MEI STATISTICS 1 EXPECTATION + VARIANCE EXERCISE 4C

1) i)

r	0	1	2	3
$P(X=r)$	0.25	0.35	0.3	0.1

$k = 0.1$

ii) $E(X) = 0 \times 0.25 + 1 \times 0.35 + 2 \times 0.3 + 3 \times 0.1$

$= 1.25$

iii) $E(X^2) = 0.25 \times 0 + 0.35 \times 1^2 + 0.3 \times 2^2 + 0.1 \times 3^2$

$= 2.45$

$\mathrm{Var}(X) = E(X^2) - (E(X))^2$

$= 2.45 - 1.25^2$

$= 0.8875$

$\mathrm{s.d}(X) = \sqrt{0.8875} = 0.9421$

2) X takes values 0, 1, 2, 3, 4, 6

1st	2nd	prob	
0	0		$\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$
0	1		$\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$
0	3		$\frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$
1	0		$\frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$
1	1		$\frac{1}{3} \times \frac{1}{3} = \frac{1}{9}$
1	3		$\frac{1}{3} \times \frac{1}{6} = \frac{1}{18}$
3	0		$\frac{1}{6} \times \frac{1}{2} = \frac{1}{12}$
3	1		$\frac{1}{6} \times \frac{1}{3} = \frac{1}{18}$
3	3		$\frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$

r	0	1	2	3	4	6
$P(X=r)$	$\frac{9}{36}$	$\frac{12}{36}$	$\frac{4}{36}$	$\frac{6}{36}$	$\frac{4}{36}$	$\frac{1}{36}$

i) $P(X=4) = P(3,1) + P(1,3)$

$= \frac{1}{6} \times \frac{1}{3} + \frac{1}{3} \times \frac{1}{6}$

$= \frac{1}{18} + \frac{1}{18} = \frac{1}{9}$

ii) See previous column

iii) $E(X) = \frac{1}{36}\left[9 \times 0 + 12 \times 1 + 4 \times 2 + 6 \times 3 + 4 \times 4 + 1 \times 6\right]$

$= \frac{60}{36} = \frac{5}{3}$

$E(X^2) = \frac{1}{36}\left[9 \times 0 + 12 \times 1^2 + 4 \times 2^2 + 6 \times 3^2 + 4 \times 4^2 + 1 \times 6^2\right]$

$= \frac{182}{36} = \frac{91}{18}$

$\mathrm{Var}(X) = E(X^2) - (E(X))^2$

$= \frac{91}{18} - \left(\frac{5}{3}\right)^2$

$= \frac{41}{18}$

3) i)

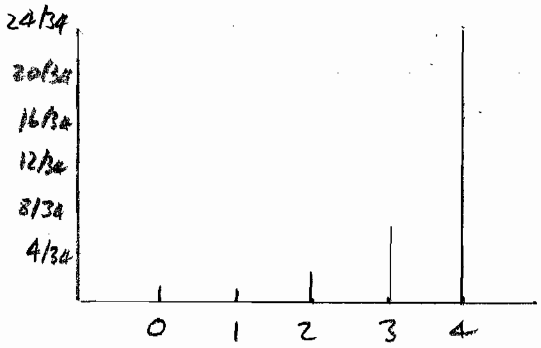
r	0	1	2	3	4
$P(X=r)$	k	k	$2k$	$6k$	$24k$

$\sum P(X=r) = 1 \Rightarrow 34k = 1$

$\Rightarrow k = \frac{1}{34}$

3 i)

r	0	1	2	3	4
$P(X=r)$	$\frac{1}{34}$	$\frac{1}{34}$	$\frac{2}{34}$	$\frac{6}{34}$	$\frac{24}{34}$



3 ii)

$$E(X) = \frac{1}{34}\left[1\times 0 + 1\times 1 + 2\times 2 + 6\times 3 + 24\times 4\right]$$

$$= \frac{119}{34} = 3.5$$

$$E(X^2) = \frac{1}{34}\left[1\times 0 + 1\times 1^2 + 2\times 2^2 + 6\times 3^2 + 24\times 4^2\right]$$

$$= 13.147$$

$$Var(X) = E(X^2) - (E(X))^2$$

$$= 13.147 - 3.5^2$$

$$= 0.897$$

iii)

$$P(X_1 = X_2) = \left(\frac{1}{34}\right)^2 + \left(\frac{1}{34}\right)^2 + \left(\frac{2}{34}\right)^2 + \left(\frac{6}{34}\right)^2 + \left(\frac{24}{34}\right)^2$$

$$= \frac{618}{1156} \approx 0.5346$$

just > 0.5

iv) Given $X_1 = X_2$

Find $P(X_1 = X_2 = 4)$

$$= \frac{\left(\frac{24}{34}\right)^2}{0.5346020761}$$

$$= 0.932$$

4)

i)

20 days @ £80
2 days @ £120
1 day @ £160
5 days @ £0

r	0	80	120	160
$P(X=r)$	$\frac{5}{28}$	$\frac{20}{28}$	$\frac{2}{28}$	$\frac{1}{28}$

ii)

$$E(X) = \frac{1}{28}\left[5\times 0 + 20\times 80 + 2\times 120 + 1\times 160\right]$$

$$= \frac{2000}{28} = £71.43$$

$$E(X^2) = \frac{1}{28}\left[5\times 0 + 20\times 80^2 + 2\times 120^2 + 1\times 160^2\right]$$

$$= \frac{182400}{28} = 6514$$

$$Var(X) = E(X^2) - (E(X))^2$$

$$= 6514 - 5102$$

$$= 1412$$

4 iii)

Option 1	WK1	WK2	WK3	WK4
		SAT		SAT
				SUN

Option 2 WK1 WK2 WK3 WK4
SAT SAT
SUN

Either the Sunday working coincides with the Saturday working or it does not.

Option 1 Weekly Wage

r	400	520	680
$P(X=r)$	$\frac{2}{4}$	$\frac{1}{4}$	$\frac{1}{4}$

$E(X) = \frac{1}{2}\times 400 + \frac{1}{4}\times 520 + \frac{1}{4}\times 680$

$E(X) = £500$

$E(X^2) = \frac{1}{2}\times 400^2 + \frac{1}{4}\times 520^2 + \frac{1}{4}\times 680^2$

$= 263200$

$Var(X) = E(X^2) - (E(X))^2$

$= 263200 - 250000$

$= 13200$

Option 2 Weekly Wage

r	400	520	560
$P(X=r)$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{1}{4}$

$E(X) = \frac{1}{4}\times 400 + \frac{1}{2}\times 520 + \frac{1}{4}\times 560$

$= £500$

$E(X^2) = \frac{1}{4}\times 400^2 + \frac{1}{2}\times 520^2 + \frac{1}{4}\times 560^2$

$= 253,600$

$Var(X) = E(X^2) - (E(X))^2$

$= 253,600 - 250,000$

$= 3,600$

5)

i)

r	0	1	2	3	4	5	6	7	8	9+
$P(X=r)$	·03	·10	·13	·16	·18	·17	·12	·09	·02	0

ii) $E(X) = 0.3\times 0 + 0.10\times 1 + 0.13\times 2 + 0.16\times 3 + 0.18\times 4 + 0.17\times 5 + 0.12\times 6 + 0.09\times 7 + 0.02\times 8$

$E(X) = 4.12$

$E(X^2) = 0.3\times 0 + 0.10\times 1^2 + 0.13\times 2^2 + 0.16\times 3^2 + 0.18\times 4^2 + 0.17\times 5^2 + 0.12\times 6^2 + 0.09\times 7^2 + 0.02\times 8^2$

$E(X^2) = 19.2$

$Var(X) = E(X^2) - (E(X))^2$

$= 19.2 - 4.12^2$

$= 2.2256$

ii) $P(X=r) = kr(8-r)$

r	0	1	2	3	4	5	6	7	8
$P(X=r)$	0	7k	12k	15k	16k	15k	12k	7k	0

iii) $\Rightarrow 84k = 1 \Rightarrow k = \frac{1}{84}$

5 iv) cont)

$$E(X) = \frac{1}{84}\left[7\times1 + 12\times2 + 15\times3 + 16\times4 + 15\times5 + 12\times6 + 7\times7\right]$$

$$= \frac{336}{84} = 4$$

$$E(X^2) = \frac{1}{84}\left[7\times1^2 + 12\times2^2 + 15\times3^2 + 16\times4^2 + 15\times5^2 + 12\times6^2 + 7\times7^2\right]$$

$$= \frac{1596}{84} = 19$$

$$Var(X) = E(X^2) - (E(X))^2$$

$$= 19 - 4^2 = 3$$

5 v)

Model reasonable: mean accurate, though variance a little high in model. Both model and actual distributions fairly symmetrical

6)

i) $P(X=r) = k(r+1)(5-r)^2$

r	0	1	2	3
P(X=r)	25k	32k	27k	16k

$\Rightarrow 100k = 1 \Rightarrow k = 0.01$

ii) $E(X) = 0.01\left[32\times1 + 27\times2 + 16\times3\right]$

$E(X) = 1.34$

$$E(X^2) = 0.01\left[32\times1 + 27\times2^2 + 16\times3^2\right]$$

$$E(X^2) = 2.84$$

$$Var(X) = E(X^2) - (E(X))^2$$

$$= 2.84 - 1.34^2$$

$$= 1.0444$$

iii)

a) P(Fail to score in first 2 games) =

$0.25\times0.25 = \frac{1}{16}$

or 0.0625

b) $P(3,1) + P(2,2) + P(1,3)$

$= .16\times.32 + .27\times.27 + .32\times.16$

$= 0.1753$

Assume probabilities constant and not affected by form or opposition.

iv)

i) Players could have changed

ii) Opposition could cause probs to vary from game to game.

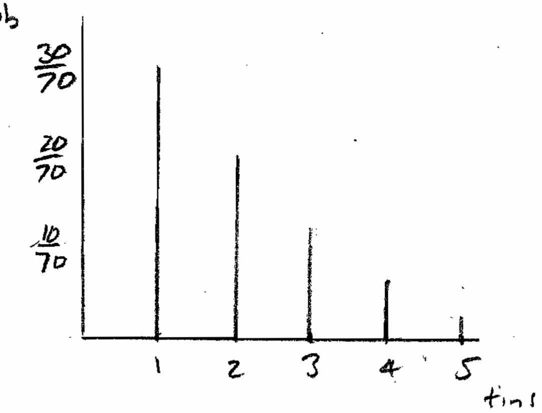
7) $P(X=r) = k(6-r)(7-r)$

r	1	2	3	4	5
P(X=r)	30k	20k	12k	6k	2k

$\Rightarrow 70k = 1 \Rightarrow k = \frac{1}{70}$

r	1	2	3	4	5
P(X=r)	$\frac{30}{70}$	$\frac{20}{70}$	$\frac{12}{70}$	$\frac{6}{70}$	$\frac{2}{70}$

7ii)



Modal value = 1
Positively skewed

tin $= \frac{4}{7} \times \frac{3}{6} \times \frac{3}{5}$
rh rh tuna

$= \frac{36}{210} = \frac{12}{70}$

as in given distribution

iii)

$E(X) = \frac{1}{70}\left[30\times1 + 20\times2 + 12\times3 + 6\times4 + 2\times5\right]$

$\mu = \frac{140}{70} = 2$

$E(X^2) = \frac{1}{70}\left[30\times1 + 20\times2^2 + 12\times3^2 + 6\times4^2 + 2\times5^2\right]$

$= \frac{364}{70} = 5.2$

$Var(X) = E(X^2) - (E(X))^2$

$= 5.2 - 2^2$

$= 1.2$

s.d $= \sqrt{1.2} = 1.0954$

iv)

Expected cans 1 rhubarb
1 tuna

$= 50p + £1.20 = £1.70$

v)

4 rhubarb 3 tuna

Prob first tuna opened is third

8)

i) $P(X=1) = p(\text{1st right}) \times p(\text{2nd wrong})$

$= 0.7 \times 0.3 = 0.21$

$P(X=0) = 0.3$

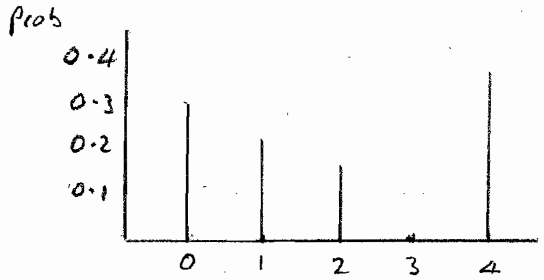
$P(X=2) = 0.7 \times 0.7 \times 0.3$

$= 0.147$

$P(X=4) = 0.7^3 = 0.343$

ii)

r	0	1	2	4
P(X=r)	0.3	0.21	0.147	0.343



iii)

$E(X) = 0.21\times1 + 0.147\times2 + 0.343\times4$

$E(X) = 1.876$

$E(X^2) = 0.21\times1 + 0.147\times2^2 + 0.343\times4^2$

8 iii) cont)

$E(x^2) = 6.286$

$Var(X) = E(x^2) - (E(X))^2$

$= 6.286 - 1.876^2$

$= 2.766624$

$s.d = \sqrt{2.766624}$

$s.d = 1.663$

8 iv)

P(higher 2nd than 1st round)

$= P(0) \times P(>0) + P(1) \times P(>1) + P(2) \times P(>2)$

$= 0.3 \times 0.7 + 0.21 \times 0.49 + 0.147 \times 0.343$

$= 0.363321$

9)

i) $P(X=1) = \frac{1}{13}$

$P(X=2) = \frac{12}{13} \times \frac{1}{12} = \frac{1}{13}$

$P(X=3) = 1 - P(X=1) - P(X=2)$

$= 1 - \frac{1}{13} - \frac{1}{13} = \frac{11}{13}$

ii)

r	1	2	3
P(X=r)	$\frac{1}{13}$	$\frac{1}{13}$	$\frac{11}{13}$

$E(X) = \frac{1}{13}\left[1 \times 1 + 1 \times 2 + 11 \times 3\right]$

$= \frac{36}{13}$

$E(x^2) = \frac{1}{13}\left[1 \times 1 + 1 \times 2^2 + 11 \times 3^2\right]$

$= \frac{104}{13} = 8$

$Var(x) = E(x^2) - E(x)^2$

$= 8 - \left(\frac{36}{13}\right)^2$

$= 0.3314$

iii)

P(3rd guess correct)

$= \frac{12}{13} \times \frac{11}{12} \times \frac{1}{11} = \frac{1}{13}$

r	50	25	15	0
P(Y=r)	$\frac{1}{13}$	$\frac{1}{13}$	$\frac{1}{13}$	$\frac{10}{13}$

$E(X) = \frac{1}{13}\left[50 + 25 + 15\right]$

$E(x) = \frac{90}{13}$ pence

iv)

Takings $10 \times 200 = 2000$ pence

Payout $= 200 \times \frac{90}{13} = 1385$ pence

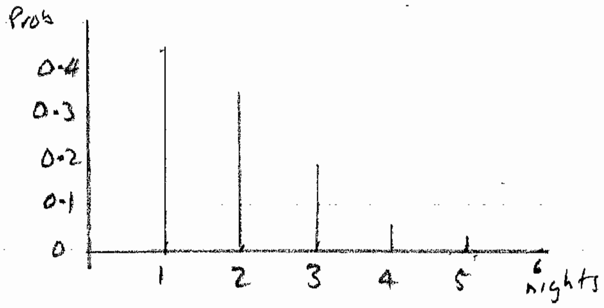
Profit ≈ 615 pence

$= £6.15p$

10)

r	1	2	3	4	5	6+
P(X=r)	·42	·33	·18	·05	·02	0

i



ii)

$E(X) = 1 \times \cdot 42 + 2 \times \cdot 33 + 3 \times \cdot 18 + 4 \times \cdot 05 + 5 \times \cdot 02$

$E(X) = 1.92$ = mean

$E(X^2) = \cdot 42 \times 1^2 + \cdot 33 \times 2^2 + \cdot 18 \times 3^2 + \cdot 05 \times 4^2 + \cdot 02 \times 5^2$

$E(X^2) = 4.66$

$Var(X) = E(X^2) - (E(X))^2$

$= 4.66 - 1.92^2$

$Var(X) = 0.9736$

$s.d = \sqrt{0.9736}$

$s.d = 0.9867$

iii)

$P(X > 2) = 0.18 + 0.05 + 0.02$

$= 0.25$

iv)

Find $P(X \geq 3 \mid X \geq 2)$

$= \dfrac{P(X \geq 3 \cap X \geq 2)}{P(X \geq 2)}$

$= \dfrac{P(X \geq 3)}{P(X \geq 2)}$

$= \dfrac{0.25}{0.58}$

$= 0.431$

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