1 i) $\int \cos^2 x \, dx$

$= \int \frac{1}{2}(1+\cos 2x)\, dx$

$= \frac{x}{2} + \frac{1}{4}\sin 2x + c$

ii) $\int \sin^2 3x \, dx$

$= \int \frac{1}{2}(1 - \cos 6x)\, dx$

$= \frac{x}{2} - \frac{1}{12}\sin 6x + c$

iii) $\int \sec^2 x \, dx$

$= \tan x + c$

iv) $\int \sin^3 x \, dx$

$= \int \sin x(\sin^2 x)\, dx$

$= \int \sin x(1 - \cos^2 x)\, dx$

$= \int \sin x - \sin x \cos^2 x \, dx$

$= -\cos x + \frac{1}{3}\cos^3 x + c$

v) $\int \sin^4 x \, dx$

$= \int \sin^2 x \sin^2 x \, dx$

$= \int \frac{1}{2}(1 - \cos 2x)\frac{1}{2}(1 - \cos 2x)\, dx$

$= \frac{1}{4}\int (1 - 2\cos 2x + \cos^2 2x)\, dx$

$= \frac{1}{4}\int \left(1 - 2\cos 2x + \frac{1}{2}(1+\cos 4x)\right) dx$

$= \frac{1}{4}\int \left(\frac{3}{2} - 2\cos 2x + \frac{1}{2}\cos 4x\right) dx$

$= \frac{1}{4}\left[\frac{3x}{2} - \sin 2x + \frac{1}{8}\sin 4x\right] + c$

$= \frac{3x}{8} - \frac{1}{4}\sin 2x + \frac{1}{32}\sin 4x + c$

vi) $\int \cos^5 x \, dx$

$= \int \cos x \cos^4 x \, dx$

$= \int \cos x(\cos^2 x)(\cos^2 x)\, dx$

$= \int \cos x(1 - \sin^2 x)(1 - \sin^2 x)\, dx$

$= \int \cos x(1 - 2\sin^2 x + \sin^4 x)\, dx$

1 vi) (cont)
$$=\int\left(\cos x - 2\cos x\sin^2 x + \cos x\sin^4 x\right)dx$$
$$= \sin x - \frac{2}{3}\sin^3 x + \frac{1}{5}\sin^5 x + c$$

vii)
$$\int \tan 2x \, dx$$
$$= \int \frac{\sin 2x}{\cos 2x}\, dx$$
$$= -\frac{1}{2}\int \frac{-2\sin 2x}{\cos 2x}\, dx$$
$$= -\frac{1}{2}\ln|\cos 2x| + c$$

viii)
$$\int \cot x \, dx$$
$$= \int \frac{\cos x}{\sin x}\, dx$$
$$= \ln|\sin x| + c$$

ix)
$$\int(\cot x + \tan x)\, dx$$
$$= \int\left(\frac{\cos x}{\sin x} + \frac{\sin x}{\cos x}\right) dx$$
$$= \ln(\sin x) - \ln(\cos x) + c$$
$$= \ln\left(\frac{\sin x}{\cos x}\right) + c$$
$$= \ln|\tan x| + c$$

2) i)
$$\int \sin x \cos^2 x \, dx$$
$$= -\frac{1}{3}\cos^3 x + c$$

ii)
$$\int \cos^2 3x \, dx = \int \frac{1}{2}(1+\cos 6x)\, dx$$
$$= \int\left(\frac{1}{2} + \frac{1}{2}\cos 6x\right) dx$$
$$= \frac{x}{2} + \frac{1}{12}\sin 6x + c$$

iii)
$$\int \sin 5x \cos 2x \, dx$$
$$\tfrac{1}{2}(\theta+\phi) = 5 \qquad \tfrac{1}{2}(\theta-\phi) = 2$$
$$\theta+\phi = 10 \qquad \theta-\phi = 4$$
$$2\theta = 14 \quad \Rightarrow \theta = 7$$
$$\Rightarrow \phi = 3$$
$$\int \frac{1}{2}(\sin 7x + \sin 3x)\, dx$$
$$= -\frac{1}{14}\cos 7x - \frac{1}{6}\cos 3x + c$$

iv)
$$\int (1+\sin x)^2\, dx$$
$$= \int\left(1 + 2\sin x + \sin^2 x\right) dx$$
$$= \int\left(1 + 2\sin x + \frac{1}{2}(1-\cos 2x)\right) dx$$
$$= \int\left(\frac{3}{2} + 2\sin x - \frac{1}{2}\cos 2x\right) dx$$

2iv) (cont)

$$= \frac{3x}{2} - 2\cos x - \frac{1}{4}\sin 2x + c$$

v)

$$\int (\sin x + \cos x)^2 \, dx$$

$$= \int \left(\sin^2 x + 2\sin x \cos x + \cos^2 x \right) dx$$

$$= \int (1 + \sin 2x) \, dx$$

$$= x - \frac{1}{2}\cos 2x + c$$

2vi)

$$\int \sec^2 x \tan x \, dx$$

$$= \frac{1}{2}\tan^2 x + c$$

3i)

$$\int_0^{\frac{\pi}{4}} \cos x \sin^3 x \, dx$$

$$= \left[\frac{1}{4}\sin^4 x \right]_0^{\frac{\pi}{4}}$$

$$= \frac{1}{4}\left(\left(\frac{1}{\sqrt{2}} \right)^4 - 0 \right)$$

$$= \frac{1}{16}$$

3ii)

$$\int_0^{\frac{\pi}{4}} \frac{\sec^2 x}{1 + \tan x} \, dx$$

$$= \left[\ln |1 + \tan x| \right]_0^{\frac{\pi}{4}}$$

$$= \ln 2 - \ln 1 = \ln 2$$

3iii)

$$\int_0^{\sqrt{\pi}} x \sin(x^2) \, dx$$

Let $u = x^2$

$$\frac{du}{dx} = 2x$$

$$du = 2x \, dx$$

$$\frac{1}{2} du = x \, dx$$

when $x = \sqrt{\pi}$, $u = \pi$

when $x = 0$, $u = 0$

$$\int_0^{\pi} \frac{1}{2} \sin u \, du$$

$$= \frac{1}{2} \left[-\cos u \right]_0^{\pi}$$

$$= \frac{1}{2} \left[1 - (-1) \right] = 1$$

3iv)

$$\int_0^{\frac{\pi}{4}} \tan^2 x \, dx$$

Let $u = \tan x$

$$\frac{du}{dx} = \sec^2 x$$

$$du = \sec^2 x \, dx$$

$$du = (1 + \tan^2 x) \, dx$$

$$du = (1 + u^2) \, dx$$

$$\frac{1}{1+u^2} du = dx$$

when $x = \frac{\pi}{4}$, $u = 1$

when $x = 0$, $u = 0$

3iv) cont

$$\int_0^1 \frac{u^2}{1+u^2}\,du$$

$$u^2+1 \overline{\big) u^2+0}^{\;1}$$
$$\frac{u^2+1}{-1}$$

$$\int_0^1 1-\frac{1}{1+u^2}\,du$$

$$= \left[u - \tan^{-1}u\right]_0^1$$

$$= \left(1-\frac{\pi}{4}\right)-(0-0)$$

$$= 1-\frac{\pi}{4}$$

3v)

$$\int_0^{\frac{\pi}{2}} \sin 6x \cos 4x\,dx$$

$\frac{1}{2}(\theta+\phi) = 6x \qquad \frac{1}{2}(\theta-\phi) = 4x$

$\theta+\phi = 12x \qquad \theta-\phi = 8x$

$\Rightarrow 2\theta = 20x \Rightarrow \theta = 10x$
$\phi = 2x$

$$= \int_0^{\frac{\pi}{2}} \frac{1}{2}\left(\sin 10x + \sin 2x\right)dx$$

$$= \frac{1}{2}\left[-\frac{1}{10}\cos 10x - \frac{1}{2}\cos 2x\right]_0^{\frac{\pi}{2}}$$

$$= \frac{1}{2}\left[\left(-\frac{1}{10}\cos 5\pi - \frac{1}{2}\cos\pi\right) - \left(-\frac{1}{10}\cos 0 - \frac{1}{2}\cos 0\right)\right]$$

$$= \frac{1}{2}\left[\left(\frac{1}{10}+\frac{1}{2}\right)-\left(-\frac{1}{10}-\frac{1}{2}\right)\right]$$

$$= \frac{1}{2}\left[\frac{12}{10}\right] = \frac{6}{10} = \frac{3}{5}$$

3vi)

$$\int_0^{\frac{\pi}{4}} \sin 3x \sin 4x\,dx$$

$$\cos\theta - \cos\phi = -2\sin\left(\frac{\theta+\phi}{2}\right)\sin\left(\frac{\theta-\phi}{2}\right)$$

$\frac{1}{2}(\theta+\phi) = 4x \qquad \frac{1}{2}(\theta-\phi) = 3x$

$\theta+\phi = 8x \qquad \theta-\phi = 6x$

$\Rightarrow 2\theta = 14x \Rightarrow \theta = 7x$
$\phi = x$

$$\cos 7x - \cos x = -2\sin 4x \sin 3x$$

$$-\frac{1}{2}\int_0^{\frac{\pi}{4}} (\cos 7x - \cos x)\,dx$$

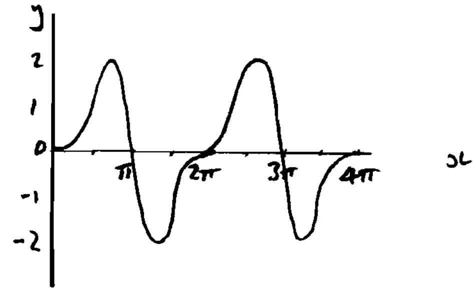
$$= -\frac{1}{2}\left[\frac{1}{7}\sin 7x - \sin x\right]_0^{\frac{\pi}{4}}$$

$$= -\frac{1}{2}\left[\left(\frac{1}{7}\sin\frac{7\pi}{4} - \sin\frac{\pi}{4}\right)-(0-0)\right]$$

$$= -\frac{1}{2}\left[\frac{1}{7}\left(-\frac{1}{\sqrt{2}}\right) - \frac{1}{\sqrt{2}}\right]$$

$$= \frac{1}{14\sqrt{2}} + \frac{1}{2\sqrt{2}} = \frac{8}{14\sqrt{2}} = \frac{8\sqrt{2}}{28} = \frac{2\sqrt{2}}{7}$$

4) i)



ii) $\int_0^{\pi} \sin x(\cos x - 1)^2 \, dx$

$= \int_0^{\pi} \sin x(\cos^2 x - 2\cos x + 1) \, dx$

$= \int_0^{\pi} \sin x \cos^2 x - 2\sin x \cos x + \sin x \, dx$

$= \left[-\frac{1}{3}\cos^3 x + \cos^2 x - \cos x\right]_0^{\pi}$

$= \left(+\frac{1}{3} + 1 + 1\right) - \left(-\frac{1}{3} + 1 - 1\right)$

$= \frac{8}{3}$

5) $\int \frac{1}{1 + \cos x} \, dx$

$\cos x = 2\cos^2\left(\frac{x}{2}\right) - 1$

$= \int \frac{1}{2\cos^2\left(\frac{x}{2}\right)} \, dx$

$= \frac{1}{2} \int \sec^2\left(\frac{x}{2}\right) dx$

$= \tan\left(\frac{x}{2}\right) + c$

6) i) $\frac{d}{dx} \cos 2x = -2\sin 2x$

$\therefore \int \sin 2x \, dx$

$= -\frac{1}{2} \int -2\sin 2x \, dx$

$= -\frac{1}{2}\cos 2x + A$

ii) $\sin 2x = 2\sin x \cos x$

$\int \sin 2x \, dx = \int 2\sin x \cos x \, dx$

Let $u = \sin x$

$\frac{du}{dx} = \cos x$

$du = \cos x \, dx$

Integral becomes $\int 2u \, du$

$= u^2 + B$

$= \sin^2 x + B$

iii) $\int \sin 2x \, dx = \int 2\sin x \cos x \, dx$

Let $u = \cos x$

$\frac{du}{dx} = -\sin x$

$du = -\sin x \, dx$

Integral becomes $-\int 2u \, du = -u^2 + c$

$= -\cos^2 x + c$

6iv) $\sin^2 x = -\cos^2 x + 1$

$\sin^2 x = -\frac{1}{2}\cos 2x + \frac{1}{2}$

The three answers differ by a constant which is accounted for within the constants of integration A, B, C.

7)

Omitted constant of integration

$\int \tan x \, dx = \int \frac{\sin x}{\cos x} dx$

$= \int \sin x \sec x \, dx$

Integration by parts

Let $u = \sec x$

$u = \cos^{-1} x$

$\frac{du}{dx} = -1 \cos x^{-2}(-\sin x)$

$= \frac{\sin x}{\cos^2 x}$

$= \tan x \sec x$

Let $\frac{dv}{dx} = \sin x$

$v = -\cos x$

$\int u \frac{dv}{dx} = uv - \int v \frac{du}{dx}$

$\int \tan x \, dx = \int \sin x \sec x \, dx$

$= -\cos x \sec x + \int \cos x \sec x \tan x \, dx$

$= -1 + \int \tan x \, dx + C$