MEI MECHANICS 2 CONSERVATION OF MOMENTUM EXERCISE 6B

1)

$$m_1 u_1 + m_2 u_2 = (m_1 + m_2)V$$

$$50000 \times 200 + 500000 \times 195$$

$$= (550000)V$$

$$V = \frac{50000 \times 200 + 500000 \times 195}{550000}$$

$V = 195.45\ ms^{-1}$
in same direction

2)
i)

$$m_1 u_1 + m_2 u_2 = (m_1 + m_2)V$$

$$20 \times 10^3 \times 3 + 0 = 30 \times 10^3 V$$

$$V = \frac{60 \times 10^3}{30 \times 10^3} = 2\ ms^{-1}$$

ii)

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$20 \times 10^3 \times 3 + 0 = 20 \times 10^3 V_1 + 10 \times 10^3 \times 3$$

$$60 \times 10^3 - 30 \times 10^3 = 20 \times 10^3 V_1$$

$$\frac{30 \times 10^3}{20 \times 10^3} = V_1$$

$$V_1 = 1.5\ ms^{-1}$$

3)
i)

$$m_1 u_1 + m_2 u_2 = (m_1 + m_2)V$$

$$1000 \times 30 + 800 \times 20 = 1800V$$

$$V = \frac{30000 + 16000}{1800}$$

$$V = 25.56\ ms^{-1}$$

ii) Impulse = change in momentum

For small car change in momentum

$$= m_2 V - m_2 u_2$$

$$= m_2 (V - u_2)$$

$$= 800(25.56 - 20)$$

$$= 4448\ Ns$$

in the direction cars are travelling.

iii) -4448 Ns

ie in opposite direction.

4)
i)

$$m_1 u_1 + m_2 u_2 = (m_1 + m_2)V$$

$$5000 \times 2 + 0 = 6000V$$

$$V = \frac{10000}{6000} = 1\tfrac{2}{3}\ ms^{-1}$$

ii)

Impulse = change in momentum

For car $= m_2 V$

$$= 1000 \times 1\tfrac{2}{3}$$

$= 1667$ Ns forwards

For lorry = 1667 Ns backwards

5)
i)

$$m_1 u_1 + m_2 u_2 = (m_1 + m_2)V$$

$$50 \times 10^{-3} \times 200 + 0 = 16.05V$$

$$V = \frac{0.05 \times 200}{16.05} = 0.623\ ms^{-1}$$

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5ii) Impulse on block = change in mom.

$= m_2 v = 16 \times 0.623$

$= 9.97$ Ns

5iii) Average force = rate of change of mom.

$F = \dfrac{m_1 v - m_1 u}{t}$

$= \dfrac{0.05(0.623 - 200)}{0.01}$

$= -997$ N

(ie. in opposite direction to travel)

6) i) $5000 - 10 = 4990$ ms^{-1}

ii) $(m_1 + m_2)u = m_1 v_1 + m_2 v_2$

$50\,000 \times 5000 =$

$49995 \times v_1 + 5 \times 4990$

$v_1 = \dfrac{50000 \times 5000 - 5 \times 4990}{49995}$

$v_1 = 5000.001$ ms^{-1}

7) i) $m_1 v_1 + m_2 v_2 = 0$

$500 v_1 + 5 \times 300 = 0$

$v_1 = -\dfrac{1500}{500} = -3$ ms^{-1}

Recoils at 3 ms^{-1}

ii) Impulse of 1500 Ns too great

8) i) 0 kg ms^{-1}

ii) Impulse of Manoj on Alka

$= 35 \times 1.5$ Ns $= 52.5$ Ns

∴ Alka's change in momentum

$= 52.5$ kg ms^{-1}

$mv = 52.5$

$v = \dfrac{52.5}{50} = 1.05$ ms^{-1}

(in +ve direction, ie backwards for Alka)

Change in momentum for Manoj

$= -52.5$ kg ms^{-1}

$mv = -52.5$

$v = \dfrac{-52.5}{70} = -0.75$ ms^{-1}

(in -ve direction, ie backwards for Manoj)

iii) Alka $mv = 50 \times 1.05$

$= 52.5$ kg ms^{-1}

Manoj $mv = 70 \times -0.75$

$= -52$ kg ms^{-1}

iv) $52.5 + (-52.5)$

$= 0$ kg ms^{-1}

9) i) $5 - 4 = 1\ ms^{-1}$

ii) $(m_1 + m_2 + m_3)u = m_1 v_1 + (m_2 + m_3) v_2$

$80 \times 5 = 40 \times 1 + 40 v_2$

$\frac{400 - 40}{40} = v_2$

$v_2 = 9\ ms^{-1}$

Elisabeth and sledge have velocity $9\ ms^{-1}$

iii) Elisabeth has velocity $9 - 4 = 5\ ms^{-1}$

$(m_2 + m_3)u = m_2 v_2 + m_3 v_3$

$40 \times 9 = 30 \times 5 + 10 v_3$

$\frac{360 - 150}{10} = v_3$

$v_3 = 21\ ms^{-1}$

Sledge has velocity $21\ ms^{-1}$

iv) $(m_1 + m_2 + m_3)u = (m_1 + m_2)v_1 + m_3 v_3$

$80 \times 5 = 70 \times 1 + 10 v_3$

$\frac{400 - 70}{10} = v_3$

$v_3 = 33\ ms^{-1}$

Sledge would have velocity $33\ ms^{-1}$

10) i) loss in gpe

$= mg(AB \sin\alpha)$

$= 2000 \times 9.8 \times 50 \times 0.05$

$= 49000\ J$

ii) At B $\frac{1}{2}mv^2 = 49000$

$v^2 = \frac{2 \times 49000}{2000}$

$v = 7.0\ ms^{-1}$

iii) $m_1 u_1 + m_2 u_2 = (m_1 + m_2) v$

$2000 \times 7 + 0 = 3500 v$

$v = \frac{14000}{3500} = 4\ ms^{-1}$

iv) K.E. before collision $= 49000\ J$

K.E after $= \frac{1}{2}(m_1 + m_2)v^2$

$= \frac{1}{2} \times 3500 \times 4^2 = 28000\ J$

Loss in KE $= 49000 - 28000$

$= 21000\ J$

% loss $= \frac{21000}{49000} \times 100\ \%$

$= 42.9\%$

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11) i)

$v^2 = u^2 + 2as$

$v^2 = 0 + 2 \times 10 \times 5$

$\Rightarrow \quad v = 10 \text{ ms}^{-1}$

ii)

Impulse = Change in momentum

$J = m(v - u)$

$J = 2000(-2 - 10)$

$J = -24000 \text{ Ns}$

ie 24000 Ns upwards

iii)

24000 Ns downwards

iv)

$J = Ft$

$F = \frac{J}{t} = \frac{24000}{0.025}$

$F = 960000 \text{ N}$

v)

Momentum $mv = 24000 \text{ Kg m s}^{-1}$

$v = \frac{24000}{600} = 40 \text{ ms}^{-1}$

When it comes to rest $v = 0$

Average speed $= \frac{v+u}{2} = \frac{40+0}{2}$

$= 20 \text{ ms}^{-1}$

Distance travelled $= 20 \times t$

$= 20 \times 0.025 = 0.5 \text{ m}$

12) i)

Initial velocity of spacecraft $\begin{pmatrix} 9500 \\ 0 \end{pmatrix}$ ($\underline{j}$ up, $\underline{i}$ right)

Initial velocity of debris $\begin{pmatrix} 0 \\ 9500 \end{pmatrix}$

$m_1 u_1 + m_2 u_2 = (m_1 + m_2) v$

$1000\begin{pmatrix} 9500 \\ 0 \end{pmatrix} + 0.5\begin{pmatrix} 0 \\ 9500 \end{pmatrix} = 1000.5\begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$

$\begin{pmatrix} 9500000 \\ 4750 \end{pmatrix} = 1000.5\begin{pmatrix} v_1 \\ v_2 \end{pmatrix}$

$\underline{v} = \begin{pmatrix} 9495 \\ 4.748 \end{pmatrix}$

$|\underline{v}| = 9495 \text{ ms}^{-1}$

ii)

KE of debris before collision

$= \frac{1}{2}mu^2 = \frac{1}{2} \times 0.5 \times 9500^2$

$= 22{,}562{,}500 \text{ J}$

KE of debris after collision

$= \frac{1}{2}mv^2 = \frac{1}{2} \times 0.5 \times (9495.252374^2 + 4.748^2)$

$= 22{,}539{,}960 \text{ J}$

Lost KE $= 22{,}540 \text{ J}$

12 iii) Debris change in momentum

$= m(\mathbf{v}-\mathbf{u})$

$= 0.5\begin{pmatrix} 9495 - 0 \\ 4.748 - 9500 \end{pmatrix}$

$= \begin{pmatrix} 4745.5 \\ -9495.3 \end{pmatrix}$

$\underline{F}t = \begin{pmatrix} 4745.5 \\ -9495.3 \end{pmatrix}$

$t = \dfrac{0.12}{9500}$

$\Rightarrow \underline{F} = \dfrac{9500}{0.12}\begin{pmatrix} 4745.5 \\ -9495.3 \end{pmatrix}$

$\underline{F} = \begin{pmatrix} 375685417 \\ 751711250 \end{pmatrix}$

$|\underline{F}| = 840{,}362{,}622$

$= 8.4 \times 10^8 \text{ N}$

13)
i) $m_1u_1 + m_2u_2 = (m_1+m_2)v$

$0.1 \times 10 + 0 = 5.1v$

$v = \dfrac{1}{5.1} = 0.196 \text{ ms}^{-1}$

ii) $m_1u_1 + m_2U_2 = (m_1+m_2)V$

$0.1 \times 10 + 1 = 5.2v$

$v = \dfrac{2}{5.2} = 0.385 \text{ ms}^{-1}$

13 iii) Each snowball adds momentum of 1 kgms^{-1} to system

Total momentum $= n \times 1 = n \text{ kgms}^{-1}$

Total mass $= 5 + 0.1n$

$\therefore V = \dfrac{n}{5+0.1n} \text{ ms}^{-1}$

13 iv)

$10 - \dfrac{50}{5+0.1n}$

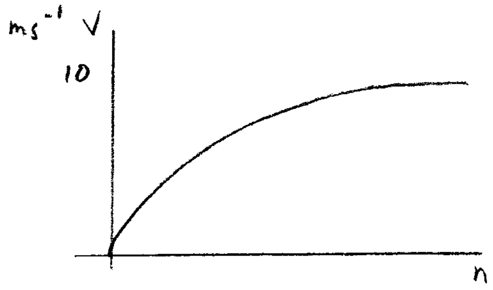
$= \dfrac{50+n-50}{5+0.1n}$

$= \dfrac{n}{5+0.1n}$

$\therefore$ we can also write

$V = 10 - \dfrac{50}{5+0.1n}$

n	0	5	10	20	50	100	1000
v	0	0·91	1.7	2·9	5	6·7	9.95



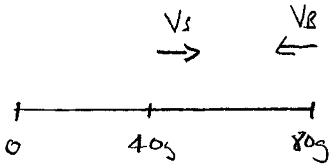
As n increases, v increases

$V \to 10 \text{ ms}^{-1}$ as $n \to \infty$

v) faster since rubber ball would have –ve momentum causing sled to have more +ve momentum

14)

i)



Let Ben have velocity V_B
Let Sleigh have velocity V_S

$$m_B V_B + m_S V_S = 0$$

$$80 V_B + 40 V_S = 0$$

$$\Rightarrow \quad V_S = -\frac{80}{40} V_B = -2V_B$$

Also $V_B - V_S = 1 \text{ ms}^{-1}$

$$\therefore \quad V_B = 1 + V_S$$

Subst

$$V_S = -2(1 + V_S)$$

$$3V_S = -2$$

$$V_S = -\frac{2}{3} \text{ ms}^{-1}$$

ie in opposite direction to Ben

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ii) Find initial centre of mass in relation to fixed point O at front of sleigh before it moves

$$120\bar{x} = 40 \times 2.5 + 80 \times 5$$

$$= 100 + 400$$

$$\bar{x} = \frac{500}{120} = 4.1\dot{6} \text{ m}$$

At time t

Position of Ben in relation to O
$= 5 - V_B t$

Since $V_B = 1 + V_S$
$= 1 - \frac{2}{3} = \frac{1}{3} \text{ ms}^{-1}$

Position of Ben
$= 5 - \frac{1}{3}t$

Position of sleigh centre of mass

$$= 2.5 + \frac{2}{3}t$$

Overall centre of mass given by

$$120\bar{x} = \left(5 - \tfrac{1}{3}t\right)80 + \left(2.5 + \tfrac{2}{3}t\right)40$$

$$= 400 - \frac{80t}{3} + 100 + \frac{80t}{3}$$

$$= 500$$

$$\Rightarrow \bar{x} = 4.1\dot{6} \text{ as before}$$

$\therefore$ $\bar{x}$ does not move during Ben's walk along sleigh

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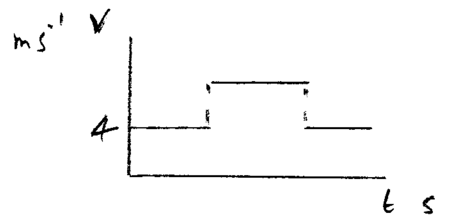
iii)

15 i) Sledge continues at $4\ ms^{-1}$

since $(m_1+m_2+m_3+m_4)\times 4$

$= m_1\times 4 + m_2\times 4 + (m_3+m_4)\times 4$

ii)



iii) Total mass $= 55+25+5+5 = 90\ kg$

$F = ma$

$-5 = 90a$

$a = -\frac{1}{18}\ ms^{-2}$

Using $v = u + at$

At rest when $v = 0$

$0 = 4 - \frac{1}{18}t$

$\Rightarrow \frac{1}{18}t = 4$

$t = 72\ s$

iv) $(m_1+m_2+m_3+m_4)u$

$= m_1v_1 + m_2v_2 + (m_3+m_4)V$

$0 = -2\times 5 - 2\times 5 + 80V$

$20 = 80V$

$V = 0.25\ ms^{-1}$

v) $(m_1+m_2+m_3+m_4)u$

$= m_1V_1 + (m_2+m_3+m_4)V$

$0 = -2\times 5 + 85V$

$\frac{10}{85} = V$

Throw 2nd bag

$(m_2+m_3+m_4)u = m_2V_2 + (m_3+m_4)V$

$85\times\frac{10}{85} = -2\times 5 + 80V$

$10 + 10 = 80V$

$V = 0.25\ ms^{-1}$

vi) $(m_1+m_2+m_3+m_4)u$

$= (m_1+m_2)V_1 + (m_3+m_4)V$

$0 = 10\times(V-2) + (80)V$

$0 = 10V - 20 + 80V$

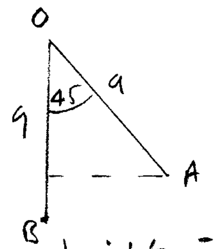
$20 = 90V$

$V = \frac{20}{90} = \frac{2}{9}\ ms^{-1}$

$V = 0.222\ ms^{-1}$

16)

i)



Loss in height $= 9 - 9\cos 45$

$= 2.636\ m$

Loss in gpe $= mgh$

$= 50 \times 9.8 \times 2.636$

$= 1292\ J$

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ii)

No work done because tension is always perpendicular to direction of motion.

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iii)

Gain in ke = loss in gpe

$\frac{1}{2}mv^2 = 1292$

$v^2 = \frac{1292}{25}$

$v = 7.19\ ms^{-1}$

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iv)

Gain in KE + Work done by resistance

= loss in gpe

(Force × arc length)

Gain in KE $= 1292 - 20 \times \frac{1}{8} \times 2\pi \times 9$

$= 1292 - 141$

$= 1151\ J$

Original KE $= \frac{1}{2} \times 50 \times 1^2 = 25\ J$

Final K.E $= 1176\ J$

$\frac{1}{2}mv^2 = 1176$

$v^2 = \frac{1176}{25}$

$\Rightarrow v = 6.85857$

$v \approx 6.86\ ms^{-1}$

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v)

$m_1 u_1 + m_2 u_2 = (m_1 + m_2)V$

$50 \times 6.86 + 70(-4) = 120V$

$63 = 120V$

$V = \frac{63}{120} = 0.525\ ms^{-1}$

$V = 0.525\ ms^{-1}$ in horizontal direction woman was travelling.