

$$1) \text{ i) } (8+6j) + (6+4j) = 14 + 10j$$

$$\text{ii) } (9-3j) + (-4+5j) = 5 + 2j$$

$$\text{iii) } (2+7j) - (5+3j) = -3 + 4j$$

$$\text{iv) } (5-j) - (6-2j) = -1 + j$$

$$\text{v) } 3(4+6j) + 9(1-2j) \\ = 12 + 18j + 9 - 18j = 21$$

$$\text{vi) } 3j(7-4j) = 21j - 12j^2 \\ = 12 + 21j$$

$$\text{vii) } (9+2j)(1+3j) = 9 + 2j + 27j + 6j^2 \\ = 9 + 29j - 6 \\ = 3 + 29j$$

$$\text{viii) } (4-j)(3+2j) = 12 - 3j + 8j - 2j^2 \\ = 12 + 5j + 2 \\ = 14 + 5j$$

$$\text{ix) } (7+3j)^2 = (7+3j)(7+3j) \\ = 49 + 21j + 21j + 9j^2 \\ = 49 + 42j - 9 \\ = 40 + 42j$$

$$\text{x) } (8+6j)(8-6j) \\ = 64 + 48j - 48j - 36j^2 \\ = 64 + 36 = 100$$

$$\text{xi) } (1+2j)(3-4j)(5+6j) \\ = [3 + 6j - 4j - 8j^2](5+6j)$$

$$= [3 + 2j + 8](5+6j)$$

$$= (11+2j)(5+6j)$$

$$= 55 + 10j + 66j + 12j^2$$

$$= 55 + 76j - 12$$

$$= 43 + 76j$$

$$\text{xii) } (3+2j)^3$$

$$= (3+2j)(3+2j)(3+2j)$$

$$= [9 + 6j + 6j + 4j^2](3+2j)$$

$$= [9 + 12j - 4](3+2j)$$

$$= (5 + 12j)(3+2j)$$

$$= 15 + 36j + 10j + 24j^2$$

$$= 15 + 46j - 24$$

$$= -9 + 46j$$

2)

$$z^2 + 2z + 2 = 0$$

$$z = \frac{-2 \pm \sqrt{4-8}}{2}$$

$$z = \frac{-2 \pm 2j}{2}$$

$$z = -1+j \text{ and } z = -1-j$$

$$\begin{aligned}
 & (-1+j)^2 + 2(-1+j) + 2 \\
 &= (-1+j)(-1+j) - 2 + 2j + 2 \\
 &= 1 - j - j + j^2 + 2j \\
 &= 1 - 1 = 0 \quad \checkmark
 \end{aligned}$$

Check $(-1-j)$ is a root

$$\begin{aligned}
 & (-1-j)^2 + 2(-1-j) + 2 \\
 &= (-1-j)(-1-j) - 2 - 2j + 2 \\
 &= 1 + j + j + j^2 - 2j \\
 &= 1 - 1 = 0 \quad \checkmark
 \end{aligned}$$

2ii)

$$\begin{aligned}
 & z^2 - 2z + 5 = 0 \\
 & z = \frac{2 \pm \sqrt{4 - 20}}{2} \\
 & z = \frac{2 \pm 4j}{2}
 \end{aligned}$$

$z = 1 + 2j$ and $z = 1 - 2j$

Check $(1+2j)$ is a root

$$\begin{aligned}
 & (1+2j)^2 - 2(1+2j) + 5 \\
 &= (1+2j)(1+2j) - 2 - 4j + 5 \\
 &= 1 + 2j + 2j + 4j^2 - 4j + 3 \\
 &= 4 - 4 = 0 \quad \checkmark
 \end{aligned}$$

Check $z = 1 - 2j$ is a root

$$(1-2j)^2 - 2(1-2j) + 5$$

$$\begin{aligned}
 &= (1-2j)^2 - 2 + 4j + 5 \\
 &= 1 - 4j + 4j^2 + 3 + 4j \\
 &= 4 - 4 = 0 \quad \checkmark
 \end{aligned}$$

2iii)

$$\begin{aligned}
 & z^2 - 4z + 13 = 0 \\
 & z = \frac{4 \pm \sqrt{16 - 52}}{2} \\
 & z = \frac{4 \pm 6j}{2}
 \end{aligned}$$

$z = 2 + 3j$ and $z = 2 - 3j$

Check $z = 2 + 3j$ is a root

$$\begin{aligned}
 & (2+3j)^2 - 4(2+3j) + 13 \\
 &= (2+3j)(2+3j) - 8 - 12j + 13 \\
 &= 4 + 6j + 6j + 9j^2 + 5 - 12j \\
 &= 9 - 9 = 0 \quad \checkmark
 \end{aligned}$$

Check $z = 2 - 3j$ is a root

$$\begin{aligned}
 & (2-3j)^2 - 4(2-3j) + 13 \\
 &= (2-3j)(2-3j) - 8 + 12j + 13 \\
 &= 4 - 6j - 6j + 9j^2 + 5 + 12j \\
 &= 9 - 9 = 0 \quad \checkmark
 \end{aligned}$$

2iv)

$$\begin{aligned}
 & z^2 + 6z + 34 = 0 \\
 & z = \frac{-6 \pm \sqrt{36 - 136}}{2}
 \end{aligned}$$

$$2iv\text{cont}) \quad z = \frac{-6 \pm 10j}{2}$$

$$z = -3 + 5j \text{ and } z = -3 - 5j$$

Check $-3 + 5j$ is a root

$$\begin{aligned} & (-3 + 5j)^2 + 6(-3 + 5j) + 34 \\ &= (-3 + 5j)(-3 + 5j) - 18 + 30j + 34 \\ &= 9 - 15j - 15j + 25j^2 + 16 + 30j \\ &= 25 - 25 = 0 \quad \checkmark \end{aligned}$$

Check $z = -3 - 5j$ is a root

$$\begin{aligned} & (-3 - 5j)^2 + 6(-3 - 5j) + 34 \\ &= (-3 - 5j)(-3 - 5j) - 18 - 30j + 34 \\ &= 9 + 15j + 15j + 25j^2 + 16 - 30j \\ &= 25 - 25 = 0 \quad \checkmark \end{aligned}$$

2v)

$$4z^2 - 4z + 17 = 0$$

$$z = \frac{4 \pm \sqrt{16 - 272}}{8}$$

$$z = \frac{4 \pm 16j}{8}$$

$$z = \frac{1}{2} + 2j \text{ and } z = \frac{1}{2} - 2j$$

Check $(\frac{1}{2} + 2j)$ is a root

$$\begin{aligned} & 4\left(\frac{1}{2} + 2j\right)^2 - 4\left(\frac{1}{2} + 2j\right) + 17 \\ &= 4\left(\frac{1}{4} + 2j + 4j^2\right) - 2 - 8j + 17 \\ &= 1 + 8j - 16 + 15 - 8j = 0 \quad \checkmark \end{aligned}$$

Check $z = (\frac{1}{2} - 2j)$ is a root

$$\begin{aligned} & 4\left(\frac{1}{2} - 2j\right)^2 - 4\left(\frac{1}{2} - 2j\right) + 17 \\ &= 4\left(\frac{1}{4} - 2j + 4j^2\right) - 2 + 8j + 17 \\ &= 1 - 8j - 16 + 15 + 8j = 0 \quad \checkmark \end{aligned}$$

2vi)

$$z^2 + 4z + 6 = 0$$

$$z = \frac{-4 \pm \sqrt{16 - 24}}{2}$$

$$z = \frac{-4 \pm 2\sqrt{2}j}{2}$$

$$z = -2 + \sqrt{2}j \text{ and } z = -2 - \sqrt{2}j$$

Check $(-2 + \sqrt{2}j)$ is a root

$$\begin{aligned} & (-2 + \sqrt{2}j)^2 + 4(-2 + \sqrt{2}j) + 6 \\ &= (4 - 4\sqrt{2}j + 2j^2) - 8 + 4\sqrt{2}j + 6 \\ &= 4 - 2 - 8 + 6 = 0 \quad \checkmark \end{aligned}$$

Check $(-2 - \sqrt{2}j)$ is a root

$$\begin{aligned} & (-2 - \sqrt{2}j)^2 + 4(-2 - \sqrt{2}j) + 6 \\ &= (4 + 4\sqrt{2}j + 2j^2) - 8 - 4\sqrt{2}j + 6 \\ &= 4 - 2 - 8 + 6 = 0 \quad \checkmark \end{aligned}$$

$$3) \quad z = 2 + 3j \quad w = 6 - 4j$$

i) $\text{Re}(z) = 2$

ii) $\text{Im}(w) = -4$

iii) $z^* = 2 - 3j$

$$3) \text{iv) } w^* = 6 + 4j$$

$$\begin{aligned} \text{v) } z^* + w^* &= 2 - 3j + 6 + 4j \\ &= 8 + j \end{aligned}$$

$$\begin{aligned} \text{vi) } z^* - w^* &= 2 - 3j - (6 + 4j) \\ &= 2 - 3j - 6 - 4j \\ &= -4 - 7j \end{aligned}$$

$$\begin{aligned} \text{vii) } \operatorname{Im}(z + z^*) &= \operatorname{Im}(2 + 3j + 2 - 3j) \\ &= \operatorname{Im}(4) = 0 \end{aligned}$$

$$\begin{aligned} \text{viii) } \operatorname{Re}(w - w^*) &= \operatorname{Re}(6 - 4j - (6 + 4j)) \\ &= \operatorname{Re}(6 - 4j - 6 - 4j) \\ &= \operatorname{Re}(-8j) = 0 \end{aligned}$$

$$\begin{aligned} \text{ix) } zz^* - ww^* &= (2 + 3j)(2 - 3j) - (6 - 4j)(6 + 4j) \\ &= [4 + 9] - [36 + 16] \\ &= 13 - 52 = -39 \end{aligned}$$

$$\begin{aligned} \text{x) } (z^3)^* &= ((2 + 3j)^3)^* \\ &= [(2 + 3j)(2 + 3j)(2 + 3j)]^* \\ &= [(4 + 12j + 9j^2)(2 + 3j)]^* \end{aligned}$$

$$\begin{aligned} &= [(-5 + 12j)(2 + 3j)]^* \\ &= [-10 + 24j - 15j + 36j^2]^* \\ &= [-10 + 9j - 36]^* \\ &= [-46 + 9j]^* \\ &= -46 - 9j \end{aligned}$$

$$\begin{aligned} \text{xi) } (z^*)^3 &= (2 - 3j)^3 \\ &= (2 - 3j)(2 - 3j)(2 - 3j) \\ &= (4 - 12j + 9j^2)(2 - 3j) \\ &= (-5 - 12j)(2 - 3j) \\ &= -10 - 24j + 15j + 36j^2 \\ &= -10 - 9j - 36 \\ &= -46 - 9j \end{aligned}$$

$$\begin{aligned} \text{xii) } zw^* - z^*w &= (2 + 3j)(6 + 4j) - (2 - 3j)(6 - 4j) \\ &= [12 + 18j + 8j + 12j^2] - [12 - 18j - 8j + 12j^2] \\ &= 26j - (-26j) \\ &= 52j \end{aligned}$$

$$4) \text{ Let } z = x + yj$$

$$\text{Then } z^* = x - yj$$

$$(z^*)^* = x - (-yj)$$

$$= x + yj$$

$$= z$$

$$5) \text{ Let } z_1 = x_1 + y_1j$$

$$\text{Let } z_2 = x_2 + y_2j$$

$$\text{Then } (z_1 + z_2)^* = (x_1 + x_2 + (y_1 + y_2)j)^*$$

$$= x_1 + x_2 - (y_1 + y_2)j$$

$$= x_1 - y_1j + x_2 - y_2j$$

$$= z_1^* + z_2^*$$

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