1) i)

$u_y = 20 \sin 30 = 10 \text{ ms}^{-1}$

$u_x = 20 \cos 30 = 17.3 \text{ ms}^{-1}$

ii) $a_y = -10 \text{ ms}^{-2}$

$a_x = 0$

iii) Vertical Motion

At Q $y = 0$

Using $s = ut + \frac{1}{2}at^2$

$0 = 10t - 5t^2$
$0 = t(10 - 5t)$

$\Rightarrow t = 0, \text{ or } t = 2$

At Q (first bounce) $t = 2$ s

x = const horiz vel × time

$= 17.3 \times 2$

$= 34.6$ m

At max height $V_x = 0$

using $v = u + at$

$0 = 10 - 10t$

$\Rightarrow t = 1$ s

Max height at $t = 1$ s

v) Using $v^2 = u^2 + 2as$

$0 = 10^2 - 20y$

$20y = 100$

Max height $y = 5$ m

2)
i)

$u_x = 50 \cos 35 = 40.96 \text{ ms}^{-1}$

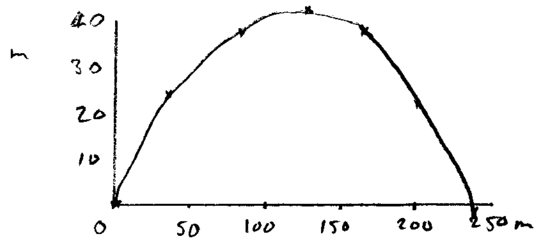
$u_y = 50 \sin 35 = 28.68 \text{ ms}^{-1}$

ii)

t	0	1	2	3	4	5	6
x	0	41	82	123	164	205	246
y	0	24	38	42	37	21	−4

use $s = ut + \frac{1}{2}at^2$

iii)



Max height approx 42m

Horizontal distance to bounce approx 240m

iv) Using $v^2 = u^2 + 2as$

At max height $v_y = 0$

2 cont)

$0 = 28.68^2 - 19.6y$

$19.6y = 28.68^2$

$y = 41.97\text{ m}$

Using $s = ut + \frac{1}{2}at^2$

At bounce $y = 0$

$0 = 28.68t - 4.9t^2$

$0 = t(28.68 - 4.9t)$

$t = 0$ or $t = \frac{28.68}{4.9} = 5.853$

At $t = 5.853$

$x = 5.853 \times 40.96$

$= 239.7\text{ m}$

Good agreement with estimates

v) Ball a particle

No spin

No air resistance

3) i)

$u_x = 19\cos 25 = 17.2\text{ ms}^{-1}$

$u_y = 19\sin 25 = 8.03\text{ ms}^{-1}$

ii) Using $s = ut + \frac{1}{2}at^2$

Hits ground when $y = 0$

$0 = 8.03t - 4.9t^2$

$0 = t(8.03 - 4.9t)$

$t = 0$ or $t = \frac{8.03}{4.9} = 1.64\text{ s}$

Hits ground after 1.64 s

iii) Horiz distance = const vely × time

$= 17.2 \times 1.64 = 28.2\text{ m}$

iv) By symmetry of motion

Time for max height $= \frac{1.64}{2} = 0.82\text{ s}$

v) Subst in $s = ut + \frac{1}{2}at^2$

$y = 8.03 \times 0.82 - 4.9 \times 0.82^2$

$y = 3.29\text{ m}$

v) Alternatively

use $v^2 = u^2 + 2as$

At max height $v_y = 0$

$0 = 8.03^2 - 19.6y$

$y = \frac{8.03^2}{19.6} = 3.29\text{ m}$

vi) Time for ball to travel 20m horizontally

$\frac{\text{Dist}}{\text{Speed}} = \frac{20}{17.2} = 1.163\text{ s}$

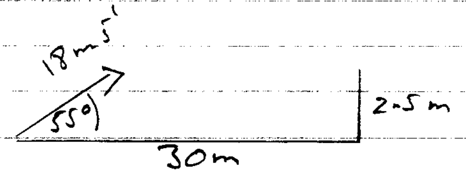
Height at $t = 1.163\text{ s}$

$y = 8.03 \times 1.163 - 4.9 \times 1.163^2$

$y = 2.71\text{ m}$

To high for player to stop ball

4)



i) Horizontally

$u_x = 18\cos 55^\circ$
$= 10.3 \text{ ms}^{-1}$

Vertically

$u_y = 18\sin 55^\circ$
$= 14.7 \text{ ms}^{-1}$

ii) Constant Horizontal Velocity

$\therefore$ time $= \frac{\text{distance}}{\text{speed}} = \frac{30}{10.3}$

$t = 2.91 s$

iii) Vertically using $s = ut + \frac{1}{2}at^2$

$y = 14.7t - 4.9t^2$

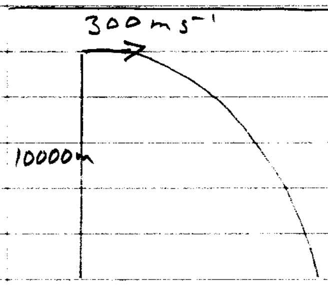
$y = 14.7 \times 2.91 - 4.9 \times 2.91^2$
$y = 1.28$ m

When ball crosses goal line it is 1.28 m above ground, $\therefore$ goes straight into goal!

iv) Goalkeeper $= 30 - 5 = 25$ m away
Time to reach keeper $= \frac{25}{10.3} = 2.43 s$

At $t = 2.43$
$y = 14.7 \times 2.43 - 4.9 \times 2.43^2$
$y = 6.79$ m
No, keeper cannot stop ball

5)



Vert $u_y = 0$

i) Using $s = ut + \frac{1}{2}at^2$
$y = 0 - 4.9t^2$
At ground
$-10000 = -4.9t^2$

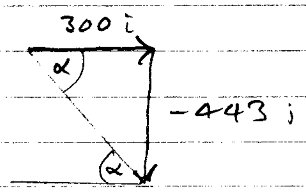
$\Rightarrow t = \sqrt{\frac{-10000}{-4.9}} = 45.2 s$

ii) Horizontal distance = speed × time
$= 300 \times 45.2$
$= 13,560$ m
$= 13.6$ km

iii) vertically
$v_y = u_y + at$
$v_y = 0 - 9.8t$
$v_y = -9.8 \times 45.2$
$v_y = -443 \text{ ms}^{-1}$
horizontally $v_x = 300 \text{ ms}^{-1}$

Speed on impact $= \sqrt{300^2 + (-443)^2}$
Speed $= 535 \text{ ms}^{-1}$

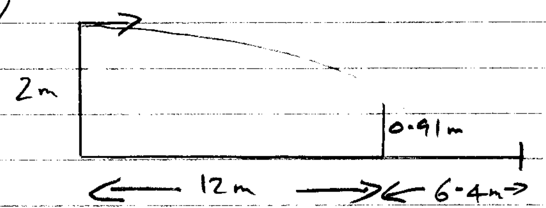
iv)



$\alpha = \tan^{-1}\frac{443}{300}$

$\alpha = 56^\circ$

6)



i) Vert $u_y = 0$
At top of net $y = -1.09$ m
Using $s = ut + \frac{1}{2}at^2$

$-1.09 = 0 \times t - 4.9t^2$
$4.9t^2 = 1.09$
$t^2 = \frac{1.09}{4.9}$

6 cont $t = 0.47\,s$

6ii) At ground $y = -2$

$$-2 = 0t - 4.9t^2$$
$$4.9t^2 = 2$$
$$t^2 = \frac{2}{4.9}$$
$$t = 0.64\,s$$

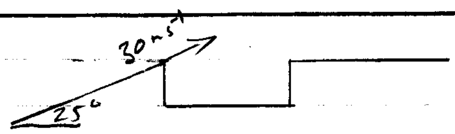
6iii) Need to travel 12 m horizontally in 0.47 s

$$\text{speed} = \frac{\text{distance}}{\text{time}} = \frac{12}{0.47}$$
$$\text{speed} = 25.5\ ms^{-1}$$

6iv) If it lands on limiting line then horizontal distance $= 12 + 6.4$
$= 18.4\,m$

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}} = \frac{18.4}{0.64}$$
$$\text{speed} = 28.8\ ms^{-1}$$

7)



$$u_y = 30 \sin 25° = 12.6785\ ms^{-1}$$

$$u_x = 30 \cos 25° = 27.1892\ ms^{-1}$$

Time to cross 50 m horizontally

$$= \frac{\text{Distance}}{\text{Speed}} = \frac{50}{27.1892} = 1.839\,s$$

At $t = 1.839$

(using $s = ut + \frac{1}{2}at^2$ vertically)

$$y = 12.6785 \times 1.839 - 4.9 \times 1.839^2$$

$$y = 6.74\ m \quad > 0$$

$\therefore$ clears gorge

7ii) If horizontal distance reduced by 40% then 50 m is 60% of an unaffected jump
Jump would have to be $50 \times \frac{100}{60}$

$$= 83.33\ m$$

$$\text{Time for jump} = \frac{\text{Distance horizontally}}{\text{speed}}$$
$$= \frac{83.33}{u \cos 25°}$$

Vertically $y = ut + \frac{1}{2}at^2$

$$y = u \sin 25 \times t - 4.9t^2$$

On landing

$$0 = u \sin 25 \times t - 4.9t^2$$
$$0 = t(u \sin 25 - 4.9t)$$
$$\Rightarrow t = 0 \quad \text{or} \quad u \sin 25 = 4.9t$$
$$t = \frac{u \sin 25}{4.9}$$

Equating times

$$\frac{u \sin 25°}{4.9} = \frac{83.33}{u \cos 25°}$$

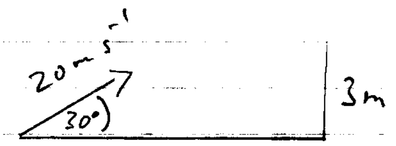
$$u^2 \sin 25° \cos 25° = 83.33 \times 4.9$$

$$u^2 = \frac{83.33 \times 4.9}{\sin 25 \cos 25}$$

$$u^2 = 1066.039976$$

$$u = 32.7\ ms^{-1}$$

8) i)



Horizontal vel^y $u_x = 20\cos 30$

$= 17.32 \text{ ms}^{-1}$

$\text{Time} = \frac{\text{Distance}}{\text{Speed}} = \frac{35}{17.32}$

$t = 2.02 \text{ s}$

ii) Find y at $t = 2.02$

Using $s = ut + \frac{1}{2}at^2$

$y = 20\sin 30 \times t - 4.9t^2$

$y = 20\sin 30 \times 2.02 - 4.9 \times 2.02^2$

$y = 0.20604$

as this is less than 3m the ball does not go over the crossbar

iii) Hits bar when $x = 35$, $y = 3$

$y = u\sin 30 \times t - 4.9t^2$

Also $x = u\cos 30\, t$

$\therefore \frac{35}{u\cos 30} = t$

Subst for t

$3 = \cancel{u}\sin 30 \times \frac{35}{\cancel{u}\cos 30} - \frac{4.9 \times 35^2}{u^2\cos^2 30}$

$3 = 35\tan 30 - \frac{6002.5}{u^2\cos^2 30}$

$\frac{6002.5}{u^2\cos^2 30} = 35\tan 30 - 3$

$\frac{6002.5}{\cos^2 30} = u^2(35\tan 30 - 3)$

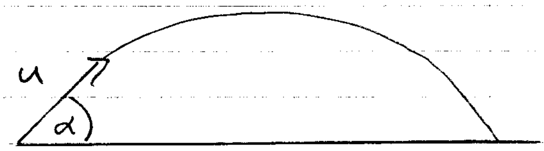
$u^2 = \frac{6002.5}{\cos^2 30(35\tan 30 - 3)}$

$u = 21.566 \text{ ms}^{-1}$

$u = 21.57 \text{ ms}^{-1}$

iv) Spin causes ball to rise more

9)



$u_y = u\sin\alpha$ $\quad u_x = u\cos\alpha$

$y = u\sin\alpha \times t - 5t^2$

$x = u\cos\alpha \times t$

when projectile lands $y = 0$

$\therefore 0 = u\sin\alpha\, t - 5t^2$

$0 = t(u\sin\alpha - 5t)$

$\Rightarrow t = 0$ or $t = \frac{u\sin\alpha}{5}$

To find range substitute for t

$x = u\cos\alpha \times \frac{u\sin\alpha}{5}$

$x = \frac{u^2\sin\alpha\cos\alpha}{5}$

i) For
$u = 20, \alpha = 30$ $\quad x = 34.6 \text{ m}$
$u = 20, \alpha = 40$ $\quad x = 39.4 \text{ m}$
$u = 20, \alpha = 45$ $\quad x = 40 \text{ m}$
$u = 20, \alpha = 50$ $\quad x = 39.4 \text{ m}$
$u = 20, \alpha = 60$ $\quad x = 34.6 \text{ m}$

9ii) $x = \dfrac{u^2 \sin\alpha \cos\alpha}{5}$

since for $0 \le \alpha \le 90$

$\sin\alpha = \cos(90-\alpha)$

and $\cos\alpha = \sin(90-\alpha)$

$\dfrac{u^2 \sin\alpha\cos\alpha}{5} = \dfrac{u^2\sin(90-\alpha)\cos(90-\alpha)}{5}$

ie Range the same for α and $90-\alpha$

9iii) If $x = 36$ and $u = 20$

$36 = 80 \sin\alpha\cos\alpha$

If $\alpha = 32.1^\circ$, $80\sin 32.1^\circ \cos 32.1^\circ = 36.01$

$\therefore$ $\alpha = 32.1^\circ$ is suitable

$\alpha = 90 - 32.1 = 57.9^\circ$ is the other suitable angle

9iv) When error is $+0.5^\circ$

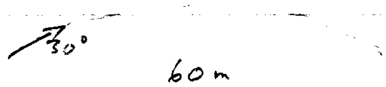
$80\sin\alpha\cos\alpha$
$= 80\sin 32.6^\circ\cos 32.6 = 36.31$ m

or

$80\sin 58.4^\circ\cos 58.4^\circ = 35.70$ m

Smaller angle 31 cm out to nearest cm
Larger angle 30 cm out to nearest cm

10i)



$u_y = u\sin 30^\circ$ $u_x = u\cos 30^\circ$

$x = u\cos 30 \times t$

$y = u\sin 30 \times t - 4.9t^2$

when ball lands

$0 = t(u\sin 30 - 4.9t)$

$t = \dfrac{u\sin 30}{4.9}$

$x = \dfrac{u^2\sin 30\cos 30}{4.9}$

If ball lands when $x = 60$

$\dfrac{60 \times 4.9}{\sin 30\cos 30} = u^2$

$u = 26.06\ \mathrm{ms^{-1}}$

10ii) If $x = 60$ when $y = 3.2$

$60 = u\cos 30 \times t$

$t = \dfrac{60}{u\cos 30}$

Subst for t in $y = u\sin 30 \times t - 4.9t^2$

$3.2 = \dfrac{\cancel{u}\sin 30 \times 60}{\cancel{u}\cos 30} - \dfrac{4.9\times 60^2}{u^2\cos^2 30}$

$3.2 = 60\tan 30 - \dfrac{17640}{u^2\cos^2 30}$

$\dfrac{17640}{u^2\cos^2 30} = 60\tan 30 - 3.2$

$\dfrac{17640}{\cos^2 30} = u^2(60\tan 30^\circ - 3.2)$

$u^2 = \dfrac{17640}{\cos^2 30(60\tan 30^\circ - 3.2)}$

$u^2 = 748.067$

$u = 27.35\ \mathrm{ms^{-1}}$

10 cont) iii) If $x = 60$ when $y = 0.25$

$$u^2 = \frac{17640}{\cos^2 30(60\tan 30 - 0.25)}$$

$$u = 26.15\ ms^{-1}$$

If $x = 60$ when $y = 2.1$

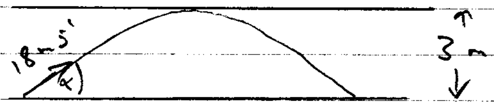
$$u^2 = \frac{17640}{\cos^2 30(60\tan 30 - 2.1)}$$

$$u = 26.88\ ms^{-1}$$

Fielder catches when

$26.15 ms^{-1} \le u \le 26.88 ms^{-1}$

11)



$u_x = 18\cos\alpha \qquad u_y = 18\sin\alpha$

Ball rises 3m

using $v^2 = u^2 + 2as$

$v_y^2 = u_y^2 - 19.6y$

At height point $v_y = 0$, $y = 3$

$$0 = u_y^2 - 19.6 \times 3$$

$$58.8 = u_y^2$$

$$u_y = 7.668$$

$\therefore\ 18\sin\alpha = 7.668$

$$\sin\alpha = \frac{7.668}{18}$$

$$\alpha = 25.2^\circ$$

Find time of flight

using $s = ut + \frac{1}{2}at^2$

$$y = 18\sin 25.2t - 4.9t^2$$

$$0 = t(18\sin 25.2 - 4.9t)$$

$$t = \frac{18\sin 25.2}{4.9}$$

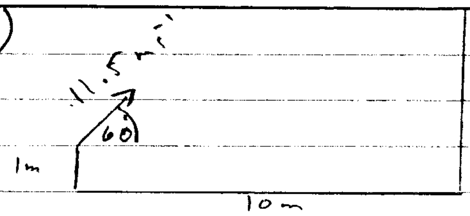
$$t = 1.564\ s$$

$$x = u\cos\alpha \times t$$

$$= 18\cos 25.2 \times 1.564$$

Range $= 25.47$ m

12)



i) Horizontal $\quad v_x = V_x = 11.5\cos 60$

$= 5.75\ ms^{-1}$

Time to reach house $= \frac{\text{Distance}}{\text{Speed}} = \frac{10}{5.75}$

$$t = 1.74\ s$$

ii) Vertically using $s = ut + \frac{1}{2}at^2 + s_0$

$$y = 11.5\sin 60 \times t - 4.9t^2 + 1$$

At $t = 1.74$

$$y = 11.5\sin 60 \times 1.74 - 4.9 \times 1.74^2 + 1$$

$$y = 3.49\ m$$

Hits Juliets window

iii) At $t = 1.74$

$v_y = u_y - 9.8t$

$= 11.5\sin 60 - 9.8 \times 1.74 = -7.093\ ms^{-1}$

12 cont iii) $V_x = 5.75\ ms^{-1}$

$$\text{Speed} = \sqrt{5.75^2 + (-7.093)^2}$$

$$= 9.13\ ms^{-1}$$

13) i)

$\uparrow 8\ ms^{-1}$

Using $v^2 = u^2 + 2as$

$$V_y^2 = 8^2 - 20y$$

At max height $V_y = 0$

$$0 = 64 - 20y$$
$$20y = 64$$
$$y = 3.2\ m$$

Cannot go higher as vertical component of velocity cannot exceed $8\ ms^{-1}$

ii) $u_y = 8\sin 30 = 4\ ms^{-1}$

Using $s = ut + \frac{1}{2}at^2$

$$y = 4t + \tfrac{1}{2}(-10)t^2$$
$$y = 4t - 5t^2$$

Lands when $y = 0$

$$0 = 4t - 5t^2$$
$$0 = t(4 - 5t)$$

$$\Rightarrow t = 0 \text{ or } t = \frac{4}{5}$$

Horizontal distance from firework when $t = \frac{4}{5}$

$$\text{Distance} = \text{Speed} \times \text{Time}$$
$$= 8\cos 30 \times 0.8$$
$$= 5.54\ m$$

iii) Using $v^2 = u^2 + 2as$

$$V_y^2 = u_y^2 - 20y$$

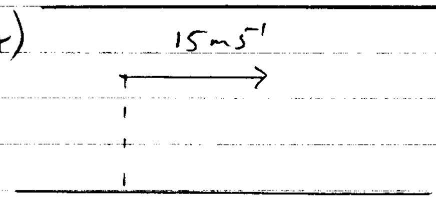
Given $y = 0$ when $V_y = 0$ at max height

$$0 = (8\sin\theta)^2 - 20 \times 2$$

$$40 = 64\sin^2\theta$$
$$\frac{40}{64} = \sin^2\theta$$
$$\Rightarrow \theta = \sin^{-1}\left(\sqrt{\frac{40}{64}}\right)$$

Angle of projection $\theta = 52.2°$

14) i)



If launched from (0,0)

$y = -19.6\ m$ when stone lands

Using $s = ut + \frac{1}{2}at^2$ vertically

$$-19.6 = 0 \times t - 4.9t^2$$

$$4.9t^2 = 19.6$$
$$t^2 = 4$$
$$t = 2\ s$$

Horizontally Distance $=$ Speed $\times$ time
$$= 15 \times 2$$
$$= 30\ m$$

ii) Vertically both stones have same initial velocity (0) and same acc ($-g$) $\therefore$ always at same height. This means that when their horizontal coords are the same they will collide.

14 iii) Collide when $y = -19.6 + 4.9$
$= -14.7$

Using $s = ut + \frac{1}{2}at^2$ vertically

$y = 0 - 4.9t^2$
$-14.7 = -4.9t^2$

$t = \sqrt{\frac{-14.7}{-4.9}} = 1.732\text{s}$

Horizontal approach speed
$= 15 + v$

Distance $= 50$ m

Using speed $= \frac{\text{distance}}{\text{time}}$

$15 + v = \frac{50}{1.732}$

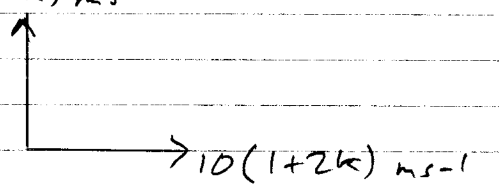
$v = \frac{50}{1.732} - 15$

$v = 13.9 \text{ ms}^{-1}$

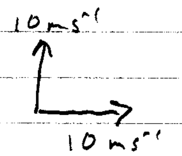
14 iv) Time to hit ground $= 2$ s
(from part (ii))

Do not collide if
Approach speed $\times 2 < 50$ m
$(15 + v)2 < 50$
$30 + 2v < 50$
$2v < 20$
$v < 10 \text{ ms}^{-1}$

15)



a) $k = 0$

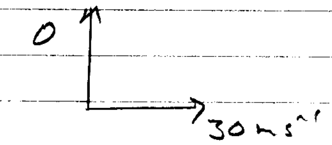


Speed $= \sqrt{10^2 + 10^2} = \sqrt{200}$

$= 14.1 \text{ ms}^{-1}$

Angle of projection $= 45°$

b) $k = 1$



Speed $= 30 \text{ ms}^{-1}$
Direction $=$ horizontal

ii)
Horizontally $x = 10(1+2k)t$

Vertically $y = 10(1-k)t - 5t^2$

iii) Hits ground when $y = 0$

$0 = 10(1-k)t - 5t^2$
$= t(10 - 10k - 5t)$
$\Rightarrow t = 0$ or $10 - 10k - 5t = 0$
$10 - 10k = 5t$
$2 - 2k = t$

$t = 2 - 2k$ or $2(1-k)$

iv) when $t = 2(1-k)$
$x = 10(1+2k) \times 2(1-k)$
$= 20(1+2k)(1-k)$
$= 20(1 + 2k - k - 2k^2)$
$= 20(1 + k - 2k^2)$ m

Given $20(1 + k - 2k^2) = \frac{5}{2}\left(9 - 16(k - \frac{1}{4})^2\right)$

Max range $= \frac{5}{2} \times 9 = 22.5$ m