

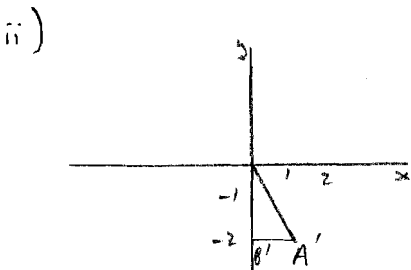
Enlargement scale factor 3, centre (0,0)

$$A(1,2) \quad A'(3,6)$$

$$B(0,2) \quad B'(0,6)$$

$$(x', y') = (3x, 3y)$$

$$\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$$



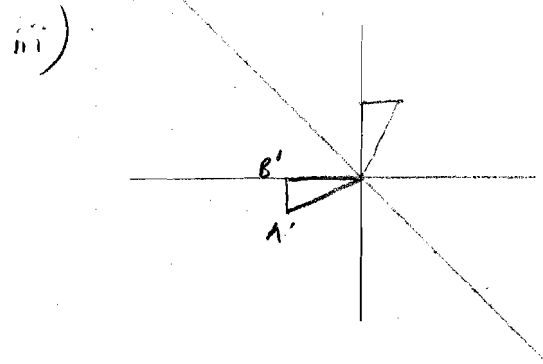
Reflection in x axis

$$A(1,2) \quad A'(1,-2)$$

$$B(0,2) \quad B'(0,-2)$$

$$(x', y') = (x, -y)$$

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$



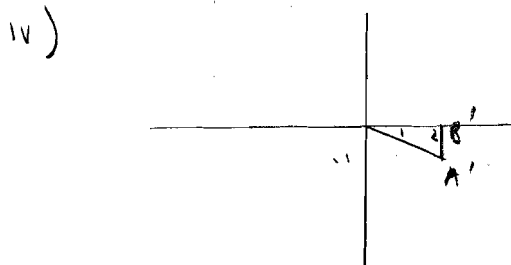
Reflection in  $x + y = 0$

$$A(1,2) \quad A'(-2,-1)$$

$$B(0,2) \quad B'(-2,0)$$

$$(x', y') = (-y, -x)$$

$$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$$



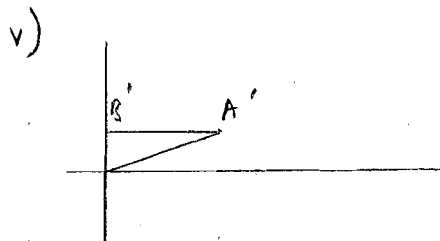
Rotation  $90^\circ$  clockwise about O

$$A(1,2) \quad A'(2,-1)$$

$$B(0,2) \quad B'(2,0)$$

$$(x', y') = (y, -x)$$

$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$



$$A(1,2) \quad A'(3,1)$$

$$B(0,2) \quad B'(0,1)$$

$$(x', y') = (3x, \frac{1}{2}y)$$

$$\begin{pmatrix} 3 & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$$

2)  $\begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$

$A(1,2)$   $A'(\cos 2\theta - 2\sin 2\theta, \sin 2\theta + 2\cos 2\theta)$

$A'(0.256, 2.221)$

$B(0,2)$   $B'(-2\sin 2\theta, 2\cos 2\theta)$

$B'(-0.684, 1.879)$

3) i)  $\begin{pmatrix} \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & \frac{1}{2} \end{pmatrix} = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$   
 $\theta = 60^\circ$  anti-clockwise about  $(0,0)$

ii)  $\begin{pmatrix} 0.574 & -0.819 \\ 0.819 & 0.574 \end{pmatrix}$

$= \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$

$\theta = \cos^{-1}(0.574)$

$\theta = 54.97^\circ$  anti-clockwise about  $(0,0)$

iii)  $\begin{pmatrix} -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$

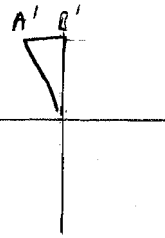
$\theta = 225^\circ$  anti-clockwise about  $(0,0)$

iv)  $\begin{pmatrix} -\frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & -\frac{\sqrt{3}}{2} \end{pmatrix}$

$\theta = 150^\circ$  anti-clockwise about  $(0,0)$

4) i)  $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$   $(x',y') = (-x, y)$

Reflection in y axis

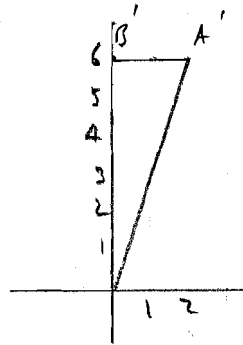


$A(1,2)$   $A'(-1,2)$

$B(0,2)$   $B'(0,2)$

4) ii)  $\begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$   $(x',y') = (2x, 3y)$

Two way stretch scale factor 2 horizontally, 3 vertically



$A(1,2)$   $A'(2,6)$

$B(0,2)$   $B'(0,6)$

4) iii)  $\begin{pmatrix} \frac{1}{2} & 0 \\ 0 & \frac{1}{2} \end{pmatrix}$   $(x',y') = (\frac{1}{2}x, \frac{1}{2}y)$

Enlargement by scale factor  $\frac{1}{2}$  about  $(0,0)$

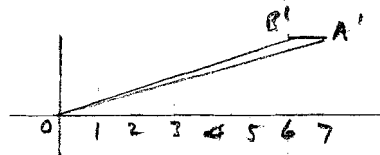


$A(1,2)$   $A'(\frac{1}{2}, 1)$

$B(0,2)$   $B'(0,1)$

4) iv)  $\begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix}$   $(x',y') = (x+3y, y)$

Shear parallel to x axis

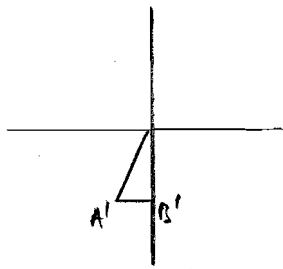


$A(1,2)$   $A'(7,2)$

$B(0,2)$   $B'(6,2)$

4v)  $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$   $(x', y') = (-x, -y)$

Rotation by  $180^\circ$  about  $(0,0)$



$A(1,2)$   $A'(-1,-2)$

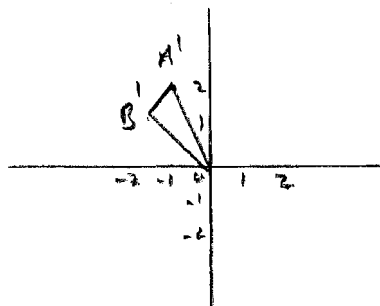
$B(0,2)$   $B'(0,-2)$

4vi)  $\begin{pmatrix} 0.6 & -0.8 \\ 0.8 & 0.6 \end{pmatrix}$

$\theta = \cos^{-1} 0.6 = 53.1^\circ$

Rotation by  $53.1^\circ$  anti-clockwise about  $(0,0)$

$(x', y') = (0.6x - 0.8y, 0.8x + 0.6y)$



$A(1,2)$   $A'(-1, 2)$   
 $B(0,2)$   $B'(-1.6, 1.2)$

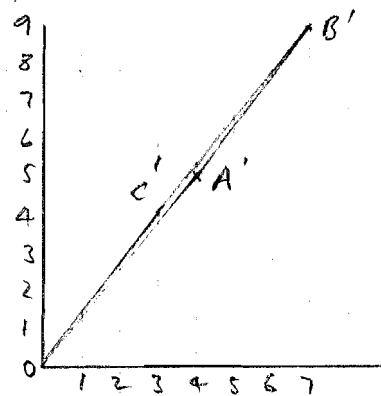
$A'(-1, 2)$

$B(0,2)$   $B'(-1.6, 1.2)$

5)  $\begin{pmatrix} 4 & 3 \\ 5 & 4 \end{pmatrix} \begin{pmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix}$

$= \begin{pmatrix} 0 & 4 & 7 & 3 \\ 0 & 5 & 9 & 4 \end{pmatrix}$

$A'(4,5)$   $B'(7,9)$   $C'(3,4)$



Area under  $OC'$  =  $\frac{1}{2} \times 3 \times 4 = 6$

Area under  $C'B'$  =  $\frac{1}{2} (4+9) \times 4 = 26$

Area under  $OC'B'$  =  $26 + 6 = 32 \text{ units}^2$

Area under  $OA'$  =  $\frac{1}{2} \times 4 \times 5 = 10$

Area under  $A'B'$  =  $\frac{1}{2} (5+9) \times 3 = 21$

Area under  $OA'B'$  =  $10 + 21 = 31 \text{ units}^2$

$\therefore$  Area within transformed shape  
 $= 32 - 31 = 1 \text{ unit}^2$

6)  $P \rightarrow P'$

each point on  $y=x$

line joining point to image  $\parallel$  to y axis

$(x, y) \rightarrow (x, x)$

Matrix is  $\begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}$

7) i)  $\begin{pmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{pmatrix}$  ii)  $\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$  iii)  $\begin{pmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

8) i) Rotate  $90^\circ$  clockwise about x axis

ii) Reflection in plane  $z=0$

iii) 3 way stretch by scale factors

2, 3 and  $\frac{1}{2}$   $\parallel$  to x, y and z axes