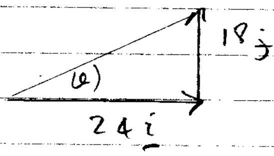
MEI MECHANICS 1 FORCES EXERCISE 7B

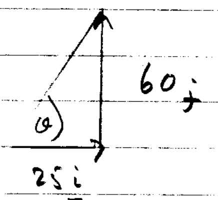
1) i) Sheuli $24\underline{i} + 18\underline{j}$



Magnitude $= \sqrt{24^2 + 18^2} = 30N$

$\theta = \tan^{-1}\frac{18}{24} = 36.9^\circ$

Veronica $25\underline{i} + 60\underline{j}$



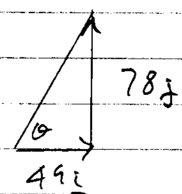
Magnitude $= \sqrt{25^2 + 60^2} = 65$ N

$\theta = \tan^{-1}\frac{60}{25} = 67.4^\circ$

ii) Omitted.

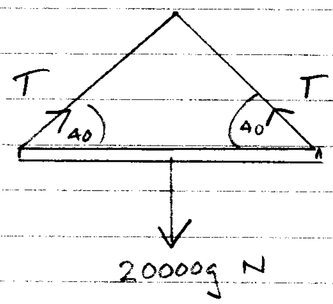
iii) Resultant $= 49\underline{i} + 78\underline{j}$

Magnitude $= \sqrt{49^2 + 78^2} = 92.1$N



$\theta = \tan^{-1}\left(\frac{78}{49}\right) = 57.9^\circ$

2) i)



ii)
Horizontal component of tension in each rope $= T\cos 40$

Vertical component of tension in each rope $= T\sin 40$

iii)
$T\sin 40 + T\sin 40 = 20000 \times 9.8$

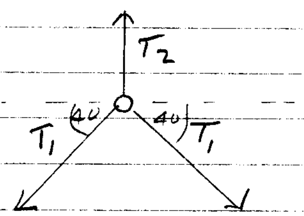
$2T\sin 40 = 196{,}000$

$T = \frac{196\,000}{2\sin 40}$

$T = 152{,}461$ N

$T = 152.5$ kN to 1 d.p.

iv)



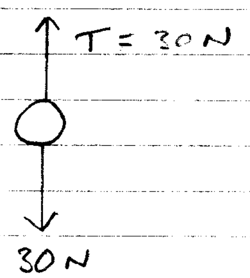
v)
$T_2 = T_1\sin 40 + T_1\sin 40$

$T_2 = 20000g$ N

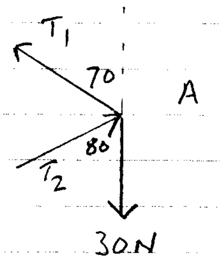
$T_2 = 196{,}000$ N

vi)
Resolve vertically for whole system

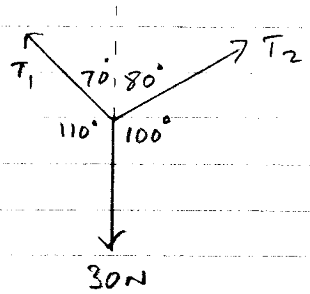
3) a)



3b)



Redraw



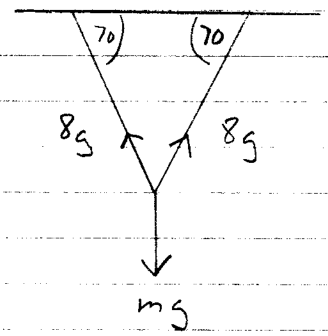
Lami's theorem gives

$$\frac{T_1}{\sin 100} = \frac{T_2}{\sin 110} = \frac{30}{\sin 150}$$

$$T_1 = \frac{30 \sin 100°}{\sin 150°} = 59.1\text{ N} \quad \text{tension}$$

$$T_2 = \frac{30 \sin 110}{\sin 150} = 56.4\text{ N} \quad \text{compression}$$

4)
i)



$$8g \sin 70 + 8g \sin 70 = mg$$

$$16g \sin 70 = mg$$

$$m = 16 \sin 70 \text{ kg}$$

$$m = 15.04\text{ kg}$$

Fish is 15.04 kg.

ii) Both balances beyond their limit and therefore read 10 kg each

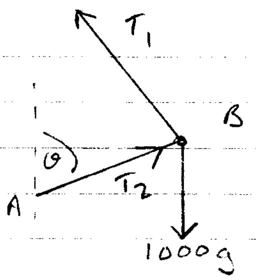
iii) Balances share weight equally

Both read $\frac{15.04 + 0.25}{2}$

$= 7.645$ kg

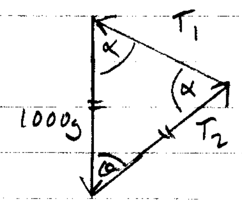
iv) Method B no good
Method A need to measure angles
Method C need to know mass of stick
Method C easier to understand for non-mathematicians

5)
i)



ii) Rod AB must be in thrust to oppose the horizontal component of T_1. Otherwise B would move left.

iii)



b)
If $\theta = 60$, $\alpha = \frac{180-60}{2} = 60°$

Then $T_1 = T_2 = 1000g = 9800$ N

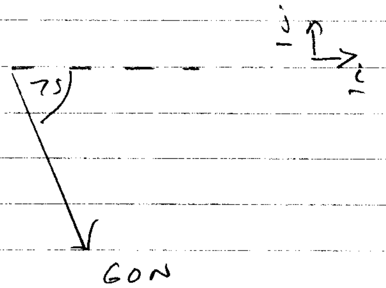
a) If $\theta = 90$, $\alpha = \frac{180-90}{2} = 45°$

Lami's Theorem $\frac{T_1}{\sin 90} = \frac{T_2}{\sin 45} = \frac{1000g}{\sin 45}$

$T_1 = 13{,}859$ N $\quad T_2 = 9800$ N

6)

i)



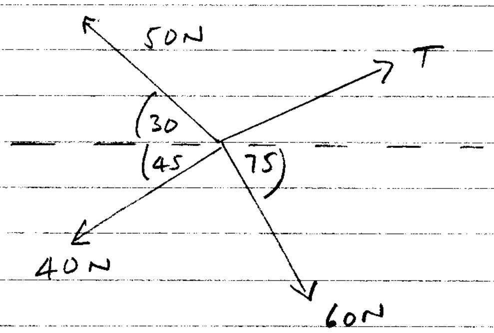
In component form the 60 N force is given by

$$60\cos 75\, \underline{i} - 60\sin 75\, \underline{j}$$

$$= 15.5\,\underline{i} - 58.0\,\underline{j}$$

to 3 sig fig

ii)



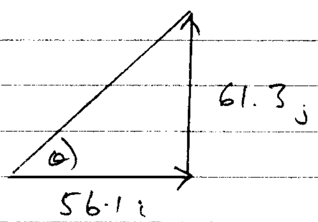
Resultant of the 3 known forces

$$\begin{pmatrix} 15.5 \\ -58.0 \end{pmatrix} + \begin{pmatrix} -50\cos 30 \\ +50\sin 30 \end{pmatrix} + \begin{pmatrix} -40\cos 45 \\ -40\sin 45 \end{pmatrix}$$

$$= \begin{pmatrix} -56.1 \\ -61.3 \end{pmatrix} = -56.1\,\underline{i} - 61.3\,\underline{j}$$

Since system is in equilibrium

$$T = +56.1\,\underline{i} + 61.3\,\underline{j}$$



Magnitude of T

$$= \sqrt{56.1^2 + 61.3^2}$$

$$= 83.1 \text{ N}$$

$$\theta = \tan^{-1}\left(\frac{61.3}{56.1}\right) = 47.5^\circ$$

Makes an angle of 47.5° with the $\underline{i}$ direction.

iii)

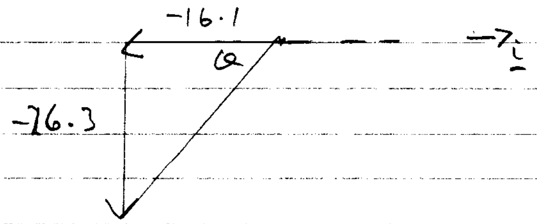
Resultant Force is due to change in T. Take T from 0 and add in new T

$$= -56.1\,\underline{i} - 61.3\,\underline{j} + 40\,\underline{i} + 35\,\underline{j}$$

$$= -16.1\,\underline{i} - 26.3\,\underline{j}$$

Magnitude of resultant

$$= \sqrt{(-16.1)^2 + (-26.3)^2} = 30.8\text{N}$$



$$\theta = \tan^{-1}\left(\frac{26.3}{16.1}\right) = 58.5^\circ$$

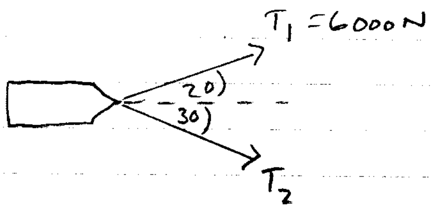
Direction $= 180 + \theta^\circ$ with $\underline{i}$ direction

$$= 238.5^\circ$$

$$\left(\text{or } -(180 - \theta) = -121.5^\circ\right)$$

7)

i)



Along line of motion components are

$6000\cos 20$ and $T_2 \cos 30$

$= 5638$ N and $T_2 \cos 30$ N

Perpendicular to line of motion components are

$6000 \sin 20$ and $T_2 \sin 30$

$= 2052$ N and $T_2 \sin 30$ N

ii) No resultant perpendicular to ℓ

$\therefore\ T_2 \sin 30 = 2052$

$T_2 = \dfrac{2052}{\sin 30} = 4104$ N

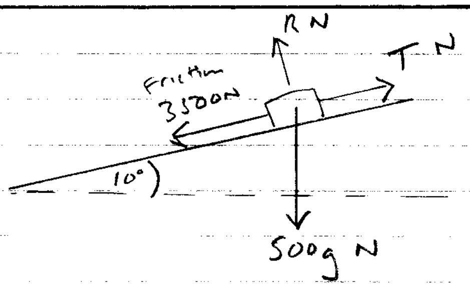
iii) If constant velocity then resultant $= 0$ in line of motion

$\therefore$ drag force $= 5638 + 4104\cos 30$

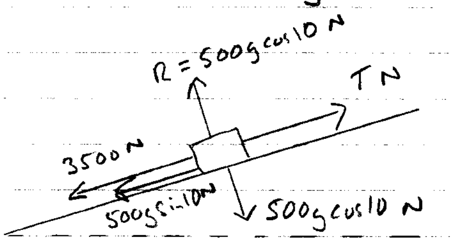
$= 9192$ N

8)

i)



ii)



iii) In equilibrium

$\therefore\ T = 3500 + 500g \sin 10°$

$T = 4351$ N

iv) Constant velocity therefore resultant force $= 0$.
Tension $= 4351$ N again

(However, we should note here that the tension have to have increased during the time the boat was accelerating from 0 to 1ms^{-1})

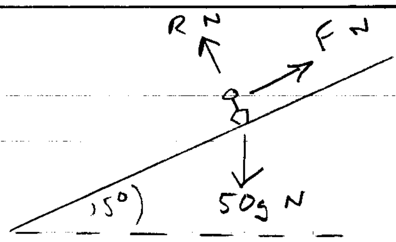
v)
Force down slope parallel to slope due to weight
$= 500g \sin 10° = 851$ N

Potential friction force up slope
$=$ up to 3500 N

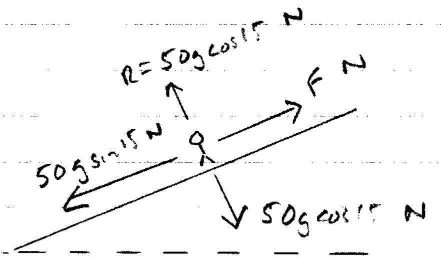
$\therefore$ boat will not slide back down if rope breaks

9)

i)



ii)



iii) Constant speed $\therefore$ resultant force $= 0$

Normal reaction $= 50g \cos 15°$

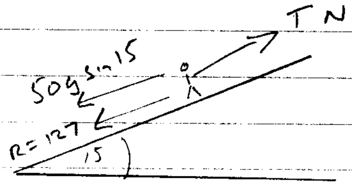
$= 473$ N

9 cont) Resistance or frictional force up the slope

$= 50g \sin 15°$

$= 127 \text{ N}$

iv)

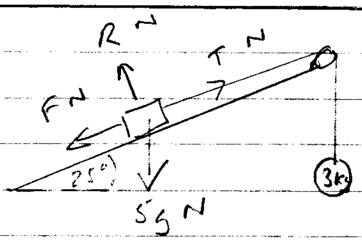


Constant speed $\therefore$ resultant $= 0$

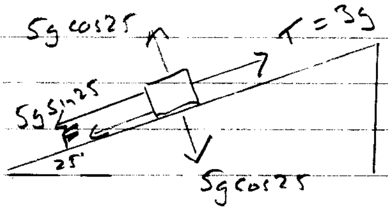
$T = 127 + 50g \sin 15$

$T = 254 \text{ N}$

10) i)



ii)



iii) Find F (the resistance to motion)

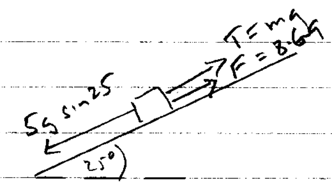
$T = 3g$ (otherwise 3 kg weight would be accelerating)

$\therefore F + 5g \sin 25 = 3g$

$F = 3g - 5g \sin 25$

$F = 8.69 \text{ N}$

iv)



$T + F = 5g \sin 25$

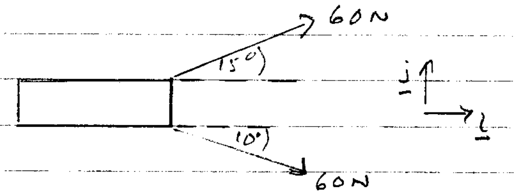
$mg + 8.69 = 5g \sin 25$

$mg = 5g \sin 25 - 8.69$

$m = \dfrac{5g \sin 25 - 8.69}{g}$

$m = 1.226 \text{ kg}$

11) i)



Top dog $60\cos 15\,\underline{i} + 60 \sin 15\,\underline{j}$

$= 57.96\,\underline{i} + 15.53\,\underline{j}$

Bottom dog

$60 \cos 10\,\underline{i} - 60 \sin 10\,\underline{j}$

$= 59.09\,\underline{i} - 10.42\,\underline{j}$

ii) a) Forward force

$= 57.96 + 59.09$

$= 117.05 \text{ N}$

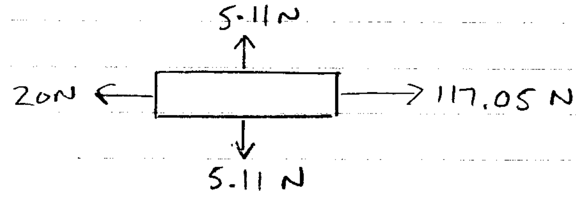
b) Sideways force

$= 15.53 - 10.42$

$= 5.11 \text{ N}$

11 cont)

iii)



Overall Resultant $117.05 - 20$

$= 97.05\ N$

in forward direction

iv) If dogs pulled forward

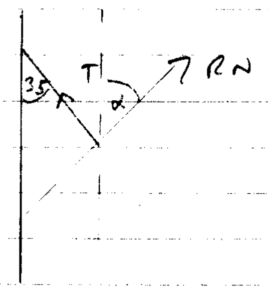
Resultant would be $60 + 60 - 20$

$= 100\ N$

$\therefore$ 2.95 N lost

12) i) Wall can push but not pull

ii)



Horizontal $T\sin 35 = R\sin\alpha$

Vertical $T\cos 35 + R\cos\alpha = 80g$

iii) Given $\alpha = 45°$

$$R = \frac{T\sin 35}{\sin 45}$$

Subst for R

$$T\cos 35 + \frac{T\sin 35}{\cancel{\sin 45}} \times \cancel{\cos 45} = 80g$$

$$T(\cos 35 + \sin 35) = 80g$$

$$T = \frac{80g}{(\cos 35 + \sin 35)}$$

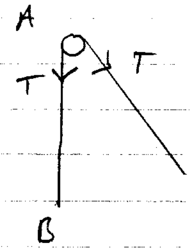
$$T = 563\ N$$

$$R = \frac{563\sin 35}{\sin 45}$$

$$R = 457\ N$$

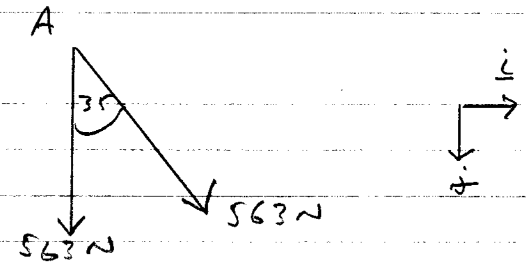
iv) 563 N down line of rope

v)



Same tension throughout rope $= 563\ N$

vi)



Resolve in $\underline{i}$ and $\underline{j}$ directions

Resultant force on A

$$563\sin 35\,\underline{i} + 563\cos 35\,\underline{j} + 563\,\underline{j}$$

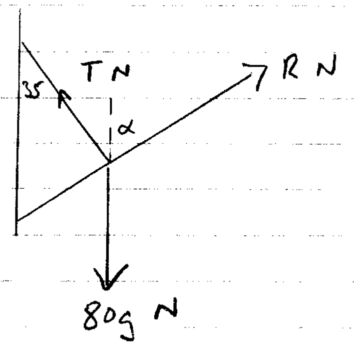
$$= 323\,\underline{i} + 1024\,\underline{j}$$

Magnitude $= \sqrt{323^2 + 1024^2}$

$= 1074\ N$

12) i) Wall can push but not pull

ii)



Horizontal Components

$$T \sin 35° = R \sin \alpha°$$

Vertical Components

$$T \cos 35° + R \cos \alpha = 80g$$

iii) Given $\alpha = 45°$

$$R \sin 45° = T \sin 35°$$

$$R = \frac{T \sin 35°}{\sin 45°}$$

Subst for R

$$T \cos 35° + \frac{T \sin 35° \cos 45°}{\sin 45°} = 80g$$

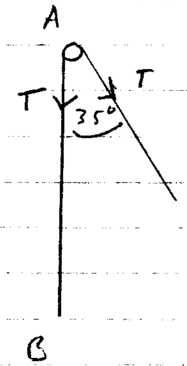
$$T\left(\cos 35° + \frac{\sin 35° \times \cancel{\frac{1}{\sqrt{2}}}}{\cancel{\frac{1}{\sqrt{2}}}}\right) = 80g$$

$$T = \frac{80g}{\cos 35° + \sin 35°} \approx 563 \text{ N}$$

$$R = \frac{T \sin 35}{\sin 45} = 457 \text{ N}$$

iv) 563 N down direction of rope

v)



Tension the same throughout rope $= 563$ N

vi) Resolve forces on pulley into $\underline{i}$, $\underline{j}$

$$= T\underline{j} + T\cos 35\,\underline{j} + T \sin 35\,\underline{i}$$

$$= 563(1 + \cos 35)\underline{j} + 563 \sin 35$$

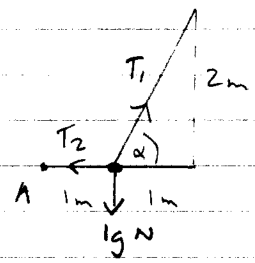
$$= 1024.18\,\underline{j} + 322.92\,\underline{i}$$

Magnitude of resultant

$$= \sqrt{1024.18^2 + 322.92^2}$$

$$= 1074 \text{ N}$$

13)



$$\alpha = \tan^{-1} 2 = 63.43°$$

Resolve vertically $T_1 \sin\alpha = 1g$

$$T_1 = \frac{9.8}{\sin 63.43°} = 10.96 \text{ N}$$

13cont) Resolve horizontally

$T_2 = T_1 \cos\alpha$

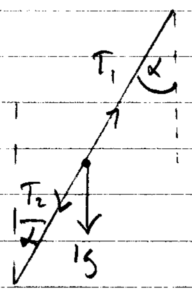
$T_2 = 10.96 \cos 63.43°$

$T_2 = 4.9$ N

ii) Path is an arc of a circle with centre A and radius 1m

Cannot lie in straight line as two parallel forces and a third force cannot form a triangle

Alternatively, suppose they were in a straight line



Then the angles marked α would be equal

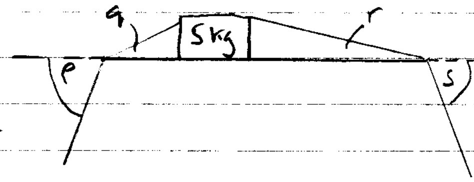
Resolving horizontally would give

$T_1 \sin\alpha = T_2 \sin\alpha$

$\Rightarrow T_1 = T_2$

∴ T_1 and T_2 cancel each other out and resultant force is 1g vertically downwards. ∴ particle would not be in equilibrium

14) i)



If q = r and parcel in equilibrium

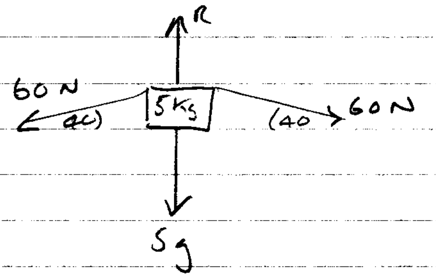
Let Spot pull with force 60 N and Scruff pull with force T N

Resolving horizontally

$60 \cos r = T \cos r$

$\Rightarrow T = 60$ N

ii)

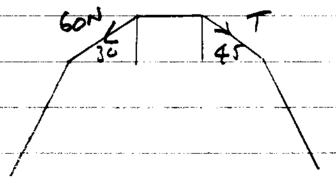


Resolving vertically

$5g + 60 \sin 40 + 60 \sin 40 = R$

$R = 126$ N

iii)



In equilibrium

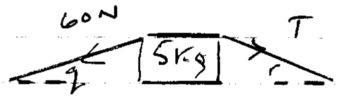
∴ horizontally

$60 \cos 30 = T \cos 45$

$T = \dfrac{60 \cos 30}{\cos 45}$

$T = 73.5$ N

14 (cont) iv)



In equilibrium

$$60 \cos q = T \cos r$$

$$T = \frac{60 \cos q}{\cos r}$$

The closer the parcel to scruff, the bigger the angle r. This makes cos r smaller and causes T to be bigger.

15)

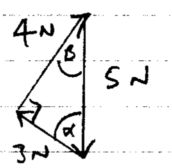
i) Since B is a smooth pulley there is a single tension throughout string section AC
$\therefore$ tension in AB = tension in AC
Likewise, tension in CD = tension in DE

Since A is in equilibrium, tension in AB = 3N and because E is in equilibrium, tension in DE = 4N

$\therefore$ tensions are as follows
AB = BC = 3N
CD = DE = 4N

ii)

Forces at C



(3, 4, 5 right-angled Δ)

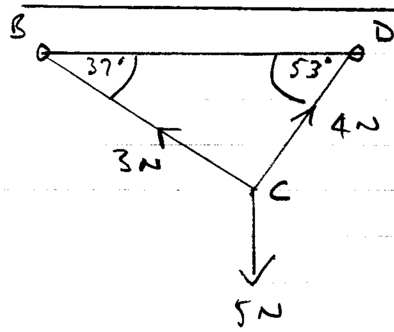
$$\alpha = \tan^{-1}\frac{4}{3} = 53.1°$$

$$\beta = 90 - \alpha = 36.9°$$

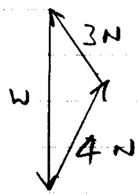
BC makes an angle of 53.1° with vertical

CD makes an angle of 36.9° with vertical

iii)



iv)

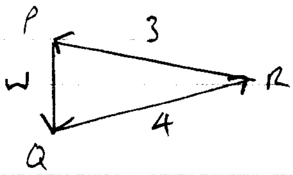


Δ angle of forces at C has sides 3, 4 and W

$$\therefore W < 3 + 4$$

$$W < 7 \text{ N}$$

v)



As W decreases $\angle QPR$ increases but $\angle QPR$ must be less than 90°

$$\therefore 4^2 < W^2 + 3^2$$
$$16 - 9 < W^2$$
$$7 < W^2$$

$$W > \sqrt{7} \text{ N}$$