

$$\begin{aligned} 1) i) \quad e^{-j\pi} &= \cos(-\pi) + j \sin(-\pi) \\ &= \cos \pi - j \sin \pi \\ &= -1 \end{aligned}$$

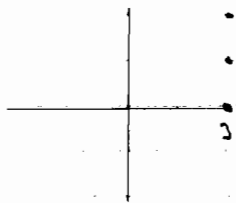
$$\begin{aligned} ii) \quad e^{j\frac{\pi}{4}} &= \cos \frac{\pi}{4} + j \sin \frac{\pi}{4} \\ &= \frac{1}{\sqrt{2}} + j \frac{1}{\sqrt{2}} \end{aligned}$$

$$\begin{aligned} iii) \quad e^{\frac{2+5j}{6}\pi} &= e^{\frac{2}{6}\pi} \left( \cos \frac{5\pi}{6} + j \sin \frac{5\pi}{6} \right) \\ &= e^{\frac{2}{6}\pi} \left( -\frac{1}{2} + j \frac{1}{2} \right) \\ &= 1.209 + 0.698j \end{aligned}$$

$$\begin{aligned} iv) \quad e^{3-4j} &= e^3 \left( \cos(-4) + j \sin(-4) \right) \\ &= e^3 \left( \cos(4) - j \sin(4) \right) \\ &= -13.129 + 15.201j \end{aligned}$$

$$\begin{aligned} 2) \quad e^z &= e^3 \\ \Rightarrow z &= 3 + 2n\pi j \end{aligned}$$

where  $n$  is an integer

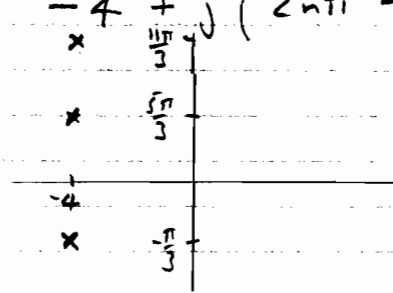


$$\begin{aligned} 3) \quad e^z &= \frac{1 - \sqrt{3}j}{2e^4} \\ e^z e^4 &= \frac{1 - \sqrt{3}j}{2} \\ e^{z+4} &= \frac{1}{2} - \frac{\sqrt{3}}{2}j \end{aligned}$$

$$\begin{aligned} e^{z+4} &= \cos\left(-\frac{\pi}{3}\right) + j \sin\left(-\frac{\pi}{3}\right) \\ e^{z+4} &= e^{-\frac{\pi}{3}j} \end{aligned}$$

$$z+4 = -\frac{\pi}{3}j + 2n\pi j$$

$$z = -4 + j \left( 2n\pi - \frac{\pi}{3} \right)$$



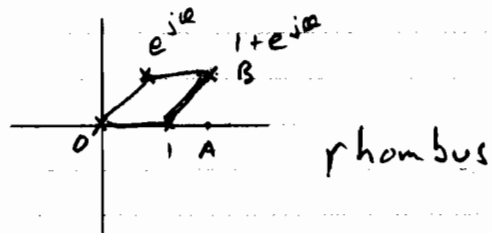
$$\begin{aligned} 4) \quad e^{z^*} &= (e^z)^* \\ e^{x-iy} &= (e^{x+iy})^* \\ e^{x-iy} &= e^x e^{-iy} \\ &= e^x (\cos y - j \sin y) \end{aligned}$$

$$\begin{aligned} \text{But } (e^{x+iy})^* &= (e^x e^{iy})^* = (e^x (\cos y + j \sin y))^* \\ &= e^x (\cos y - j \sin y) \end{aligned}$$

$\therefore$  true for all  $z$

$$\begin{aligned} 5) \quad 1 - e^{j0} &= e^{j\frac{\pi}{2}} e^{-j\frac{\pi}{2}} - e^{j\frac{\pi}{2}} e^{j\frac{\pi}{2}} \\ &= e^{j\frac{\pi}{2}} \left( e^{-j\frac{\pi}{2}} - e^{j\frac{\pi}{2}} \right) \\ &= e^{j\frac{\pi}{2}} \left( \cos \frac{\pi}{2} - j \sin \frac{\pi}{2} - \left( \cos \frac{\pi}{2} + j \sin \frac{\pi}{2} \right) \right) \\ &= e^{j\frac{\pi}{2}} \left( -2j \sin \frac{\pi}{2} \right) = -2j \sin \frac{\pi}{2} e^{j\frac{\pi}{2}} \end{aligned}$$

6)



Prove  $1 + e^{j\alpha} = 2 \cos \frac{\alpha}{2} e^{j\frac{\alpha}{2}}$

By symmetry of rhombus

$$\arg(1 + e^{j\alpha}) = \frac{\alpha}{2}$$

Find  $|1 + e^{j\alpha}|$

$$OB^2 = OA^2 + AB^2$$

$$|1 + e^{j\alpha}|^2 = (1 + \cos\alpha)^2 + \sin^2\alpha$$

$$= 1 + 2\cos\alpha + \cos^2\alpha + \sin^2\alpha$$

$$= 2 + 2\cos 2\alpha$$

$$= 2(1 + \cos 2\alpha)$$

$$= 4 \cos^2 \frac{\alpha}{2}$$

$$\therefore |1 + e^{j\alpha}| = 2 \cos \frac{\alpha}{2}$$

$$\begin{aligned} \therefore 1 + e^{j\alpha} &= 2 \cos \frac{\alpha}{2} \left( \cos \frac{\alpha}{2} + j \sin \frac{\alpha}{2} \right) \\ &= 2 \cos \frac{\alpha}{2} e^{j\frac{\alpha}{2}} \end{aligned}$$

7)

$$(1 + e^{2j\alpha})^n$$

$$= \left( 2 \cos \left( \frac{2\alpha}{2} \right) e^{j\frac{2\alpha}{2}} \right)^n$$

$$= 2^n \cos^n \alpha e^{jn\alpha}$$