MEI FP1 COMPLEX NUMBERS EXERCISE 2B

1) i) $\frac{1}{3+j} = \frac{1}{(3+j)}\frac{(3-j)}{(3-j)} = \frac{3-j}{10}$

$= \frac{3}{10} - \frac{3}{10}j$

1) ii) $\frac{1}{6-j} = \frac{1}{(6-j)}\frac{(6+j)}{(6+j)}$

$= \frac{6+j}{36+1}$

$= \frac{6}{37} + \frac{1}{37}j$

1) iii) $\frac{5j}{6-2j} = \frac{5j}{(6-2j)}\frac{(6+2j)}{(6+2j)}$

$= \frac{30j-10}{36+4} = \frac{30j-10}{40}$

$= -\frac{1}{4} + \frac{3}{4}j$

1) iv) $\frac{7+5j}{6-2j} = \frac{(7+5j)}{(6-2j)}\frac{(6+2j)}{(6+2j)}$

$= \frac{42+30j+14j-10}{36+4}$

$= \frac{32+44j}{40}$

$= \frac{4}{5} + \frac{11}{10}j$

1) v) $\frac{3+2j}{1+j} = \frac{(3+2j)}{(1+j)}\frac{(1-j)}{(1-j)}$

$= \frac{3+2j-3j+2}{1+1}$

$= \frac{5-j}{2} = \frac{5}{2} - \frac{1}{2}j$

1) vi) $\frac{47-23j}{6+j} = \frac{(47-23j)}{(6+j)}\frac{(6-j)}{(6-j)}$

$= \frac{282-138j-47j-23}{36+1}$

$= \frac{259-185j}{37}$

$= 7 - 5j$

1) vii) $\frac{2-3j}{3+2j} = \frac{(2-3j)}{(3+2j)}\frac{(3-2j)}{(3-2j)}$

$= \frac{6-9j-4j-6}{9+4}$

$= \frac{-13j}{13} = -j$

1) viii) $\frac{5-3j}{4+3j} = \frac{(5-3j)}{(4+3j)}\frac{(4-3j)}{(4-3j)}$

$= \frac{20-12j-15j-9}{16+9}$

$= \frac{11-27j}{25} = \frac{11}{25} - \frac{27}{25}j$

1) ix) $\frac{6+j}{2-5j} = \frac{(6+j)}{(2-5j)}\frac{(2+5j)}{(2+5j)}$

$= \frac{12+2j+30j-5}{4+25}$

$= \frac{7+32j}{29} = \frac{7}{29} + \frac{32}{29}j$

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1) x) $\frac{12-8j}{(2+2j)^2} = \frac{12-8j}{(4+8j-4)}$

$= \frac{12-8j}{8j} = \frac{(12-8j)}{8j}\frac{(-8j)}{(-8j)}$

$= \frac{-96j-64}{64}$

$= -1 - \frac{3}{2}j$

2) ii) $(a+bj)^2 = 21+20j$

$a^2 + 2abj + b^2j^2 = 21+20j$

$a^2 - b^2 + 2abj = 21+20j$

$\Rightarrow \quad a^2 - b^2 = 21 \quad (1)$

and $\quad 2ab = 20 \quad (2)$

From (2) $\quad a = \frac{20}{2b} = \frac{10}{b}$

Subst in (1) $\quad \frac{10^2}{b^2} - b^2 = 21$

$100 - b^4 = 21b^2$

$b^4 + 21b^2 - 100 = 0$

$(b^2-4)(b^2+25) = 0$

$\Rightarrow b^2 = 4 \Rightarrow b = \pm 2$

or $b^2 = -25$ no real roots

If $b = +2$, $a = \frac{10}{2} = 5$

If $b = -2$, $a = -5$

Since $a > 0$ solution is

$a = 5, \quad b = 2$

2) ii) $(a+bj)^2 = -40 - 42j$

$a^2 - b^2 + 2abj = -40 - 42j$

$\Rightarrow \quad a^2 - b^2 = -40 \quad (1)$

and $\quad 2ab = -42 \quad (2)$

from (2) $\quad a = \frac{-42}{2b} = \frac{-21}{b}$

Subst in (1) $\quad \left(\frac{-21}{b}\right)^2 - b^2 = -40$

$\frac{441}{b^2} - b^2 = -40$

$441 - b^4 = -40b^2$

$b^4 - 40b^2 - 441 = 0$

$(b^2-49)(b^2+9) = 0$

$\Rightarrow b^2 = 49$ or $b^2 = -9$

$\Rightarrow b = \pm 7$ or no real roots

When $b = +7$ $\quad a = \frac{-21}{7} = -3$

When $b = -7$ $\quad a = \frac{-21}{-7} = +3$

Solution for $a > 0$

$a = 3, \quad b = -7$

2) iii) $(a+bj)^2 = -5 - 12j$

$\Rightarrow a^2 - b^2 = -5$ (1)

and $2ab = -12$ (2)

from (2) $a = \frac{-12}{2b} = -\frac{6}{b}$

Subst in (1)

$\left(\frac{-6}{b}\right)^2 - b^2 = -5$

$\frac{36}{b^2} - b^2 = -5$

$36 - b^4 = -5b^2$

$b^4 - 5b^2 - 36 = 0$

$(b^2 - 9)(b^2 + 4) = 0$

$\Rightarrow b^2 = 9$ or $b^2 = -4$

$b = \pm 3$ no real roots

When $b = +3$ $a = -\frac{6}{3} = -2$

When $b = -3$ $a = \frac{-6}{-3} = +2$

Solution for $a > 0$

$a = 2, \quad b = -3$

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2) iv) $(a+bj)^2 = -9 + 40j$

$a^2 - b^2 + 2abj = -9 + 40j$

$\Rightarrow a^2 - b^2 = -9$ (1)

and $2ab = 40$ (2)

From (2) $a = \frac{40}{2b} = \frac{20}{b}$

Subst in (1)

$\left(\frac{20}{b}\right)^2 - b^2 = -9$

$\frac{400}{b^2} - b^2 = -9$

$400 - b^4 = -9b^2$

$b^4 - 9b^2 - 400 = 0$

$(b^2 + 16)(b^2 - 25) = 0$

$\Rightarrow b^2 = 25$ or $b^2 = -16$

$b = \pm 5$ no real roots

When $b = 5$ $a = \frac{20}{5} = 4$

When $b = -5$ $a = \frac{20}{-5} = -4$

Solution for $a > 0$

$a = 4, \quad b = 5$

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2) v) $(a+bj)^2 = 1 - 1.875j$

$a^2 - b^2 + 2abj = 1 - \frac{15}{8}j$

$\Rightarrow a^2 - b^2 = 1$ (1)

$2ab = -\frac{15}{8}$ (2)

From (2) $a = -\frac{15}{16b}$

Subst in (1)

$\left(-\frac{15}{16b}\right)^2 - b^2 = 1$

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2) v) cont.

$$\frac{225}{256b^2} - b^2 = 1$$

$$225 - 256b^4 = 256b^2$$

$$256b^4 + 256b^2 - 225 = 0$$

From calculator

$b^2 = 0.5625$ or $b^2 = -1.562$

$b = \pm 0.75$ no real roots

when $b = 0.75$ $a = \frac{-15}{16 \times 0.75} = \frac{-15}{12}$

when $b = -0.75$ $a = +\frac{15}{12} = \frac{5}{4}$

Solution for $a > 0$

$$a = \frac{5}{4}, \quad b = -\frac{3}{4}$$

$$1 - 4b^4 = 0$$

$$4b^4 = 1$$

$$b^4 = \frac{1}{4}$$

$$\Rightarrow b^2 = \pm\frac{1}{2}$$

Either $b^2 = \frac{1}{2}$ or $b^2 = -\frac{1}{2}$

$b = \pm\frac{1}{\sqrt{2}}$ no real roots

when $b = \frac{1}{\sqrt{2}}$, $a = \frac{1}{2 \times \frac{1}{\sqrt{2}}} = \frac{1}{\sqrt{2}}$

when $b = -\frac{1}{\sqrt{2}}$ $a = -\frac{1}{\sqrt{2}}$

Solution for $a > 0$

$$a = \frac{1}{\sqrt{2}}, \quad b = \frac{1}{\sqrt{2}}$$

2) vi)

$$(a+bj)^2 = j$$

$$a^2 - b^2 + 2abj = j$$

$$\Rightarrow a^2 - b^2 = 0 \quad (1)$$

and $2ab = 1$ (2)

From (2)

$$a = \frac{1}{2b}$$

Subst in (1)

$$\left(\frac{1}{2b}\right)^2 - b^2 = 0$$

$$\frac{1}{4b^2} - b^2 = 0$$

3)

$$\frac{a}{3+j} + \frac{b}{1+2j} = 1 - j$$

$$(1+2j)a + b(3+j) = (1-j)(1+2j)(3+j)$$

$$a + 2aj + 3b + bj = [1 - j + 2j + 2](3+j)$$

$$a + 3b + (2a+b)j = (3+j)(3+j)$$

$$a + 3b + (2a+b)j = 9 + 6j - 1$$

$$(a+3b) + (2a+b)j = 8 + 6j$$

$$\Rightarrow a + 3b = 8 \quad (1)$$

and $2a + b = 6$ (2)

From (1) $a = 8 - 3b$

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3 cont) Subst in (2)

$2(8-3b)+b=6$

$16-6b+b=6$

$16-5b=6$

$10=5b$

$b=2$

$\therefore a=8-3\times 2=2$

Solution $a=2,\ b=2$

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4) i) $(1+j)z=3+j$

$z=\frac{3+j}{1+j}$

$z=\frac{(3+j)(1-j)}{(1+j)(1-j)}$

$z=\frac{3+j-3j+1}{1+1}$

$z=\frac{4-2j}{2}$

$z=2-j$

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4) ii) $(3-4j)(z-1)=10-5j$

$z-1=\frac{10-5j}{3-4j}$

$z-1=\frac{(10-5j)(3+4j)}{(3-4j)(3+4j)}$

$z-1=\frac{30-15j+40j+20}{9+16}$

$z-1=\frac{50+25j}{25}$

$z-1=2+j$

$z=2+j+1$

$z=3+j$

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4) iii) $(2+j)(z-7+3j)=15-10j$

$z-7+3j=\frac{15-10j}{2+j}$

$z-7+3j=\frac{(15-10j)}{(2+j)}\frac{(2-j)}{(2-j)}$

$z-7+3j=\frac{30-20j-15j-10}{4+1}$

$z-7+3j=\frac{20-35j}{5}$

$z-7+3j=4-7j$

$z=4-7j+7-3j$

$z=11-10j$

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4) iv) $(3+5j)(z+2-5j)=6+3j$

$z+2-5j=\frac{6+3j}{3+5j}$

$z+2-5j=\frac{(6+3j)(3-5j)}{(3+5j)(3-5j)}$

4 iv) cont

$$z + 2 - 5j = \frac{18 + 9j - 30j + 15}{9 + 25}$$

$$z + 2 - 5j = \frac{33 - 21j}{34}$$

$$34z + 68 - 170j = 33 - 21j$$

$$34z = 33 - 68 - 21j + 170j$$

$$34z = -35 + 149j$$

$$z = \frac{-35}{34} + \frac{149}{34}j$$

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5)

$$z^2 = 2z^*$$

$$(a + bj)^2 = 2(a - bj)$$

$$a^2 - b^2 + 2abj = 2a - 2bj$$

$$\Rightarrow \quad a^2 - b^2 = 2a \qquad (1)$$

and $\quad 2ab = -2b \qquad (2)$

From (2)

$$2ab + 2b = 0$$

$$2b(a + 1) = 0$$

$$\Rightarrow \quad a = -1 \text{ or } b = 0$$

Subst $a = -1$ in (1)

$$(-1)^2 - b^2 = -2$$

$$1 - b^2 = -2$$

$$3 = b^2$$

$$b = \pm\sqrt{3}$$

Giving $\quad z = -1 + \sqrt{3}$

and $\quad z = -1 - \sqrt{3}$

When $b = 0$

$$a^2 - 0^2 = 2a$$

$$a^2 - 2a = 0$$

$$a(a - 2) = 0$$

$$\Rightarrow \quad a = 0 \text{ or } a = 2$$

Giving $\quad z = 0$

and $\quad z = 2$

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6)

$$z = x + yj$$

$$\frac{1}{z} + \frac{1}{z^*} = \frac{1}{x + yj} + \frac{1}{x - yj}$$

$$= \frac{1}{(x + yj)}\frac{(x - yj)}{(x - yj)} + \frac{1}{(x - yj)}\frac{(x + yj)}{(x + yj)}$$

$$= \frac{(x - yj) + (x + yj)}{x^2 + y^2}$$

$$= \frac{2x}{x^2 + y^2}$$

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7)

i) Let $z = x + yj$

$$\frac{z + z^*}{2} = \frac{x + yj + x - yj}{2}$$

7 i) cont $\quad \dfrac{z+z^*}{2} = \dfrac{2x}{2} = x = \mathrm{Re}(z)$

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7 ii) $\quad \dfrac{z-z^*}{2j} = \dfrac{x+yj-(x-yj)}{2j}$

$$= \frac{x+yj-x+yj}{2j}$$

$$= \frac{2yj}{2j} = y = \mathrm{Im}(z)$$

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