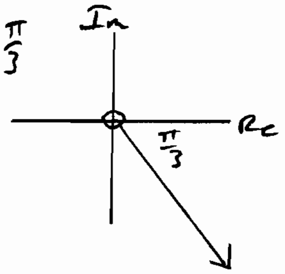
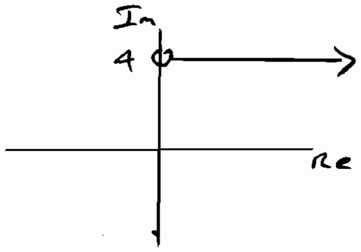
1) $\arg z = -\frac{\pi}{3}$

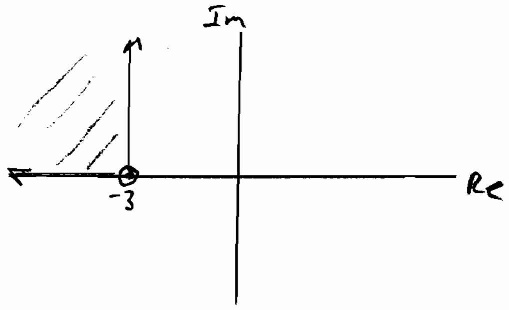



2) $\arg(z - 4j) = 0$



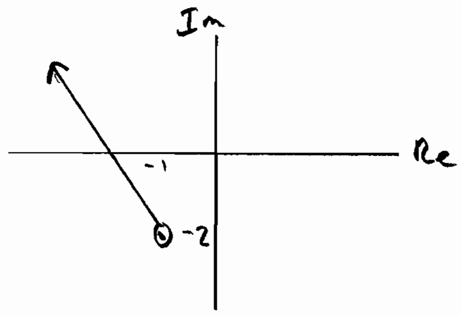

3) $\arg(z+3) \geq \frac{\pi}{2}$

$\arg(z-(-3)) \geq \frac{\pi}{2}$



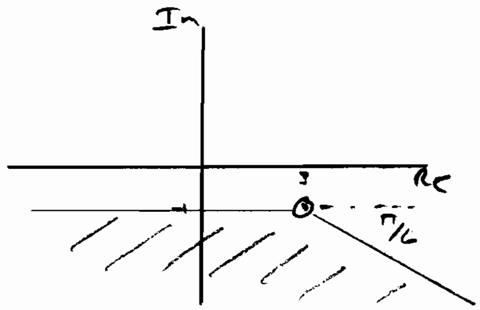

4) $\arg(z+1+2j) = \frac{3\pi}{4}$

$\arg(z-(-1-2j)) = \frac{3\pi}{4}$



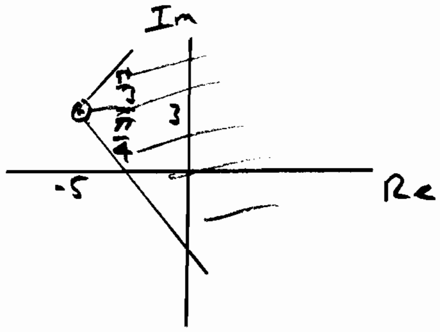

5) $\arg(z-3+j) \leq -\frac{\pi}{6}$

$\arg(z-(3-j)) \leq -\frac{\pi}{6}$

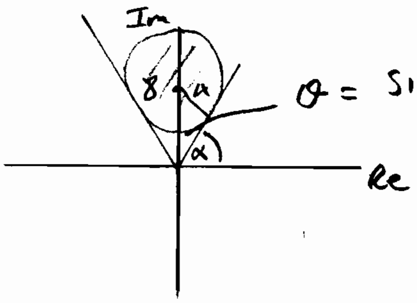



6) $-\frac{\pi}{4} \leq \arg(z+5-3j) \leq \frac{\pi}{3}$

$-\frac{\pi}{4} \leq \arg(z-(-5+3j)) \leq \frac{\pi}{3}$




7) $|z-8j| \leq 4$




$\theta = \sin^{-1}\frac{4}{8} = \frac{\pi}{6}$

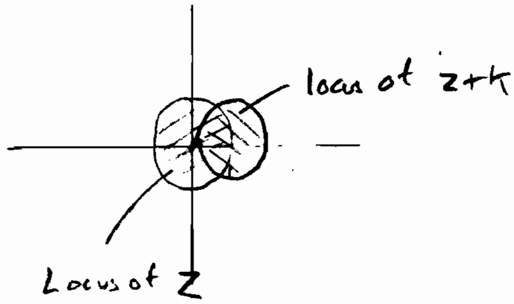
$\alpha = \frac{\pi}{2} - \frac{\pi}{6} = \frac{\pi}{3}$

Min $\arg z = \frac{\pi}{3}$

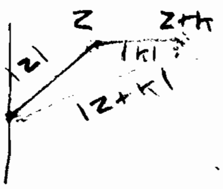
Max $\arg z = \frac{2\pi}{3}$

8) $|z| \le k$

Prove $0 \le |z+k| \le 2k$




$$|z+k| \le |z| + |k|$$




$$|z+k| \le |k| + |k|$$

$$0 \le |z+k| \le 2k$$

---

8 cont

$\arg(z+k)$

Let $z = x + yi$

$$|z|^2 = x^2 + y^2 \le k^2$$

$$\therefore x^2 \le k^2$$

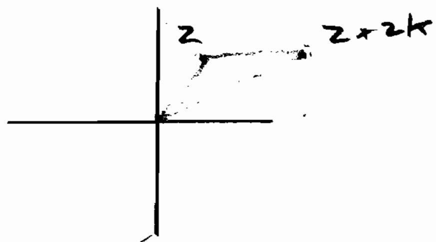
$$-k < x < k$$

$$\therefore \operatorname{Re}(z+k) = x + k > 0$$

$$-\frac{\pi}{2} < \arg(z+k) < \frac{\pi}{2}$$

---

8)

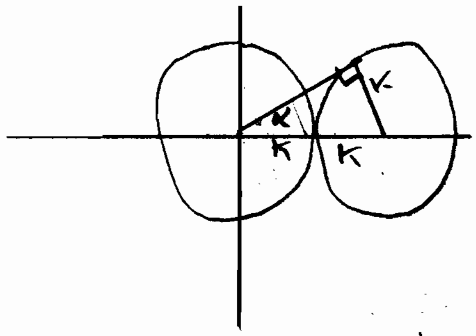



$$\text{Max } |z+2k| = 3k$$

$$\text{Min } |z+2k| = k$$

$$\text{Max } \arg(z+2k) = \frac{\pi}{6}$$

$$\text{Min } \arg(z+2k) = -\frac{\pi}{6}$$

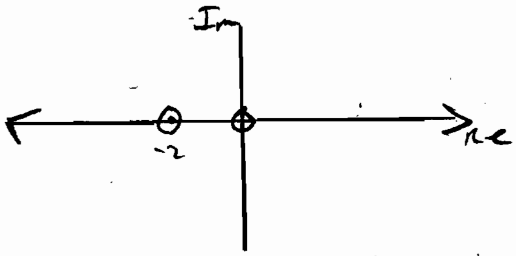



$$\alpha = \sin^{-1}\frac{k}{2k} = \sin^{-1}\frac{1}{2}$$

$$\alpha = \frac{\pi}{6}$$

---

9) $\arg z = \arg(z+2)$

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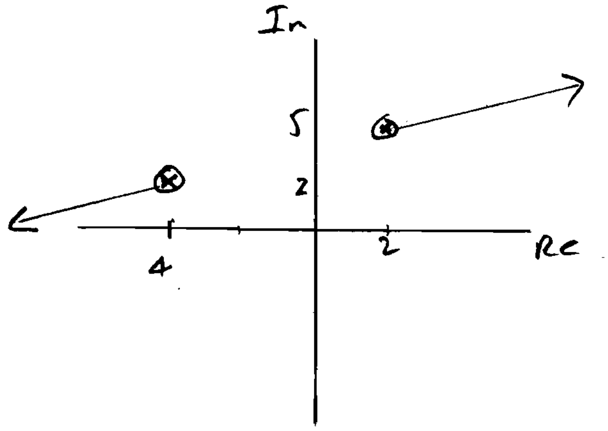
10) $\arg(z-2-5j)$

$= \arg(z+4-2j)$

$\arg(z-(2+5j))$

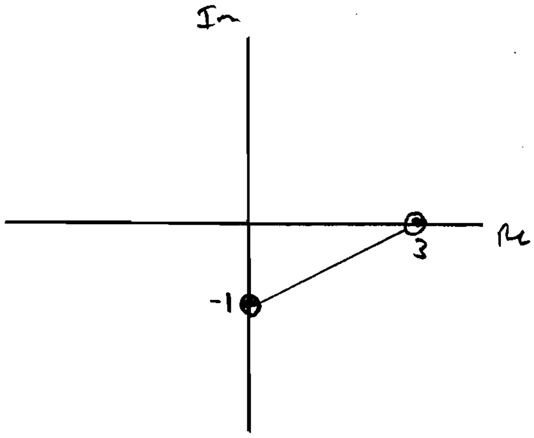
$= \arg(z-(-4+2j))$




11)

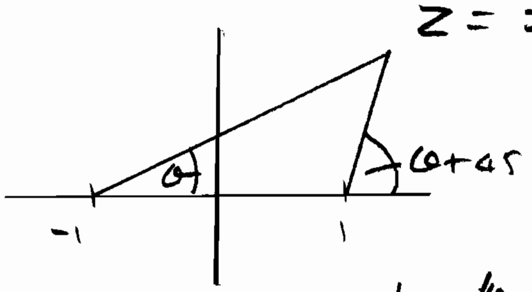
$\arg(z+j) = \arg(z-3)+\pi$

$\arg(z-(-j)) = \arg(z-3)+\pi$




12)

12




$$\tan\theta = \frac{y}{x+1} \qquad \tan(\theta+45) = \frac{y}{x-1}$$

But $\tan(\theta+45) = \dfrac{\tan\theta + \tan 45}{1 - \tan\theta\tan 45} = \dfrac{1+\tan\theta}{1-\tan\theta}$

$$= \frac{1+\frac{y}{x+1}}{1-\frac{y}{x+1}} = \frac{\frac{x+1+y}{x+1}}{\frac{x+1-y}{x+1}} = \frac{1+x+y}{1+x-y}$$

$$\therefore \quad \frac{y}{x-1} = \frac{1+x+y}{1+x-y}$$

$$y(1+x-y) = (x-1)(1+x+y)$$

$$y + \cancel{xy} - y^2 = \cancel{x} - 1 + x^2 - \cancel{x} + \cancel{xy} - y$$

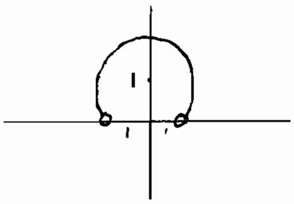
$$0 = x^2 + y^2 - 2y - 1$$

$$0 = x^2 + (y-1)^2 - 1 - 1$$

$$x^2 + (y-1)^2 = 2$$

Circle centre $(0,1)$ radius $\sqrt{2}$

But $\mathrm{Im}(z) > 0$ for $\arg(z-1) > \arg(z+1)$




Part of circle radius $\sqrt{2}$