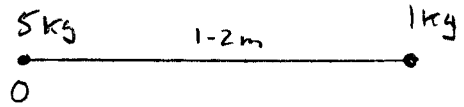
MEI MECHANICS 2 CENTRE OF MASS EXERCISE 4A

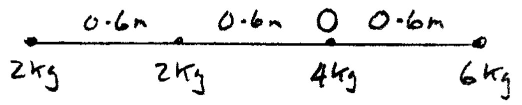
1) i)



$6\bar{x} = 5 \times 0 + 1 \times 1.2$

$\bar{x} = \frac{1.2}{6} = 0.2\text{m}$

ii)

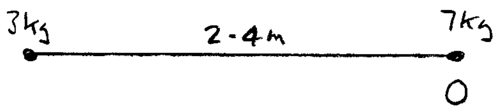


$14\bar{x} = 2(-1.2) + 2(-0.6) + 6(0.6)$

$14\bar{x} = -2.4 - 1.2 + 3.6$

$14\bar{x} = 0 \Rightarrow \bar{x} = 0$

iii)

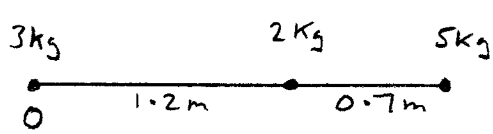


$10\bar{x} = 3(-2.4)$

$10\bar{x} = -7.2$

$\bar{x} = -0.72\text{ m}$

iv)

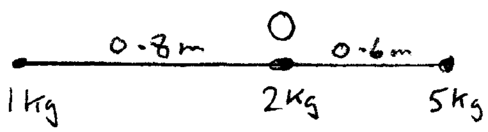


$10\bar{x} = 2 \times 1.2 + 5 \times 1.9$

$10\bar{x} = 2.4 + 9.5$

$\bar{x} = 1.19\text{ m}$

v)

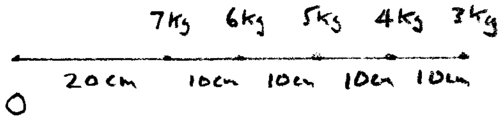


$8\bar{x} = 1(-0.8) + 5(0.6)$

$8\bar{x} = -0.8 + 3.0$

$\bar{x} = \frac{2.2}{8} = 0.275\text{ m}$

vi)



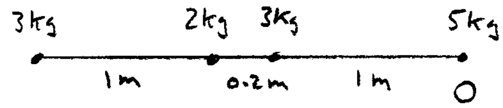
$25\bar{x} = 7(0.2) + 6(0.3) + 5(0.4)$

$+ 4(0.5) + 3(0.6)$

$25\bar{x} = 9$

$\bar{x} = 0.36\text{ m}$

vii)

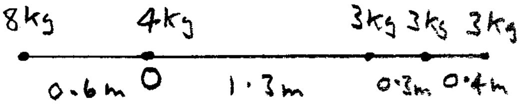


$13\bar{x} = 3(-2.2) + 2(-1.2) + 3(-1)$

$13\bar{x} = -12$

$\bar{x} = -\frac{12}{13} = -0.923\text{ m}$

viii)



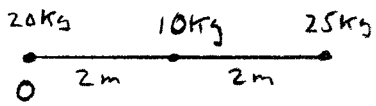
$21\bar{x} = 8(-0.6) + 3(1.3)$

$+ 3(1.6) + 3(2.0)$

$21\bar{x} = 9.9$

$\bar{x} = \frac{9.9}{21} = 0.471\text{ m}$

2)



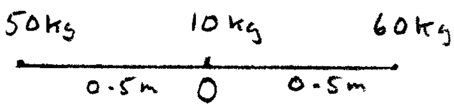
$$55\bar{x} = 10(2) + 25(4)$$

$$55\bar{x} = 120$$

$$\bar{x} = \frac{120}{55} = 2.18\text{ m}$$

$\bar{x} = 2.18$ m from end with 20kg child

3)



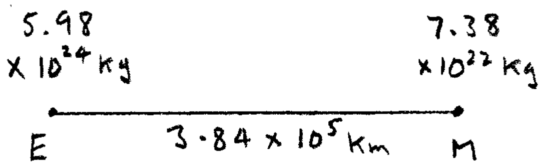
$$120\bar{x} = 50(-0.5) + 60(0.5)$$

$$120\bar{x} = 5$$

$$\bar{x} = 0.042\text{ m}$$

from centre towards 60kg mass

4)



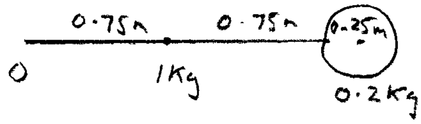
$$(5.98\times10^{24} + 7.38\times10^{22})\bar{x}$$

$$= 0 + 7.38\times10^{22} \times 3.84\times10^{5}$$

$$\bar{x} = \frac{7.38\times10^{22}\times3.84\times10^{5}}{(5.98\times10^{24}+7.38\times10^{22})}$$

$\bar{x} = 4681$ km from Earth centre

5)



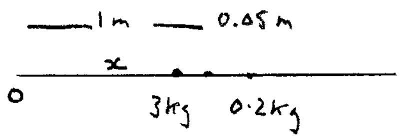
$$1.2\bar{x} = 1\times0.75 + 0.2\times1.75$$

$$1.2\bar{x} = 1.1$$

$$\bar{x} = \frac{1.1}{1.2} = 0.917\text{ m}$$

from free end of stick

6)



$$\bar{x} = 1\text{ m}$$

$$3.2\bar{x} = 3\times x + 0.2\times1.05$$

$$3.2\times1 = 3x + 0.21$$

$$2.99 = 3x$$

$$x = 0.997\text{ m}$$

from end of rod
or 0.3cm from centre of rod.

7)

Mass of disc $\propto r^2$

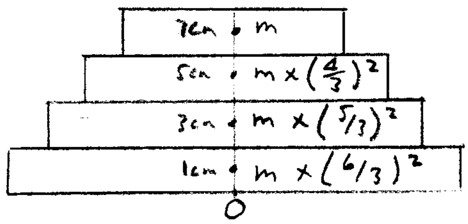
Let 3cm disc have mass m

Then 4cm disc has mass $m\times\left(\frac{4}{3}\right)^2$

5cm disc has mass $m\times\left(\frac{5}{3}\right)^2$

6cm disc has mass $m\times\left(\frac{6}{3}\right)^2$

7) cont



$$m\left(1+\left(\tfrac{4}{3}\right)^2+\left(\tfrac{5}{3}\right)^2+\left(\tfrac{6}{3}\right)^2\right)\times\bar{x}$$

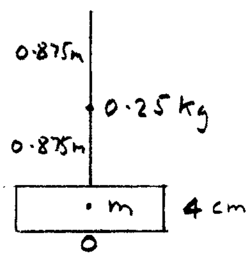
$$= 7m + 5m\times\left(\tfrac{4}{3}\right)^2 + 3m\times\left(\tfrac{5}{3}\right)^2 + 4m$$

$$\Rightarrow \bar{x} = \frac{\left(7+5\left(\frac{4}{3}\right)^2+3\left(\frac{5}{3}\right)^2+4\right)}{\left(1+\left(\frac{4}{3}\right)^2+\left(\frac{5}{3}\right)^2+4\right)}$$

$$\bar{x} = 2.95 \text{ cm}$$

Height of centre of mass = 2.95cm

8) Suppose $\bar{x}$ is 12cm above ground



$$(m+0.25)\bar{x} = m\times 0.02 + 0.25\times 0.915$$

$$0.12m + 0.25\times 0.12 = 0.02m + 0.22875$$

$$0.12m - 0.02m = 0.22875 - 0.25\times 0.12$$

$$0.1m = 0.19875$$

$$m = 1.9875 \text{ kg}$$

$$m \geq 1.99 \text{ kg} \quad (\text{to 3 s.f.})$$

for $\bar{x} \leq 12$cm

9) Let mass of pole be m

$$(m+8)\bar{x} = 1(0.5+1+1.5+2+3+4+5)+2.5m$$

$$2.44m + 8\times 2.44 = 17 + 2.5m$$

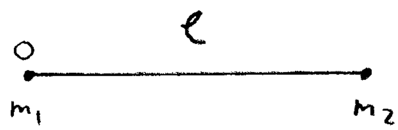
$$2.44m - 2.5m = 17 - 8\times 2.44$$

$$-0.06m = -2.52$$

$$m = \frac{-2.52}{-0.06}$$

$$m = 42 \text{ kg}$$

10)



$$(m_1+m_2)\bar{x} = m_2 l$$

$$\bar{x} = \frac{m_2 l}{m_1+m_2}$$

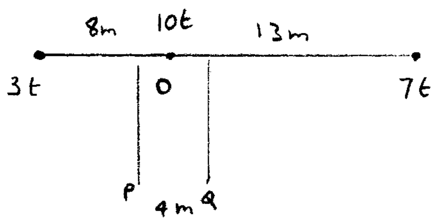
from end with mass m_1

Alternatively,

$$\bar{x} = \frac{m_1 l}{m_1+m_2}$$

from end with mass m_1

11)



11 i) cont)

$$20\bar{x} = 3(-8) + 7(13)$$

$$20\bar{x} = 67$$

$$\bar{x} = 3.35\text{ m}$$

Since supports are only 2 m either side of O, crane will fall over, pivoting about Q

11 ii)

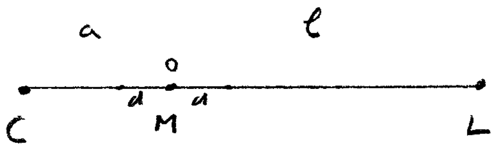
Without a load

$$13\bar{x} = 3(-8) = -24$$

$$\bar{x} = -\frac{24}{13} = -1.85\text{ m}$$

Since P and Q are 2 m either side of O, $\bar{x}$ lies between the supports so crane will not fall over.

11 iii)



With load

$$(C+M+L)\bar{x} = Ll - Ca$$

$$\bar{x} = \frac{Ll - Ca}{C+M+L}$$

but $\bar{x} < d$ if stable

$$\therefore \frac{Ll - Ca}{C+M+L} < d$$

Without load

$$(C+M)\bar{x} = -Ca$$

$$\bar{x} = \frac{-Ca}{C+M}$$

but $\bar{x} > -d$ if stable

$$\therefore \frac{-Ca}{C+M} > -d$$

$$\Rightarrow \frac{Ca}{C+M} < d \qquad (*)$$

11 ii cont)

With safe max load

$$(13+L)\bar{x} = 3(-8) + L(13)$$

but $\bar{x} < 2$ for safety

$$\Rightarrow (13+L)2 > 13L - 24$$

$$26 + 2L > 13L - 24$$

$$50 > 11L$$

$$L < 4.55 \text{ tonnes}$$

11 iv)

Find max load in terms of M, a, d, l

$\therefore$ must eliminate C

From $(*)$ above

$$Ca < d(C+M)$$

$$Ca < Cd + Md$$

11 iv) cont

$$Ca - Cd < Md$$

$$C(a-d) < Md$$

$$C < \frac{Md}{a-d}$$

Now

$$(M+C+L)\bar{x} = Ll - Ca$$

For safety $\bar{x} < d$

$$\Rightarrow (M+C+L)d > Ll - Ca$$

$$Md + Cd + Ld > Ll - Ca$$

$$Md + Cd + Ca > Ll - Ld$$

$$Md + C(d+a) > L(l-d)$$

$$\frac{Md + C(d+a)}{(l-d)} > L$$

Substituting for C

$$\frac{Md + \frac{Md}{(a-d)}(d+a)}{(l-d)} > L$$

$$\frac{Md(a-d) + Md(d+a)}{(a-d)(l-d)} > L$$

$$\frac{Mda - Md^2 + Md^2 + Mda}{(a-d)(l-d)}$$

$$\Rightarrow L < \frac{2Mda}{(a-d)(l-d)}$$
