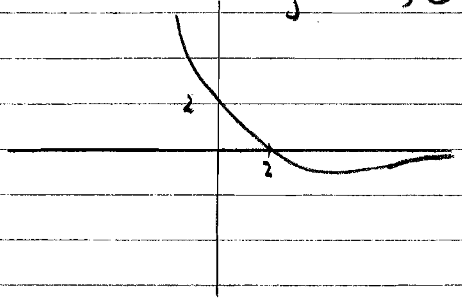
When $x = 0$, $y = (2 - 0)e^0 = 2$

When $y = 0$ $\Rightarrow (2 - x) = 0$ $\Rightarrow x = 2$

Cuts y-axis at $(0, 2)$

Cuts x-axis at $(2, 0)$

2ii) $y = (2-x)e^{-x}$



2iii) Area $= \int_0^2 (2-x)e^{-x}\,dx$

Let $u = (2-x)$ $\Rightarrow \frac{du}{dx} = -1$

Let $\frac{dv}{dx} = e^{-x}$ $\Rightarrow v = -e^{-x}$

Using $\int u\frac{dv}{dx} = uv - \int v\frac{du}{dx}$

$\int_0^2 (2-x)e^{-x}\,dx$

$= \left[-(2-x)e^{-x}\right]_0^2 - \int_0^2 (-1)(-e^{-x})\,dx$

$= \left[(x-2)e^{-x}\right]_0^2 - \int_0^2 e^{-x}\,dx$

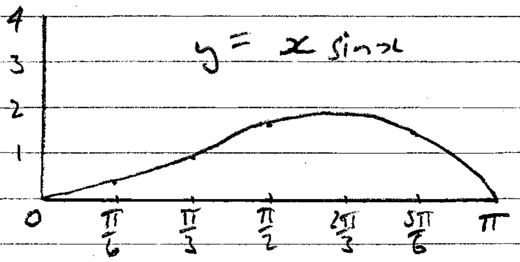
$= \left[(x-2)e^{-x}\right]_0^2 - \left[-e^{-x}\right]_0^2$

$= \left[0e^{-2} + 2e^{0}\right] - \left[-e^{-2} - -e^{0}\right]$

$= +2 + e^{-2} - 1$

$= e^{-2} + 1$

3)i) $y = x\sin x$



ii) Area $= \int_0^{\pi} x\sin x\,dx$

Let $u = x$ $\Rightarrow \frac{du}{dx} = 1$

Let $\frac{dv}{dx} = \sin x$ $\Rightarrow v = -\cos x$

Using $\int u\frac{dv}{dx} = uv - \int v\frac{du}{dx}$

$\int_0^{\pi} x\sin x\,dx$

$= \left[-x\cos x\right]_0^{\pi} + \int_0^{\pi} \cos x\,dx$

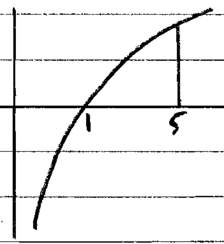
$= \left[-x\cos x\right]_0^{\pi} + \left[\sin x\right]_0^{\pi}$

$= \left[-\pi\cos\pi - 0\cos 0\right] + \left[0 - 0\right]$

$= \left[\pi - 0\right] + \left[0\right]$

$= \pi$

4) $y = \ln x$



Area $= \int_1^5 \ln x\,dx$

$= \left[x\ln x - x\right]_1^5$

$= (5\ln 5 - 5) - (1\ln 1 - 1)$

$= 5\ln 5 - 5 - (0 - 1) = 5\ln 5 - 4$

5) Area $= \int_0^{\frac{\pi}{2}} x\cos x \, dx$

Let $u = x$ $\Rightarrow \frac{du}{dx} = 1$

Let $\frac{dv}{dx} = \cos x$ $\Rightarrow v = \sin x$

Using $\int u \frac{dv}{dx} = uv - \int v \frac{du}{dx}$

$$\int_0^{\frac{\pi}{2}} x\cos x \, dx$$

$$= \left[x \sin x\right]_0^{\frac{\pi}{2}} - \int_0^{\frac{\pi}{2}} \sin x \, dx$$

$$= \left[x \sin x\right]_0^{\frac{\pi}{2}} - \left[-\cos x\right]_0^{\frac{\pi}{2}}$$

$$= \left[\frac{\pi}{2}\sin\frac{\pi}{2} - 0\right] - \left[-\cos\frac{\pi}{2} - -\cos 0\right]$$

$$= \left[\frac{\pi}{2} - 0\right] - \left[0 + 1\right]$$

$$= \frac{\pi}{2} - 1$$

6)

i) $y = x\sqrt{x+1}$

When $y = 0$ either $x = 0$ or $x = -1$

Area $= \left| \int_{-1}^{0} x\sqrt{x+1} \, dx \right|$

$$\int_{-1}^{0} x\sqrt{x+1} \, dx$$

Let $u = x$ $\Rightarrow \frac{du}{dx} = 1$

Let $\frac{dv}{dx} = (x+1)^{\frac{1}{2}}$ $\Rightarrow v = \frac{2}{3}(x+1)^{3/2}$

Using $\int u \frac{dv}{dx} = uv - \int v \frac{du}{dx}$

$$\int_{-1}^{0} x\sqrt{x+1} \, dx$$

$$= \left[\frac{2}{3}x(x+1)^{3/2}\right]_{-1}^{0} - \int_{-1}^{0} \frac{2}{3}(x+1)^{\frac{3}{2}} dx$$

$$= \left[\frac{2}{3}x(x+1)^{\frac{3}{2}}\right]_{-1}^{0} - \left[\frac{2}{5}\frac{2}{3}(x+1)^{5/2}\right]_{-1}^{0}$$

$$= \left[0 - -\frac{2}{3}(0)^{\frac{3}{2}}\right] - \left[\frac{4}{15}(1)^{5/2} - \frac{4}{15}0^{5/2}\right]$$

$$= \left[0 + 0\right] - \left[\frac{4}{15} - 0\right]$$

$$= -\frac{4}{15}$$

$\therefore$ Area $\left| -\frac{4}{15} \right|$

$$= \frac{4}{15} \text{ units}^2$$

ii) By substitution $u = x+1$

$$\int_{-1}^{0} x\sqrt{x+1} \, dx$$

Let $u = x+1$

$\Rightarrow \frac{du}{dx} = 1$

$\Rightarrow du = dx$

Also $x = u - 1$

when $x = 0$, $u = 1$

when $x = -1$, $u = 0$

6 cont) ii)

$$\int_{-1}^{0} x\sqrt{x+1}\,dx$$

$$= \int_{0}^{1} (u-1)u^{\frac{1}{2}}\,du$$

$$= \int_{0}^{1} \left(u^{3/2} - u^{\frac{1}{2}}\right) du$$

$$= \left[\frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2}\right]_0^1$$

$$= \left[\frac{2}{5} - \frac{2}{3}\right] - \left[0 - 0\right]$$

$$= \frac{6}{15} - \frac{10}{15} = -\frac{4}{15}$$

$$\text{Area} = \left|-\frac{4}{15}\right| = \frac{4}{15}\text{ units}^2$$

7)

$$y = x^2 \ln 2x$$

$y = 0 \Rightarrow \ln 2x = 0$

since fn not defined for $x = 0$

$$\ln 2x = 0$$

$$\Rightarrow 2x = 1$$

$$x = \frac{1}{2}$$

$$\text{Area} = \int_{\frac{1}{2}}^{1} x^2 \ln 2x\,dx$$

Let $u = \ln 2x$ Let $\frac{dv}{dx} = x^2$

$$\Rightarrow \frac{du}{dx} = \frac{2}{2x} \qquad \Rightarrow v = \frac{x^3}{3}$$

$$\Rightarrow \frac{du}{dx} = \frac{1}{x}$$

Using $\int u\frac{dv}{dx} = uv - \int v\frac{du}{dx}$

$$\int_{\frac{1}{2}}^{1} x^2 \ln 2x\,dx$$

$$= \left[\frac{x^3}{3}\ln 2x\right]_{\frac{1}{2}}^{1} - \int_{\frac{1}{2}}^{1} \frac{x^3}{3}\frac{1}{x}\,dx$$

$$= \left[\frac{x^3}{3}\ln 2x\right]_{\frac{1}{2}}^{1} - \int_{\frac{1}{2}}^{1} \frac{x^2}{3}\,dx$$

$$= \left[\frac{x^3}{3}\ln 2x\right]_{\frac{1}{2}}^{1} - \left[\frac{x^3}{9}\right]_{\frac{1}{2}}^{1}$$

$$= \left[\frac{1}{3}\ln 2 - \frac{1}{24}\ln 1\right] - \left[\frac{1}{9} - \frac{1}{72}\right]$$

$$= \frac{1}{3}\ln 2 - \frac{1}{9} + \frac{1}{72}$$

$$= \frac{1}{3}\ln 2 - \frac{7}{72}$$

$= 0.134$ units^2

to 3 sig fig.

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