MEI FP1 MATRICES EXERCISE 1C

$\underline{A} = \begin{pmatrix} 3 & 1 \\ 2 & 4 \end{pmatrix}$ $\underline{B} = \begin{pmatrix} -3 & 7 \\ 2 & 5 \end{pmatrix}$ $\underline{C} = \begin{pmatrix} 2 & 3 & 4 \\ 5 & 7 & 1 \end{pmatrix}$

$\underline{D} = \begin{pmatrix} 3 & 4 \\ 7 & 0 \\ 1 & -2 \end{pmatrix}$ $\underline{E} = \begin{pmatrix} 4 & 7 \\ 3 & -2 \\ 1 & 5 \end{pmatrix}$ $\underline{F} = \begin{pmatrix} 3 & 7 & -5 \\ 2 & 6 & 0 \\ -1 & 4 & 8 \end{pmatrix}$

1) i) $\underline{AB} = \begin{pmatrix} -7 & 26 \\ 2 & 34 \end{pmatrix}$

ii) $\underline{CA}$ $\quad \underline{C}_{2\times3}$ $\quad \underline{A}_{2\times2}$

$\underline{CA}$ does not exist

iii) $\underline{BC}$ $\quad B_{2\times2}\ C_{2\times3} \Rightarrow BC_{2\times3}$

$\underline{BC} = \begin{pmatrix} 29 & 40 & -5 \\ 29 & 41 & 13 \end{pmatrix}$

iv) $\underline{C}_{2\times3}\ \underline{D}_{3\times2}$ $\quad \underline{CD}_{2\times2}$

$\underline{CD} = \begin{pmatrix} 31 & 0 \\ 65 & 18 \end{pmatrix}$

v) $\underline{D}_{3\times2}\ \underline{C}_{2\times3}$ $\quad \underline{DC}_{3\times3}$

$\underline{DC} = \begin{pmatrix} 26 & 37 & 16 \\ 14 & 21 & 28 \\ -8 & -11 & 2 \end{pmatrix}$

vi) $\underline{A}_{2\times2}\ \underline{F}_{3\times3}$

$\underline{AF}$ does not exist

vii) $B_{2\times2}\ E_{3\times2}$ $\quad \underline{BE}$ does not exist

viii) $4\underline{F} + \underline{EC}$

$\underline{E}_{3\times2}\ \underline{C}_{2\times3}$ $\quad \underline{EC}_{3\times3}$

$\underline{EC} = \begin{pmatrix} 43 & 61 & 23 \\ -4 & -5 & 10 \\ 27 & 38 & 9 \end{pmatrix}$

$4\underline{F} = \begin{pmatrix} 12 & 28 & -20 \\ 8 & 24 & 0 \\ -4 & 16 & 32 \end{pmatrix}$

$4\underline{F} + \underline{EC} = \begin{pmatrix} 55 & 89 & 3 \\ 4 & 19 & 10 \\ 23 & 54 & 41 \end{pmatrix}$

ix) $\underline{E}_{3\times2}\ \underline{A}_{2\times2}$ $\quad \underline{EA}_{3\times2}$

$\underline{EA} = \begin{pmatrix} 26 & 32 \\ 5 & -5 \\ 13 & 21 \end{pmatrix}$

x) $\underline{F}_{3\times3}\ \underline{E}_{3\times2}$ $\quad \underline{FE}_{3\times2}$

$\underline{FE} = \begin{pmatrix} 28 & -18 \\ 26 & 2 \\ 16 & 25 \end{pmatrix}$

xi) $\underline{E}_{3\times2}\ \underline{F}_{3\times3}$

$\underline{EF}$ does not exist

MEI FP1 MATRICES EXERCISE 1C

1 (cont) xii) $\underline{A}^2$ $\quad A_{2\times2} \quad A_{2\times2}$

$\underline{A}^2_{2\times2}$

$$\begin{pmatrix} 3 & 1 \\ 2 & 4 \end{pmatrix}\begin{pmatrix} 3 & 1 \\ 2 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} 11 & 7 \\ 14 & 18 \end{pmatrix}$$

2)

$$\underline{AB} = \begin{pmatrix} 3 & 1 \\ 2 & 4 \end{pmatrix}\begin{pmatrix} -3 & 7 \\ 2 & 5 \end{pmatrix}$$

$$\underline{AB} = \begin{pmatrix} -7 & 26 \\ 2 & 34 \end{pmatrix}$$

$$\underline{BA} = \begin{pmatrix} -3 & 7 \\ 2 & 5 \end{pmatrix}\begin{pmatrix} 3 & 1 \\ 2 & 4 \end{pmatrix}$$

$$\underline{BA} = \begin{pmatrix} 11 & 25 \\ 16 & 22 \end{pmatrix}$$

$\therefore \underline{AB} \neq \underline{BA}$

$\underline{A}_{2\times2} \quad \underline{D}_{3\times2}$

$\therefore$ $\underline{AD}$ does not exist

$$\underline{DA} = \begin{pmatrix} 3 & 4 \\ 7 & 0 \\ 1 & -2 \end{pmatrix}\begin{pmatrix} 3 & 1 \\ 2 & 4 \end{pmatrix}$$

$$\underline{DA} = \begin{pmatrix} 17 & 19 \\ 21 & 7 \\ -1 & -7 \end{pmatrix}$$

$\therefore \underline{AD} \neq \underline{DA}$

Matrix multiplication is not commutative

3)

$$\underline{AC} = \begin{pmatrix} 3 & 1 \\ 2 & 4 \end{pmatrix}\begin{pmatrix} 2 & 3 & 4 \\ 5 & 7 & 1 \end{pmatrix}$$

$$\underline{AC} = \begin{pmatrix} 11 & 16 & 13 \\ 24 & 34 & 12 \end{pmatrix}$$

$$(\underline{AC})\underline{F} = \begin{pmatrix} 11 & 16 & 13 \\ 24 & 34 & 12 \end{pmatrix}\begin{pmatrix} 3 & 7 & -5 \\ 2 & 6 & 0 \\ -1 & 4 & 8 \end{pmatrix}$$

$$(\underline{AC})\underline{F} = \begin{pmatrix} 52 & 225 & 49 \\ 128 & 420 & -24 \end{pmatrix}$$

$$(\underline{CF}) = \begin{pmatrix} 2 & 3 & 4 \\ 5 & 7 & 1 \end{pmatrix}\begin{pmatrix} 3 & 7 & -5 \\ 2 & 6 & 0 \\ -1 & 4 & 8 \end{pmatrix}$$

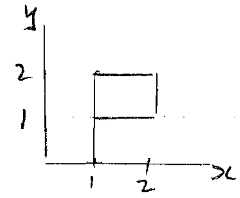
$$= \begin{pmatrix} 8 & 48 & 22 \\ 28 & 81 & -17 \end{pmatrix}$$

$$\underline{A}(\underline{CF}) = \begin{pmatrix} 3 & 1 \\ 2 & 4 \end{pmatrix}\begin{pmatrix} 8 & 48 & 22 \\ 28 & 81 & -17 \end{pmatrix}$$

$$\underline{A}(\underline{CF}) = \begin{pmatrix} 52 & 225 & 49 \\ 128 & 420 & -24 \end{pmatrix}$$

$\therefore (\underline{AC})\underline{F} = \underline{A}(\underline{CF})$

4)



i) $(1,0), (1,1), (1,2), (2,2), (2,1)$

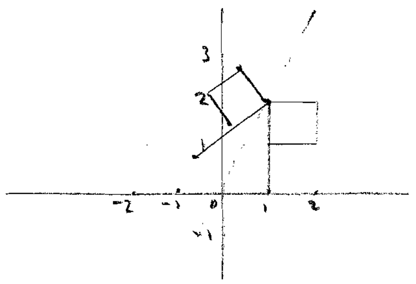
$$\begin{pmatrix} 1 & 1 & 1 & 2 & 2 \\ 0 & 1 & 2 & 2 & 1 \end{pmatrix}$$

ii)

$$\begin{pmatrix} -0.6 & 0.8 \\ 0.8 & 0.6 \end{pmatrix}\begin{pmatrix} 1 & 1 & 1 & 2 & 2 \\ 0 & 1 & 2 & 2 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} -0.6 & 0.2 & 1 & 0.4 & -0.4 \\ 0.8 & 1.4 & 2 & 2.8 & 2.2 \end{pmatrix}$$

iii



Reflection in line $y = 2x$

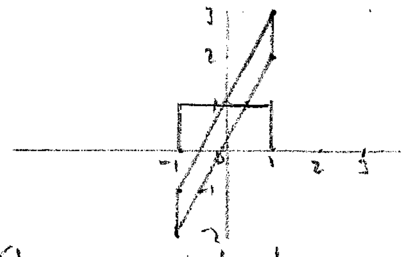
5) i) $(1,0)$ $(1,1)$ $(-1,1)$ $(-1,0)$

$$\begin{pmatrix} 1 & 1 & -1 & -1 \\ 0 & 1 & 1 & 0 \end{pmatrix}$$

ii) $$\begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 & -1 & -1 \\ 0 & 1 & 1 & 0 \end{pmatrix}$$

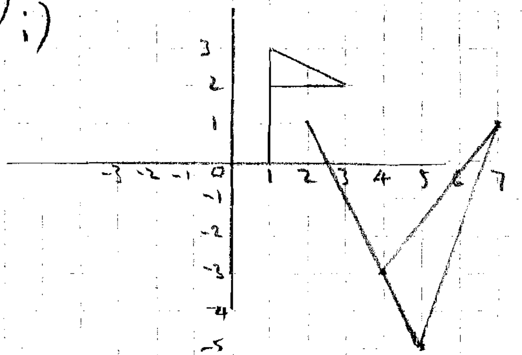
$$= \begin{pmatrix} 1 & 1 & -1 & -1 \\ 2 & 3 & -1 & -2 \end{pmatrix}$$

iii)



Shear parallel to y axis

6) i)



$$S = \begin{pmatrix} 1 & 1 & 1 & 3 \\ 0 & 2 & 3 & 1 \end{pmatrix}$$

$$MS = \begin{pmatrix} 2 & 1 \\ 1 & -2 \end{pmatrix}\begin{pmatrix} 1 & 1 & 1 & 3 \\ 0 & 2 & 3 & 1 \end{pmatrix}$$

ii)

$$S' = \begin{pmatrix} 2 & 4 & 5 & 7 \\ 1 & -3 & -5 & 1 \end{pmatrix}$$

iii)

$$\underline{M}^2 = \begin{pmatrix} 2 & 1 \\ 1 & -2 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 1 & -2 \end{pmatrix}$$

$$= \begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix}$$

iv) Enlargement by a scale factor of 5 about (0,0)

7)
i)

$$\underline{S} = \begin{pmatrix} 5 & 0 & 1 & 6 & 2 \\ 7 & 8 & 4 & 3 & 9 \end{pmatrix}$$

$$\underline{D}\,\underline{S} = \begin{pmatrix} 12 & 8 & 5 & 9 & 11 \end{pmatrix}$$

Will need $\underline{D} = \begin{pmatrix} 1 & 1 \end{pmatrix}$

ii)

$$\underline{S}\,\underline{N} = \begin{pmatrix} 14 \\ 31 \end{pmatrix}$$

Will need

$$\underline{N} = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$$

iii) Let $\underline{C} = (28, 21)$

Then $\underline{C}(\underline{S}\,\underline{N}) = (28 \times 14 + 31 \times 21)$

$= (1043)$

Total cost

$\underline{C}\,\underline{S}\,\underline{N} = (£10.43)$

8) i)

$$\begin{array}{cc} & \text{To} \\ & \begin{array}{cccc} A & B & C & D \end{array} \\ \underline{M} \quad \text{From} \begin{array}{c} A \\ B \\ C \\ D \end{array} & \begin{pmatrix} 1 & 1 & 2 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 \end{pmatrix} \end{array}$$

ii)

$$\begin{pmatrix} 1 & 1 & 2 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 1 & 2 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 4 & 3 & 3 & 4 \\ 2 & 2 & 2 & 2 \\ 2 & 1 & 5 & 0 \\ 1 & 1 & 0 & 2 \end{pmatrix}$$

Two stage routes between towns $\underline{M}^2$

iii) Three stage routes between towns $\underline{M}^3$

9) i) $\underline{A}\,\underline{s}_0 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} a \\ b \\ c \end{pmatrix}$

$= \begin{pmatrix} b \\ a \\ c \end{pmatrix}$

ii) $\underline{s}_2 = \begin{pmatrix} b \\ c \\ a \end{pmatrix}$

$\therefore\ \underline{B}\begin{pmatrix} b \\ a \\ c \end{pmatrix}$ has to give $\begin{pmatrix} b \\ c \\ a \end{pmatrix}$

$\therefore\ \underline{B} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$

iii) $\underline{M} = \underline{B}\,\underline{A} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

$\underline{M} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$

$\underline{M}\,\underline{s}_0 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}\begin{pmatrix} a \\ b \\ c \end{pmatrix}$

$= \begin{pmatrix} b \\ c \\ a \end{pmatrix} = \underline{s}_2$

iv) $\underline{M}^2 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$

$= \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$

At Stage 4

$\underline{s}_4 = \begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}\begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} c \\ a \\ b \end{pmatrix}$

v) $\underline{M}^3 = \underline{M}\,\underline{M}^2$

$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$

$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

At stage 6 strands are back in original order

//