1) $1 \quad |1| = 1 \quad \arg(1) = 0$

2) $-2 \quad |-2| = 2 \quad \arg(-2) = \pi$

3) $3j \quad |3j| = 3 \quad \arg(3j) = \frac{\pi}{2}$

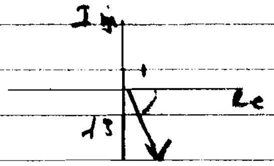
4) $-4j \quad |-4j| = 4 \quad \arg(-4j) = -\frac{\pi}{2}$

5) $1+j \quad |1+j| = \sqrt{2} \quad \arg(1+j) = \frac{\pi}{4}$

6) $-5-5j \quad |-5-5j| = 5\sqrt{2}$

$\arg(-5-5j) = -\frac{3\pi}{4}$

7) $1-\sqrt{3}j \quad |1-\sqrt{3}j| = 2$



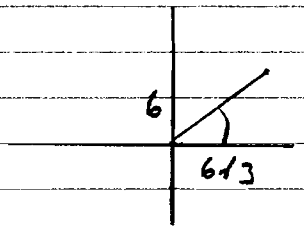
$\arg(1-\sqrt{3}j) = -\tan^{-1}\left(\frac{\sqrt{3}}{1}\right)$

$= -\frac{\pi}{3}$

8) $6\sqrt{3}+6j$

$|6\sqrt{3}+6j| = \sqrt{108+36}$

$= 12$

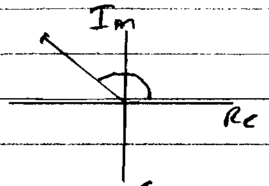


$\arg(6\sqrt{3}+6j) = \tan^{-1}\left(\frac{6}{6\sqrt{3}}\right)$

$= \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$

9) $-\sqrt{18}+\sqrt{18}j$

$|-\sqrt{18}+\sqrt{18}j| = \sqrt{18+18} = 6$



$\arg(-\sqrt{18}+\sqrt{18}j) = \frac{3\pi}{4}$

10) $8\left(\cos\frac{\pi}{5} + j\sin\frac{\pi}{5}\right)$

$\left|8\left(\cos\frac{\pi}{5} + j\sin\frac{\pi}{5}\right)\right| = 8$

$\arg\left(8\left(\cos\frac{\pi}{5} + j\sin\frac{\pi}{5}\right)\right) = \frac{\pi}{5}$

11) $\frac{\cos 2.3 + j\sin 2.3}{4}$

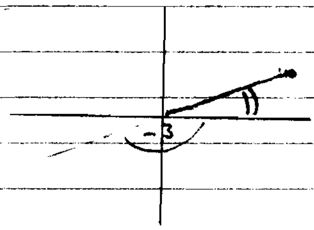
$\left|\frac{\cos 2.3 + j\sin 2.3}{4}\right| = \frac{1}{4}$

$\arg\left(\frac{\cos 2.3 + j\sin 2.3}{4}\right) = 2.3 \text{ rads}$

12) $-3(\cos(-3) + j\sin(-3))$

$|-3(\cos(-3) + j\sin(-3))| = 3$

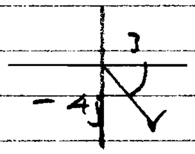
$\arg(-3(\cos(-3) + j\sin(-3))) = \pi - 3$



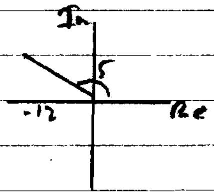
13) $3-4j \quad |3-4j| = 5$

$\arg(3-4j) = -\tan^{-1}\frac{4}{3}$

$= -0.927 \text{ rad}$



14) $-12+5j \quad |-12+5j| = 13$



$\arg(-12+5j) = \pi - \tan^{-1}\frac{5}{12}$

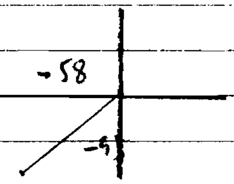
$= 2.747 \text{ rads}$

15) $4+7j \quad |4+7j| = \sqrt{65} = 8.062$

$\arg(4+7j) = \tan^{-1}\frac{7}{4}$

$= 1.052 \text{ rads}$

16) $-58-93j \quad |-58-93j| = 109.604$



$\arg(-58-93j)$

$= -\pi + \tan^{-1}\frac{93}{58}$

$= -2.128 \text{ rads}$

17) $\arg(5+2j) = \alpha$

i) $\arg(-5-2j) = \alpha - \pi$

ii) $\arg(5-2j) = -\alpha$

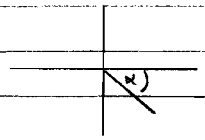
iii) $\arg(-5+2j) = \pi - \alpha$

iv) $\arg(2+5j) = \frac{\pi}{2} - \alpha$

v) $\arg(-2+5j) = \pi - \left(\frac{\pi}{2} - \alpha\right)$

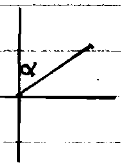
$= \frac{\pi}{2} + \alpha$

18) $\cos\alpha - j\sin\alpha$



$= \cos(-\alpha) + j\sin(-\alpha)$

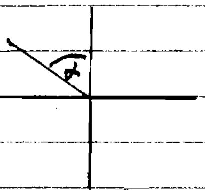
19) $3(\sin\alpha + j\cos\alpha)$



$= 3\left(\cos\left(\frac{\pi}{2}-\alpha\right) + j\sin\left(\frac{\pi}{2}-\alpha\right)\right)$

20) $j(\cos\alpha + j\sin\alpha)$

$= -\sin\alpha + j\cos\alpha$

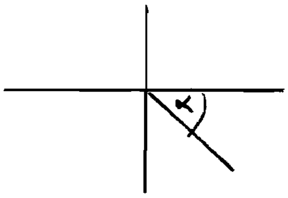


$= \cos\left(\frac{\pi}{2}+\alpha\right) + j\left(\sin\frac{\pi}{2}+\alpha\right)$

21)
$$\frac{10}{\cos\alpha + j\sin\alpha}$$

$$= \frac{10}{(\cos\alpha + j\sin\alpha)} \times \frac{(\cos\alpha - j\sin\alpha)}{(\cos\alpha - j\sin\alpha)}$$

$$= \frac{10(\cos\alpha - j\sin\alpha)}{1}$$



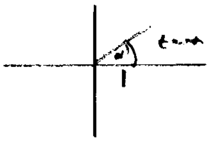
$$= \underline{10\big(\cos(-\alpha) + j\sin(-\alpha)\big)}$$

22)
$$1 + j\tan\alpha$$

$$|1 + j\tan\alpha| = \sqrt{1^2 + \tan^2\alpha}$$

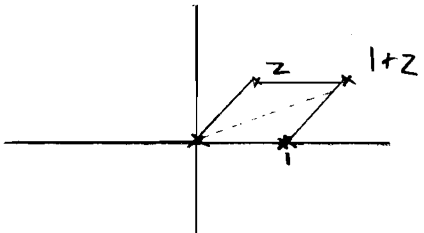
$$= \sqrt{\sec^2\alpha} = \sec\alpha$$

$$\arg(1 + j\tan\alpha) = \alpha$$



$$= \underline{\sec\alpha(\cos\alpha + j\sin\alpha)}$$

23)
i)



Rhombus

$$|1 + \cos\theta + j\sin\theta| =$$

$$\sqrt{(1+\cos\theta)^2 + \sin^2\theta}$$

$$= \sqrt{1 + 2\cos\theta + \cos^2\theta + \sin^2\theta}$$

$$= \sqrt{2 + 2\cos\theta} = \sqrt{2(1+\cos\theta)}$$

$$= \sqrt{2 \times 2\cos^2\tfrac{\theta}{2}} = \sqrt{4\cos^2\tfrac{\theta}{2}}$$

$$= \underline{2\cos\tfrac{\theta}{2}}$$

$$\arg(1 + \cos\theta + j\sin\theta) = \frac{\theta}{2}$$

(by symmetry)

23 ii)

$$1 + \cos\theta + j\sin\theta$$

$$= 2\cos^2\tfrac{\theta}{2} + j\,2\sin\tfrac{\theta}{2}\cos\tfrac{\theta}{2}$$

$$= 2\cos\tfrac{\theta}{2}\left(\cos\tfrac{\theta}{2} + j\sin\tfrac{\theta}{2}\right)$$

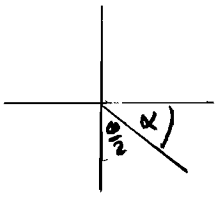
23 iii)

$$1 - \cos\theta - j\sin\theta$$

$$1 - \left(1 - 2\sin^2\tfrac{\theta}{2}\right) - j\,2\sin\tfrac{\theta}{2}\cos\tfrac{\theta}{2}$$

$$= 2\sin^2\tfrac{\theta}{2} - 2j\sin\tfrac{\theta}{2}\cos\tfrac{\theta}{2}$$

$$= 2\sin\tfrac{\theta}{2}\left(\sin\tfrac{\theta}{2} - j\cos\tfrac{\theta}{2}\right)$$



$$\alpha = \frac{\pi}{2} - \frac{\theta}{2}$$

$$-\alpha = \frac{\theta}{2} - \frac{\pi}{2}$$

Modulus $= 2\sin\frac{\theta}{2}$

arg $= \frac{\theta}{2} - \frac{\pi}{2}$

24) $\alpha = -1 + 2j$

i) $\alpha^2 = (-1+2j)(-1+2j)$

$= 1 - 4j - 4$

$\underline{\alpha^2 = -3 - 4j}$

$\alpha^3 = (-3-4j)(-1+2j)$

$= 3 + 4j - 6j + 8$

$\underline{\alpha^3 = 11 - 2j}$

$z^3 + 7z^2 + 15z + 25 = 0$

$11 - 2j + 7(-3-4j)$

$+ 15(-1+2j) + 25$

$= 11 - 2j - 21 - 28j$

$-15 + 30j + 25$

$= 11 - 21 - 15 + 25 - 2j - 28j + 30j$

$= 0$

$\therefore (-1+2j)$ is a root

ii)

$-1-2j$ is a root

$(z-(-1+2j))(z-(-1-2j))$

$= ((z+1) - 2j)((z+1) + 2j)$

$= (z+1)^2 - (2j)^2$

$= z^2 + 2z + 1 + 4$

$= z^2 + 2z + 5$

$$\begin{array}{r|l} & z + 5 \\ z^2+2z+5 & z^3 + 7z^2 + 15z + 25 \\ & \underline{z^3 + 2z^2 + 5z} \\ & \quad 5z^2 + 10z + 25 \\ & \quad \underline{5z^2 + 10z + 25} \end{array}$$

$z^3 + 7z^2 + 15z + 25$

$= (z^2 + 2z + 5)(z + 5)$

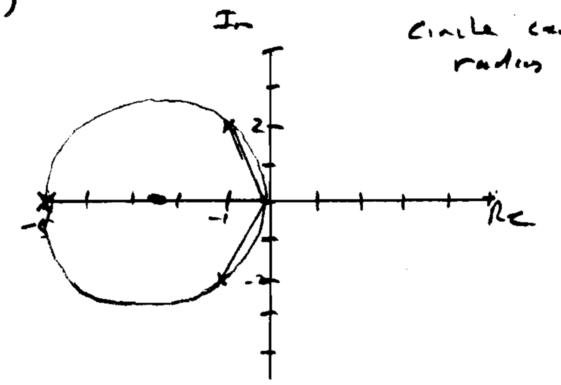
Roots are

$z = -1 + 2j$

$z = -1 - 2j$

$z = -5$

iii)



Circle centre $-\frac{5}{2}$
radius $\frac{5}{2}$

$|-5| = 5$, $\arg(-5) = \pi$

$|-1+2j| = \sqrt{5}$

$\arg(-1+2j) = \pi - \tan^{-1} 2$

$= 2.034$ rads

$|-1-2j| = \sqrt{5}$

$\arg(-1-2j) = -2.034$ rads

iv) $\left|-5 + \frac{5}{2}\right| = \left|-\frac{5}{2}\right| = \frac{5}{2}$

$\left|-1 + 2j + \frac{5}{2}\right| = \left|\frac{3}{2} + 2j\right|$

$= \sqrt{\left(\frac{3}{2}\right)^2 + 2^2} = \sqrt{\frac{9}{4} + \frac{16}{4}} = \sqrt{\frac{25}{4}} = \frac{5}{2}$

24 iv cont) $|-1-2j+5/2| = |\frac{3}{2}-2j|$

$= \sqrt{(\frac{3}{2})^2+(-2)^2}$

$= \sqrt{\frac{9}{4}+4} = \sqrt{\frac{9}{4}+\frac{16}{4}}$

$= \sqrt{\frac{25}{4}} = 5/2$

$\therefore$ all 3 roots on locus

Locus of $|z+\frac{5}{2}| = \frac{5}{2}$

is locus of $|z-(-\frac{5}{2})| = \frac{5}{2}$

Circle centre $-\frac{5}{2}$ radius $\frac{5}{2}$