1) i) $e^{-j\pi} = \cos(-\pi) + j\sin(-\pi)$

$= \cos\pi - j\sin\pi$

$= -1$

ii) $e^{j\frac{\pi}{4}} = \cos\frac{\pi}{4} + j\sin\frac{\pi}{4}$

$= \frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}$

iii) $e^{\frac{2+5j\pi}{6}} = e^{\frac{2}{6}}\left(\cos\frac{5\pi}{6} + j\sin\frac{5\pi}{6}\right)$

$= e^{\frac{2}{6}}\left(-\frac{\sqrt{3}}{2} + \frac{1}{2}j\right)$

$= 1.209 + 0.698$

iv) $e^{3-4j} = e^3\left(\cos(-4) + j\sin(4)\right)$

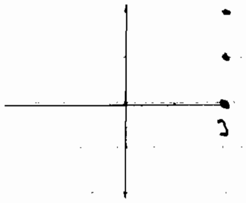
$= e^3\left(\cos(4) - j\sin(4)\right)$

$= -13.129 + 15.201j$

2) $e^z = e^3$

$\Rightarrow z = 3 + 2n\pi j$

where n is an integer



3) $e^z = \frac{1-\sqrt{3}j}{2e^4}$

$e^z e^4 = \frac{1-\sqrt{3}j}{2}$

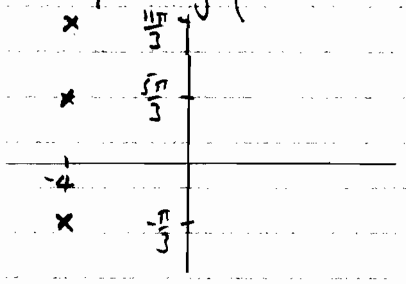
$e^{z+4} = \frac{1}{2} - \frac{\sqrt{3}}{2}j$

$e^{z+4} = \cos\left(-\frac{\pi}{3}\right) + j\sin\left(-\frac{\pi}{3}\right)$

$e^{z+4} = e^{-\frac{\pi}{3}j}$

$z+4 = -\frac{\pi}{3}j + 2n\pi j$

$z = -4 + j\left(2n\pi - \frac{\pi}{3}\right)$



4) $e^{z*} = (e^z)^*$

$e^{x-jy} = (e^{x+jy})^*$

$e^{x-jy} = e^x e^{-jy}$

$= e^x(\cos y - j\sin y)$

But $(e^{x+jy})^*$

$= (e^x e^{jy})^* = (e^x(\cos y + j\sin y))^*$

$= e^x(\cos y - j\sin y)$

$\therefore$ true for all z

5) $1 - e^{j\theta}$

$= e^{j\frac{\theta}{2}}e^{-j\frac{\theta}{2}} - e^{j\frac{\theta}{2}}e^{j\frac{\theta}{2}}$

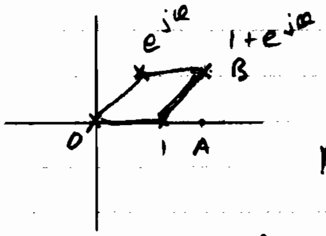
$= e^{j\frac{\theta}{2}}\left(e^{-j\frac{\theta}{2}} - e^{j\frac{\theta}{2}}\right)$

$= e^{j\frac{\theta}{2}}\left(\cos\frac{\theta}{2} - j\sin\frac{\theta}{2} - \left(\cos\frac{\theta}{2} + j\sin\frac{\theta}{2}\right)\right)$

$= e^{j\frac{\theta}{2}}\left(-2j\sin\frac{\theta}{2}\right)$

$= -2j\sin\frac{\theta}{2}\, e^{j\frac{\theta}{2}}$

6)



rhombus

Prove $1+e^{j\theta} = 2\cos\frac{\theta}{2}\, e^{j\frac{\theta}{2}}$

By symmetry of rhombus

$\arg(1+e^{j\theta}) = \frac{\theta}{2}$

Find $|1+e^{j\theta}|$

$OB^2 = OA^2 + AB^2$

$|1+e^{j\theta}|^2 = (1+\cos\theta)^2 + \sin^2\theta$

$= 1 + 2\cos\theta + \cos^2\theta + \sin^2\theta$

$= 2 + 2\cos 2\theta$

$= 2(1+\cos 2\theta)$

$= 4\cos^2\frac{\theta}{2}$

$\therefore |1+e^{j\theta}| = 2\cos\frac{\theta}{2}$

$\therefore 1+e^{j\theta} = 2\cos\frac{\theta}{2}\left(\cos\frac{\theta}{2} + j\sin\frac{\theta}{2}\right)$

$= 2\cos\frac{\theta}{2}\, e^{j\frac{\theta}{2}}$

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7) $(1+e^{2j\theta})^n$

$= \left(2\cos\left(\frac{2\theta}{2}\right) e^{j\frac{2\theta}{2}}\right)^n$

$= 2^n\cos^n\theta\, e^{jn\theta}$